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Understanding Ferrofluid Spin-Up Flows in Rotating Uniform Magnetic Fields

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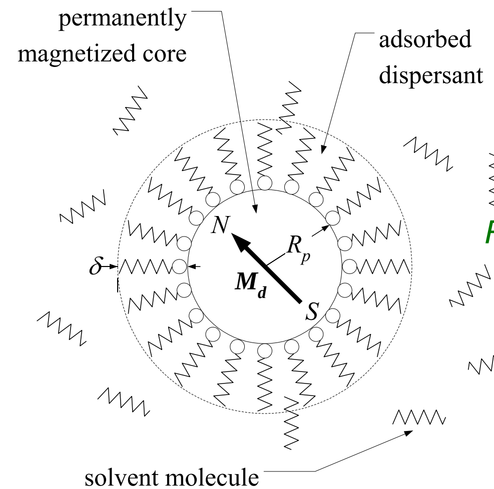
Outline

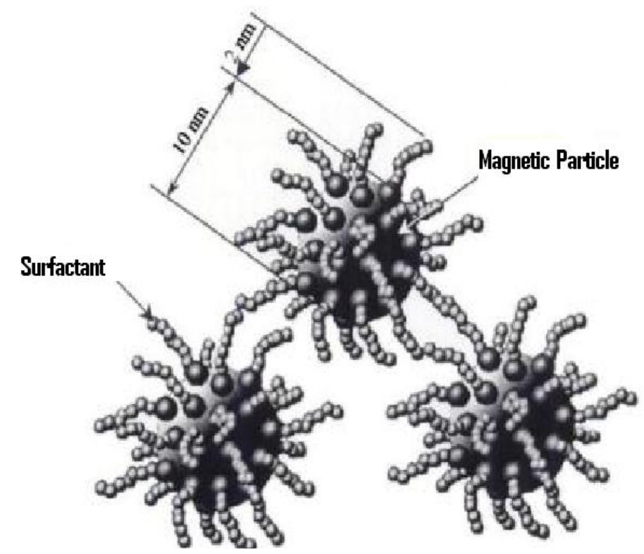
- Background to Spin-up Flows
- Uniform Rotating Fields using a Spherical Coil Assembly
- Spin diffusion Flow Modeling using COMSOL
- Experiments in Uniform Rotating Magnetic Fields (Ferrofluid Filled Sphere)
- Experiments with Non-uniform Magnetic Fields
- Simulations of Flow with Non-uniform Magnetic Fields in Infinitely Long Cylinder and Adjacent Permanent Magnet
- Conclusions

Ferrofluids

- Ferrofluids
  - Nanosized particles in carrier liquid (diameter~10nm)
  - Super-paramagnetic, single domain particles
  - Coated with a surfactant (~2nm) to prevent agglomeration
- Applications
  - Hermetic seals (hard drives)
  - Magnetic hyperthermia for cancer treatment



R_p ~ 5nm



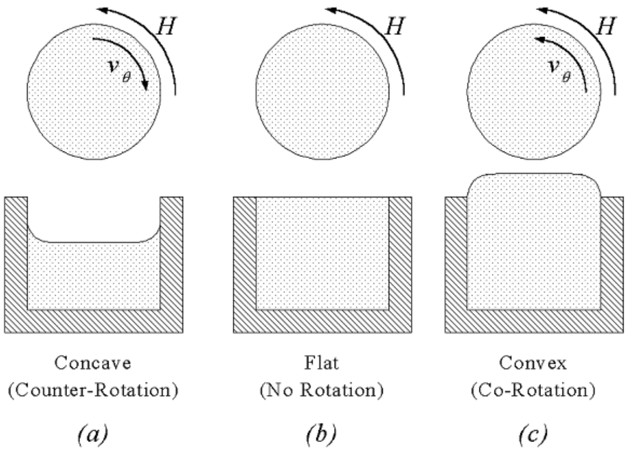
Background to Spin-up Flows

Surface Driven Flows

Spin Diffusion Theory

Ferrofluid Spin-up Experiment

- First reported by Moskowitz and Rosensweig in 1967
- Ferrofluid surface is opaque so observations were made at the free surface only
- Flow reversal on top free surface was deduced to be due to meniscus shape

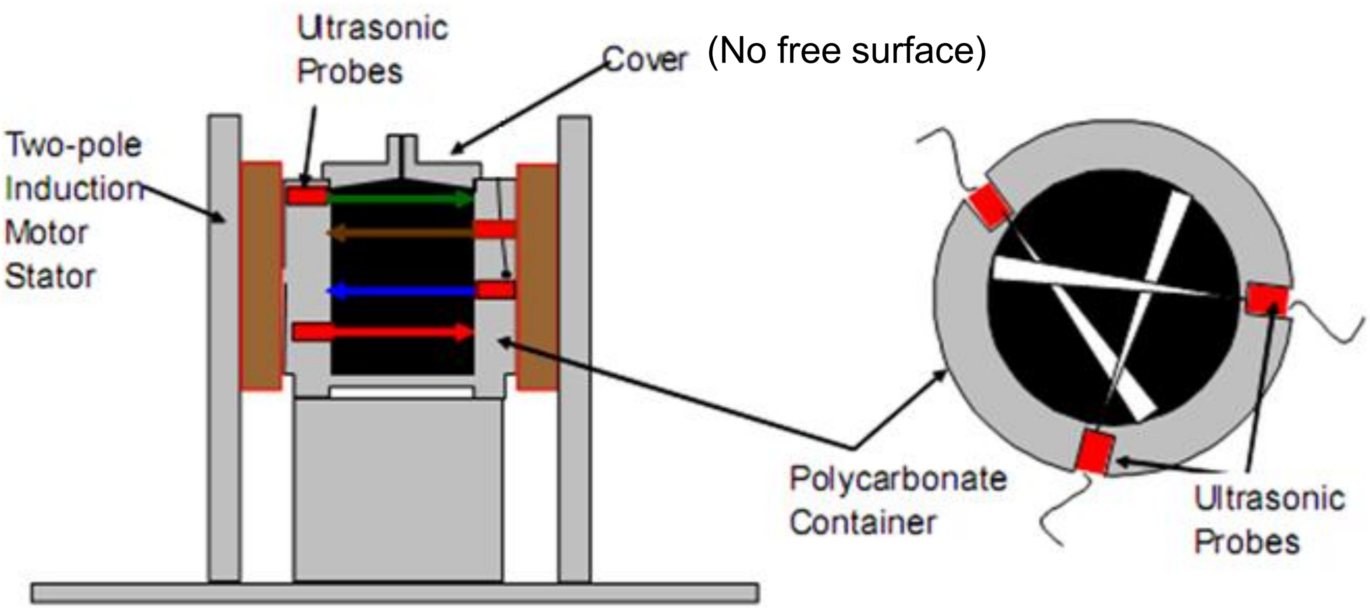


C. Rinaldi, "Continuum modeling of polarizable systems," Ph.D, Dept. of Chemical Engineering., Massachusetts Institute of Technology, Cambridge, MA, 2002.

R. Moskowitz and R. E. Rosensweig, *Nonmechanical torque-driven flow of a ferromagnetic fluid by an electromagnetic field*, Applied Physics Letters 11 (1967), no. 10, 301-303.

R. E. Rosensweig, J. Popplewell, and R. J. Johnston, *Magnetic fluid motion in rotating field*, Journal of Magnetism and Magnetic Materials 85 (1990), 171-180.

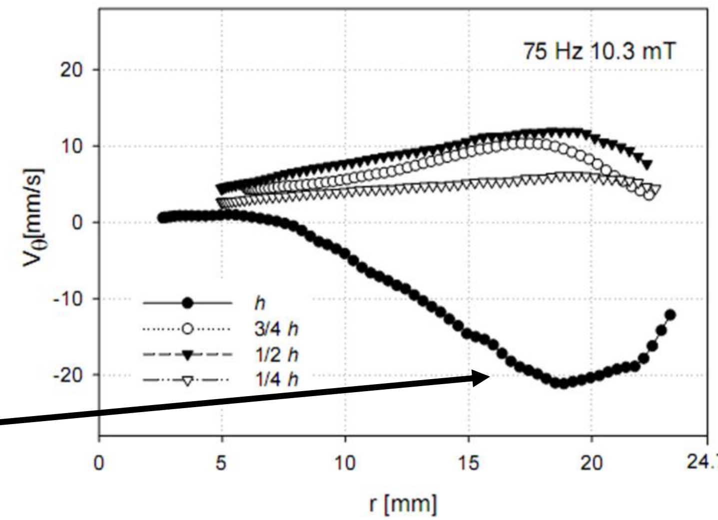
Bulk flow experiments

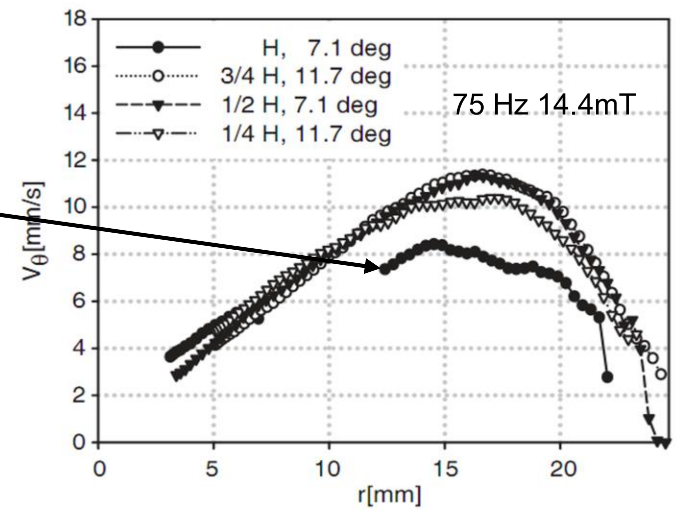


A. Chaves, C. Rinaldi, S. Elborai, X. He, and M. Zahn, *Bulk flow in ferrofluid in a uniform rotating magnetic field*, Physical Review Letters 96 (2006), no. 19, 194501-4.

Surface driven and Bulk driven flows

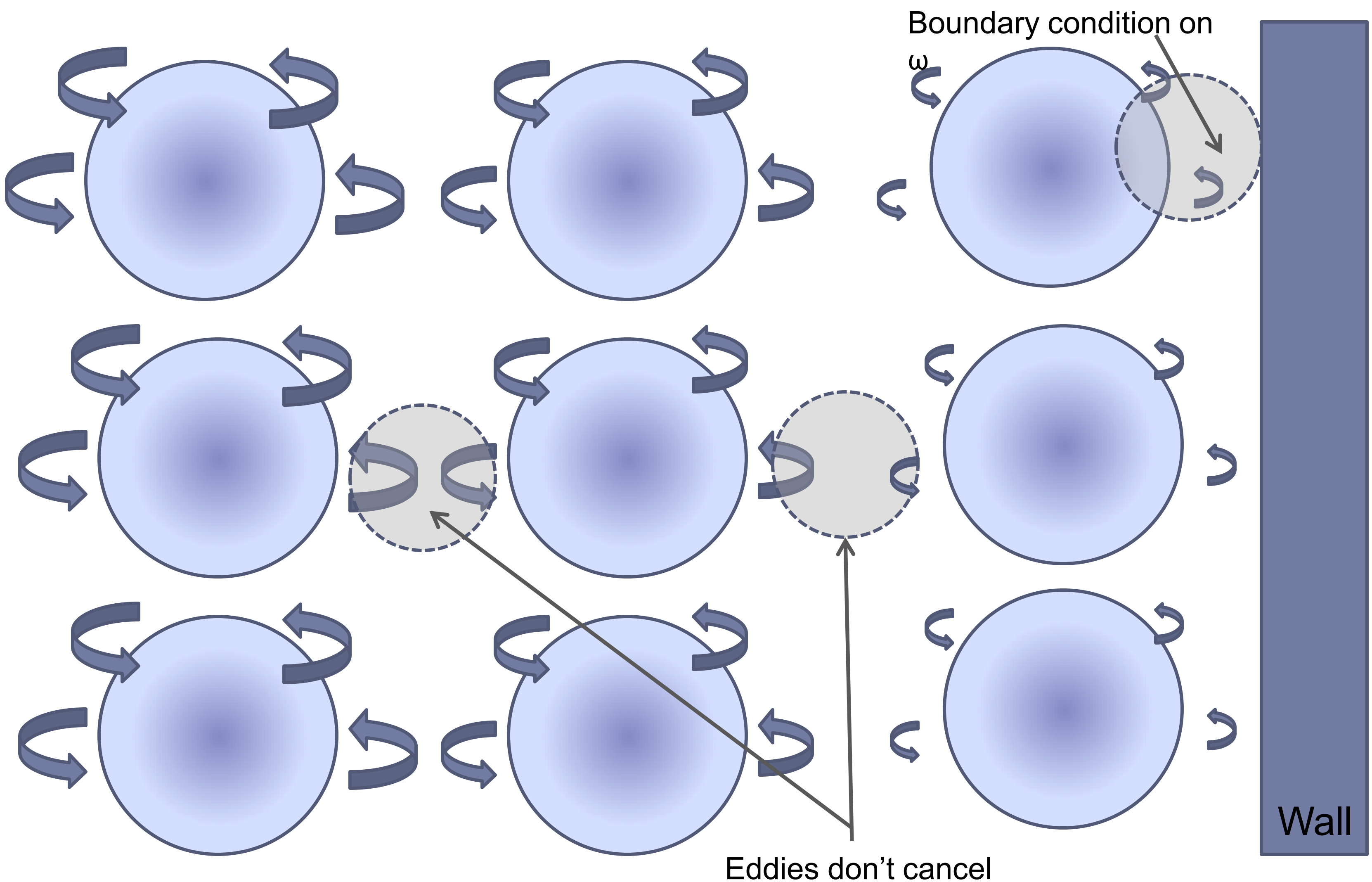
- Bulk flow velocity profiles co-rotate with the field
- If there is a free surface, there is *counter-rotation* at the surface (concave)
- If there is no free surface there is *co-rotation* near the surface





A. Chaves, C. Rinaldi, S. Elborai, X. He, and M. Zahn, *Bulk flow in ferrofluid in a uniform rotating magnetic field*, Physical Review Letters 96 (2006), no. 19, 194501-4.

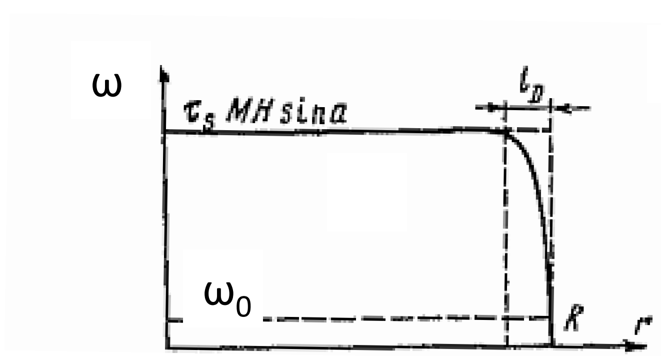
Non-uniform eddies



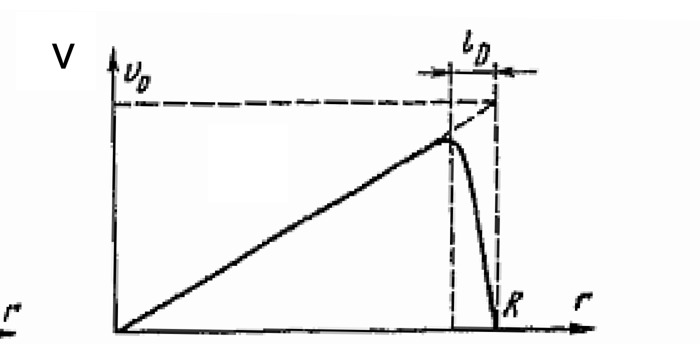
Spin-diffusion theory

- Zaitsev and Shliomis state that microscopic eddies will result in macroscopic motion in the case of non-uniform internal rotations
  - Boundary condition on spin velocity **ω** creates flow

$$\boldsymbol{\omega}(r = R_{wall}) = 0$$



ω as a function of radius



'Macroscopic' velocity 'v' as a function of radius

V. M. Zaitsev and M. I. Shliomis, *Entrainment of ferromagnetic suspension by a rotating field*, Journal of Applied Mechanics and Technical Physics 10 (1969), no. 5, 696-700.

Spin-diffusion Governing Equations

- Extended Navier-Stokes Equation

Incompressible flow

=0 =0

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\bullet\nabla)\mathbf{v}\right]=-\nabla p+\mu_0(\mathbf{M}\bullet\nabla)\mathbf{H}+2\zeta\nabla\times\boldsymbol{\omega}+(\lambda+\eta-\zeta)\nabla(\nabla\bullet\mathbf{v})+(\zeta+\eta)\nabla^2\mathbf{v}$$

Neglecting Inertia

  - Boundary condition on **v**, $\mathbf{v}(r=R_{wall})=0$

- Conservation of internal angular momentum

=0 =0

$$J\left[\frac{\partial \boldsymbol{\omega}}{\partial t}+(\mathbf{v}\bullet\nabla)\boldsymbol{\omega}\right]=\mu_0(\boldsymbol{M}\times\boldsymbol{H})+2\zeta(\nabla\times\mathbf{v}-2\boldsymbol{\omega})+(\lambda'+\eta')\nabla(\nabla\bullet\boldsymbol{\omega})+\eta'\nabla^2\boldsymbol{\omega} \qquad \zeta=\frac{3}{2}\eta\phi$$

Neglecting Inertia

  - Boundary condition on **ω** unless η'=0, $\boldsymbol{\omega}(r=R_{wall})=0$

ρ [kg/m³] is the ferrofluid mass density, p [N/m²] is the fluid pressure, ζ [Ns/m²] is the vortex viscosity, η [Ns/m²] is the dynamic shear viscosity, λ [Ns/m²] is the bulk viscosity, ω [s⁻¹] is the spin velocity of the ferrofluid, v is the velocity of the ferrofluid, J [kg/m] is the moment of inertia density, η' [Ns] is the shear coefficient of spin viscosity and λ'[Ns] is the bulk coefficient of spin viscosity, ϕ[%] is the magnetic particle volume fraction

Problems with Spin-diffusion theory

- Theoretical determination of *η'* [N-s] (≤1×10^{-18}) is many orders of magnitude smaller[1,2] than experimentally (≈10^{-8}-10^{-12}) fitted values[3,4,5]

1) K. R. Schumacher, *et al., "Experiment and simulation of laminar and turbulent ferrofluid pipe flow in an oscillating magnetic field," Physical Review E, vol. 67, p. 026308, 2003.*
2) R.E. Rosensweig, Ferrohydrodynamics, Dover Publications, 1997.
3) S. Elborai, "Ferrofluid surface and volume flows in uniform rotating magnetic fields," Ph.D thesis, Massachusetts Institute of Technology, Cambridge, MA, 2006
4) X. He, "Ferrohydrodynamic flows in uniform and non-uniform rotating magnetic fields," Ph.D thesis, Massachusetts Institute of Technology, Cambridge, MA, 2006.
5) A. Chaves, C. Rinaldi, S. Elborai, X. He, and M. Zahn, *Bulk flow in ferrofluid in a uniform rotating magnetic field*, Physical Review Letters 96 (2006), no. 19, 194501-4.

- Many authors as a result consider spin-diffusion effect to be negligible (*η'≈0*)

Shliomis[6], and Pshenichnikov[7] state that spin-up flow is a result of non-uniformities in the rotating magnetic field or magnetic properties when *η'≈0*

With *η'≈0* in a perfectly uniform magnetic field there should be no flow

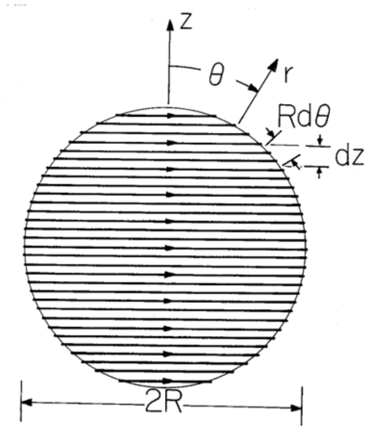
6) M. I. Shliomis, T. P. Lyubimova, and D. V. Lyubimov, *Ferrohydrodynamics: An essay on the progress of ideas*, 1988, pp. 275-290
7) A. F. Pshenichnikov, A. V. Lebedev, and M. I. Shliomis, *On the rotational effect in nonuniform magnetic fluids*, Magnetohydrodynamics 36 (2000), no. 4.

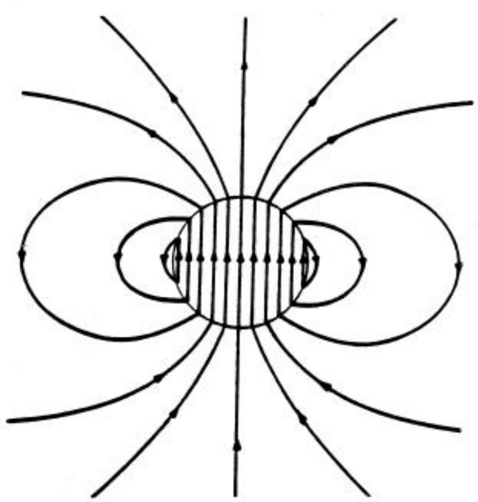
Uniform Rotating Fields Using a Spherical Coil Assembly

Motivation

- To investigate spin-up flow as a result of applied uniform and non-uniform magnetic fields
  - A ferrofluid-filled sphere in an external uniform field will have equal demagnetizing fields in all directions resulting in a uniform internal field
  - Use of permanent magnet and current carrying coil to create non-uniform fields
  - The external uniform rotating field will be generated using two spherical coils known as ‘fluxballs’

Fluxball





- N turns of wire uniformly spaced in z
- Surface Current Density $K_\phi = \dfrac{NI \sin\theta}{2R}$

$$\nabla \times \overrightarrow{H} = 0 \rightarrow \overrightarrow{H} = -\nabla\Phi$$

$$\overrightarrow{M} = \chi \overrightarrow{H}$$

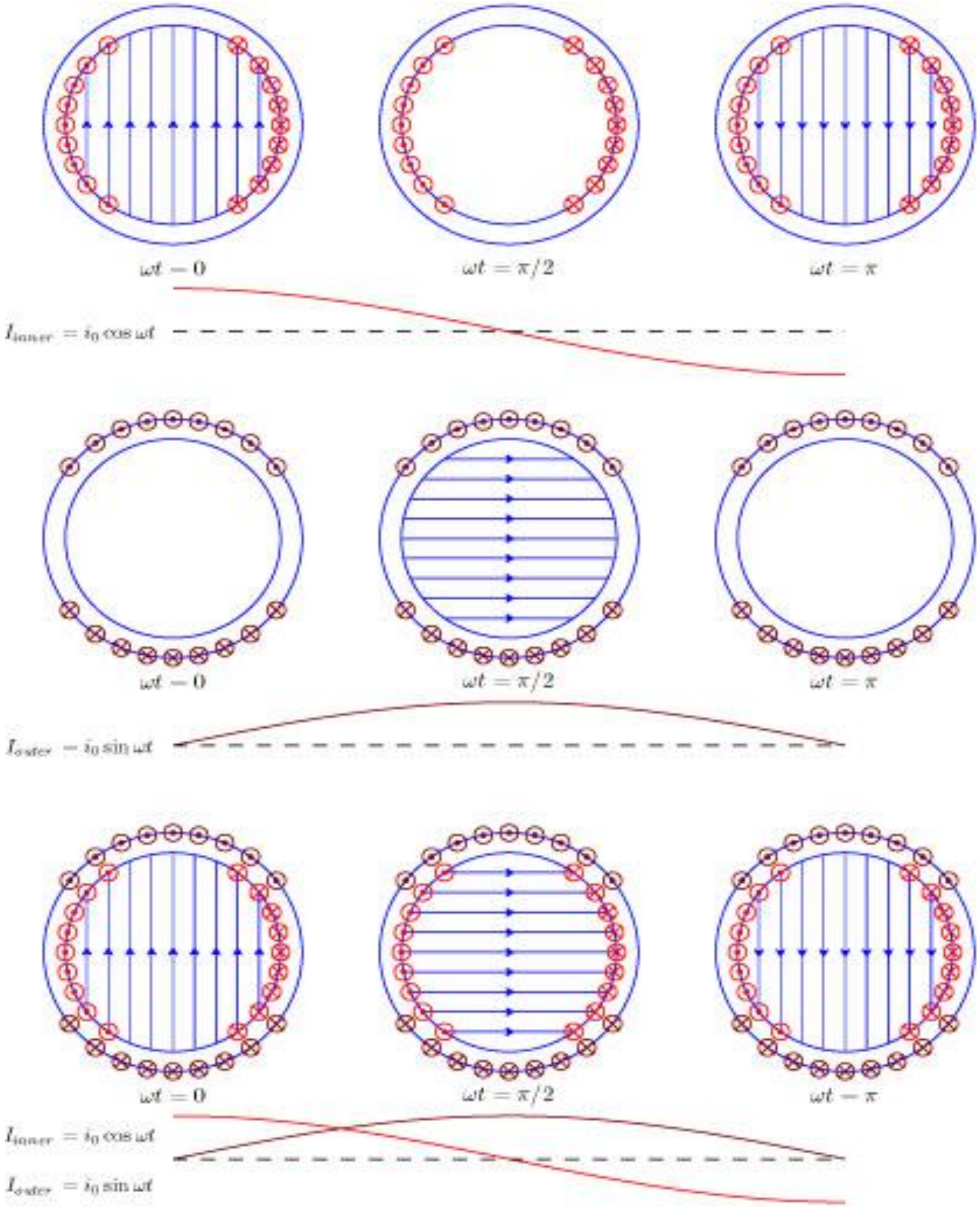
$$\nabla \cdot \overrightarrow{B} = 0 \rightarrow \nabla \cdot \overrightarrow{H} = -\nabla \cdot \overrightarrow{M} = -\chi \nabla \cdot \overrightarrow{H}$$

$$(1+\chi)\nabla^2\Phi = 0 \rightarrow \nabla^2\Phi = 0$$

- Solution to Laplace’s equation

$$\overrightarrow{H}(r<R) = \frac{NI}{((3+\chi)R)} i_z$$

H.A. Haus and J.R. Melcher, *Electromagnetic Fields and Energy* , 1989.



Rotating fields in Fluxball

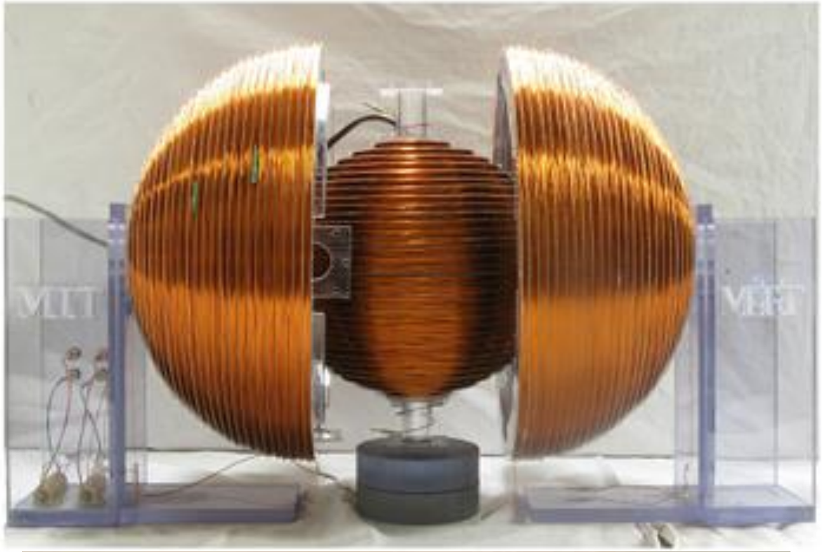
Orthogonally placed fluxballs

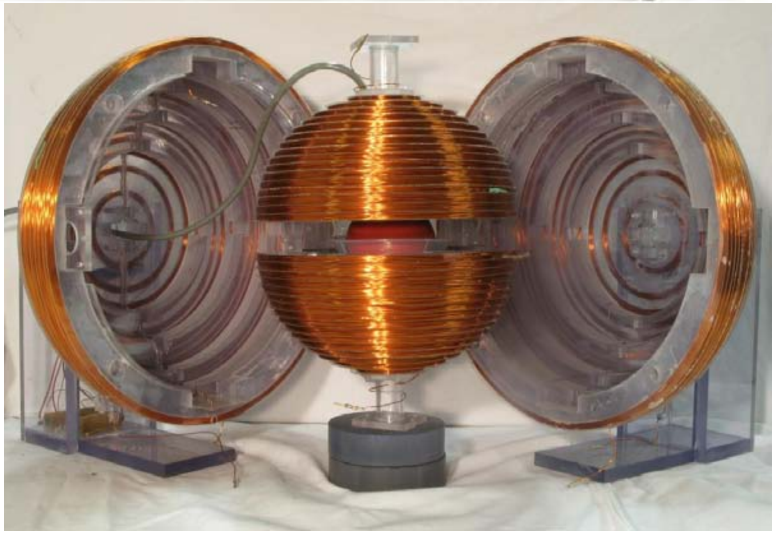
Excited by sinusoidal signals out of phase by 90^{O}

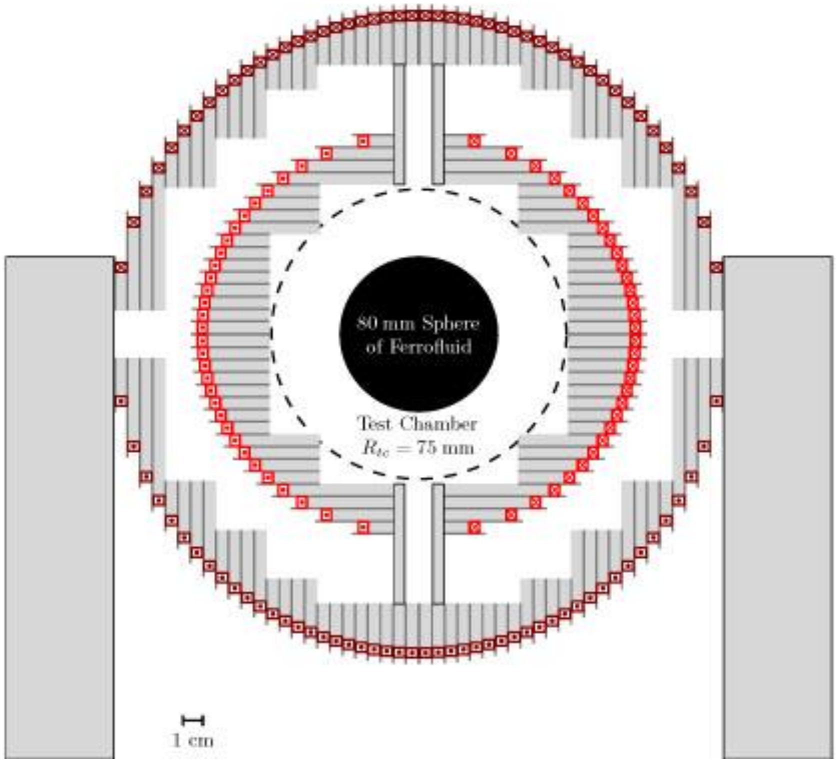
Generates a rotating magnetic field

Clinton Lawler, *A two-phase spherical electric machine for generating rotating uniform magnetic fields*, Master of Science, Massachusetts Institute of Technology, 2007

Fluxball setup







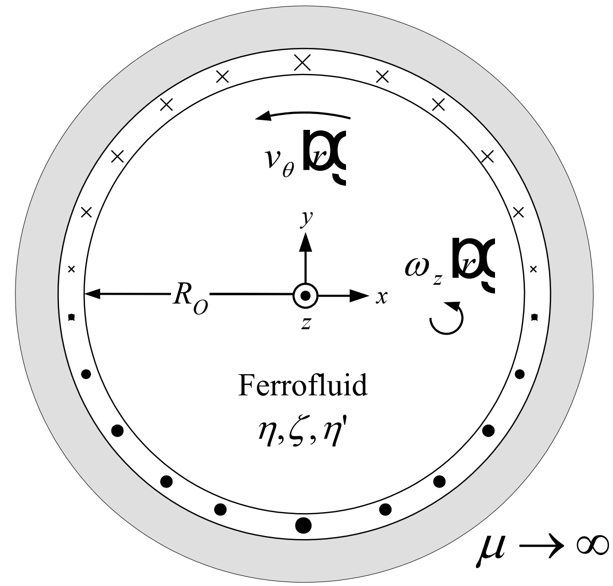
Clinton Lawler, *A two-phase spherical electric machine for generating rotating uniform magnetic fields*, Master of Science, Massachusetts Institute of Technology, 2007

Spin-up Flow Modeling (COMSOL Multiphysics η' large and $\eta'=0$)

Modeling Ferrofluid Spin-up in cylinder

- 2D model assumes no variation in z (∞ long cylinder)
- 3 phase 2 pole with infinite μ stator
- Current distribution ‘K’ generates a uniform rotating magnetic field
- Boundary conditions

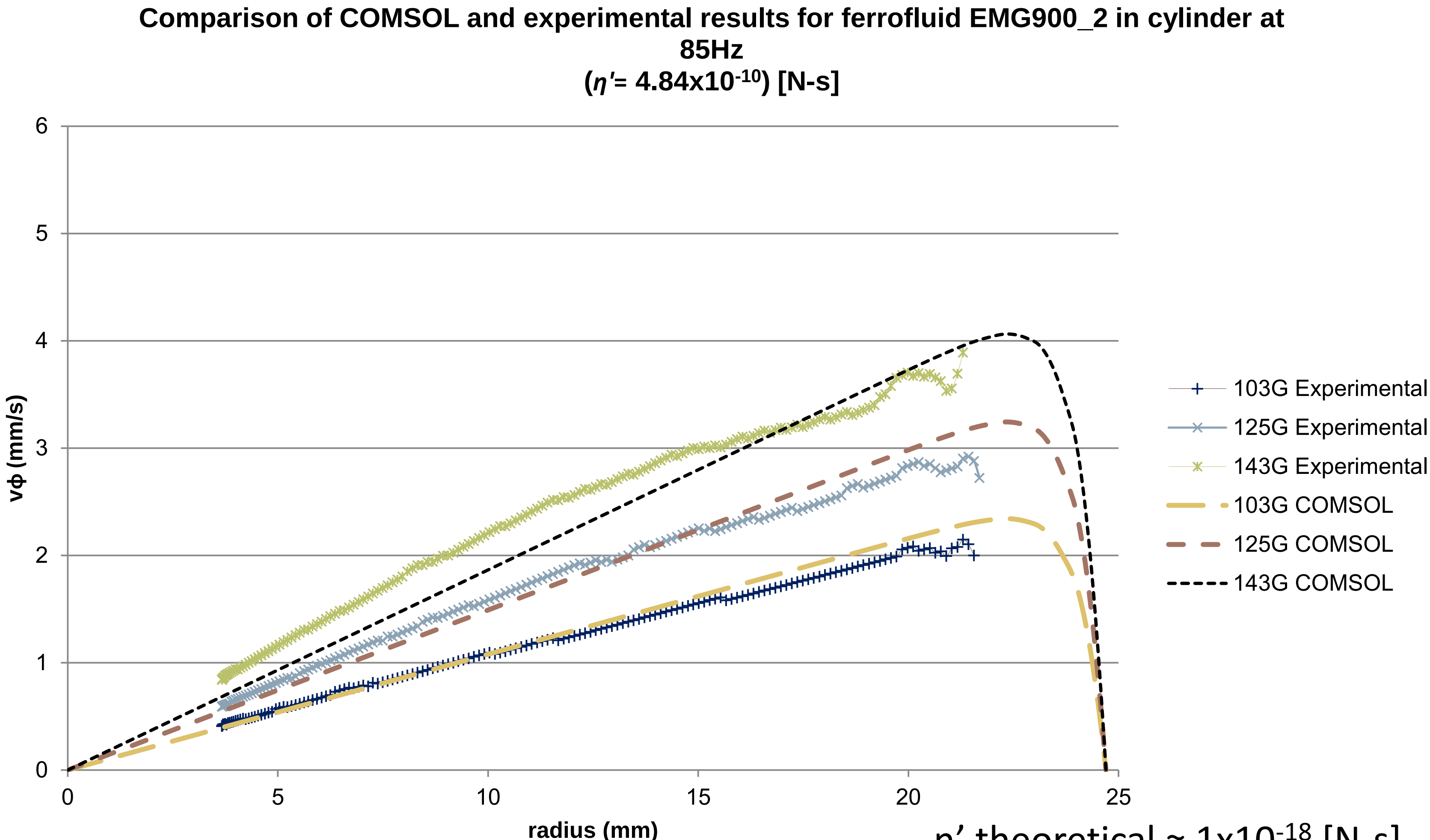
$$\mathbf{v}(r = R_0) = \boldsymbol{\omega}(r = R_0) = 0$$



	EMG900_2	MSGW11	EFH1
χ	1.19	0.56	1.59
$\mu_0 M_s$ (G)	239	154	421
η (Ns/m²)	0.0045	0.00202	0.00727

Arlex Chaves, Markus Zahn, and Carlos Rinaldi, *Spin-up flow of ferrofluids: Asymptotic theory and experimental measurements*, Phys. Fluids 20,

Simulation of cylinder experiment by Chaves

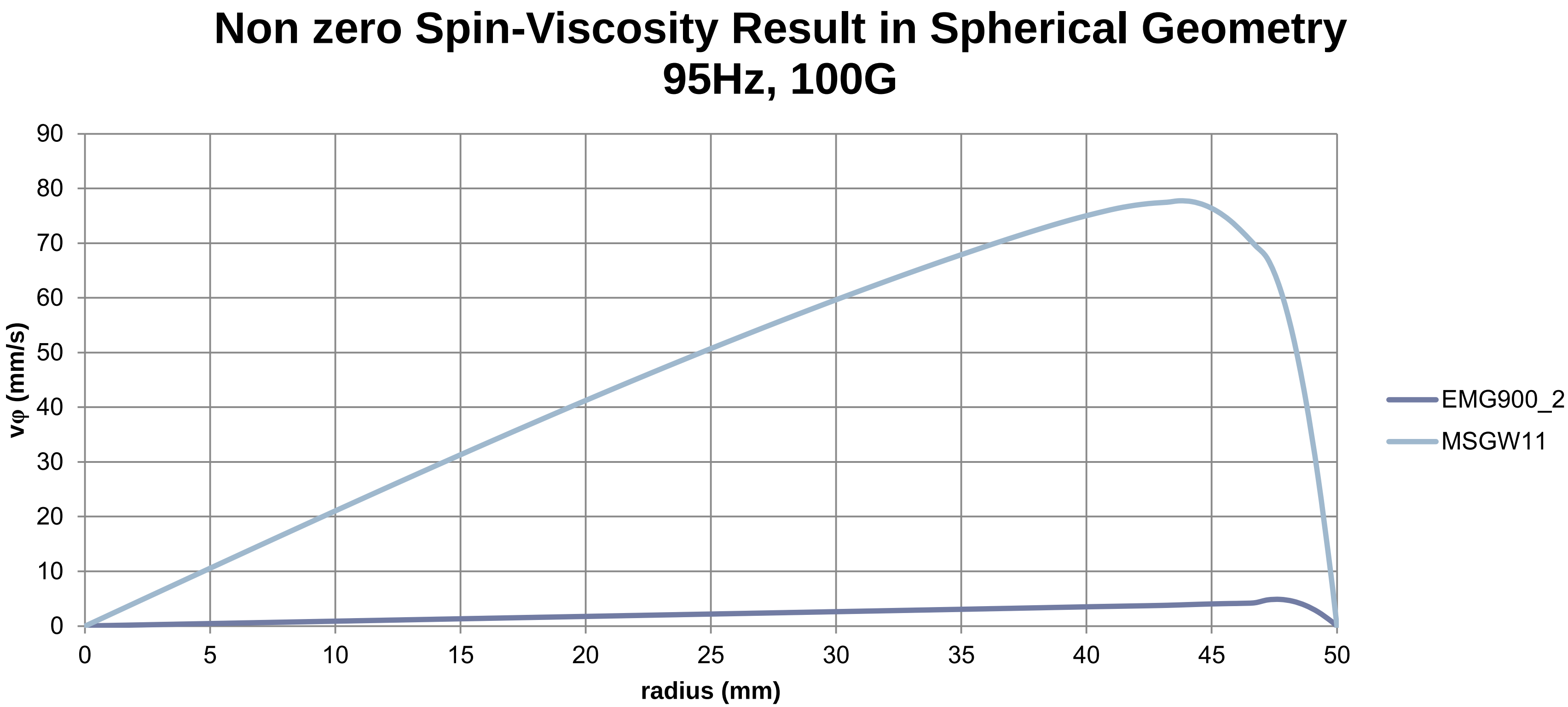


η' theoretical ≈ 1×10^{-18} [N-s]

Arlex Chaves, Markus Zahn, and Carlos Rinaldi, *Spin-up flow of ferrofluids: Asymptotic theory and experimental measurements*, Phys. Fluids 20, 053102 (2008)

K. R. Schumacher*, et al., "Experiment and simulation of laminar and turbulent ferrofluid pipe flow in an oscillating magnetic field," Physical Review E, vol. 67, p.*

COMSOL Simulations with η’≠0

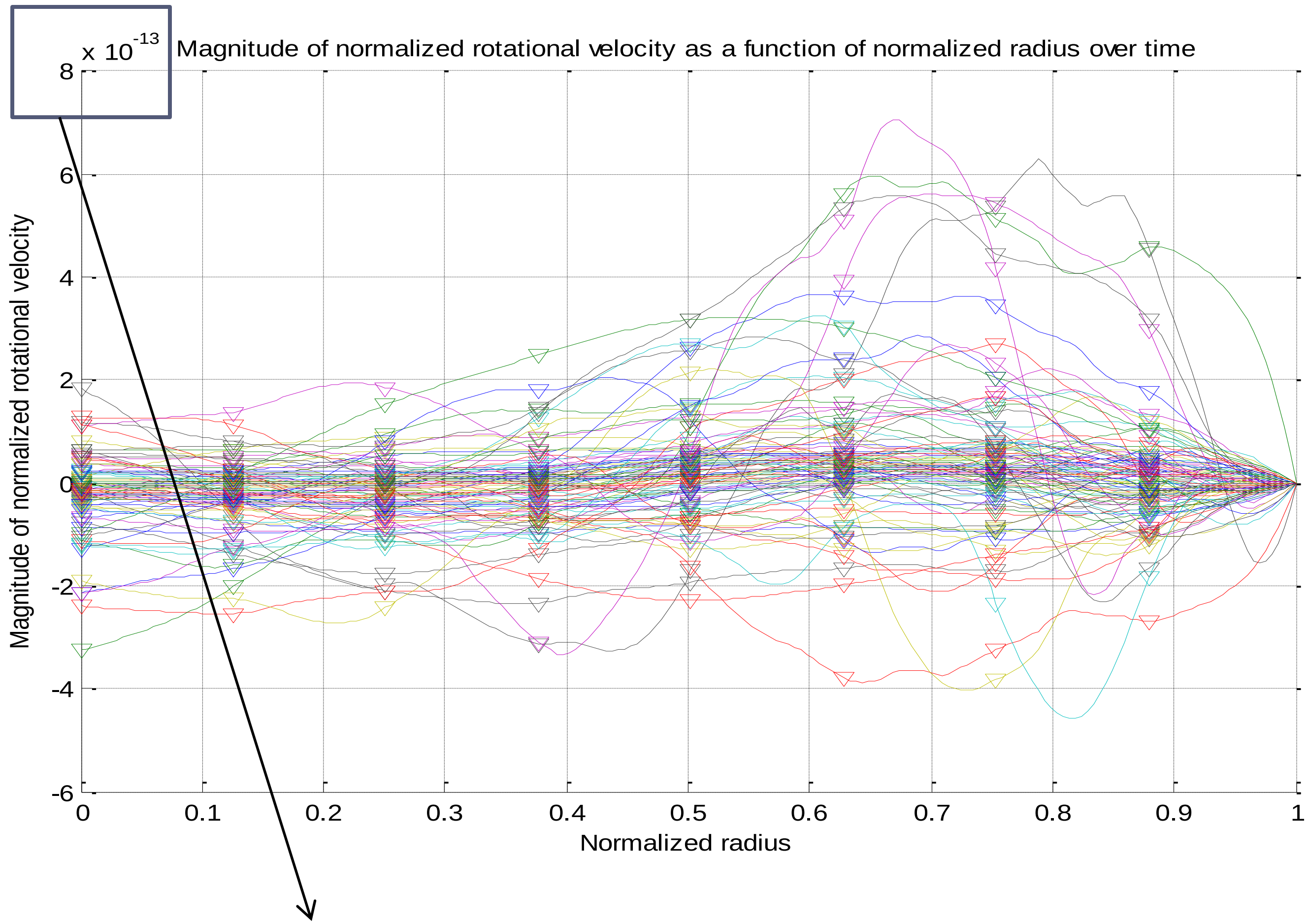


EMG900_2, η’=4.84x10^{-10} [N-s], Max Velocity ≈ 5 mm/s

MSGW11, η’=4.78x10^{-9} [N-s], Max Velocity ≈ 78 mm/s

S. Elborai, "Ferrofluid surface and volume flows in uniform rotating magnetic fields," Ph.D thesis, Massachusetts Institute of Technology, Cambridge, MA, 2006.
A. Chaves, *et al.*, *"Spin-up flow of ferrofluids: Asymptotic theory and experimental measurements," vol. 20, p. 053102, 2008.*
X. He, "Ferrohydrodynamic flows in uniform and non-uniform rotating magnetic fields," Ph.D thesis, Massachusetts Institute of Technology, Cambridge, MA, 2006.

Simulations of spherical case with η'=0

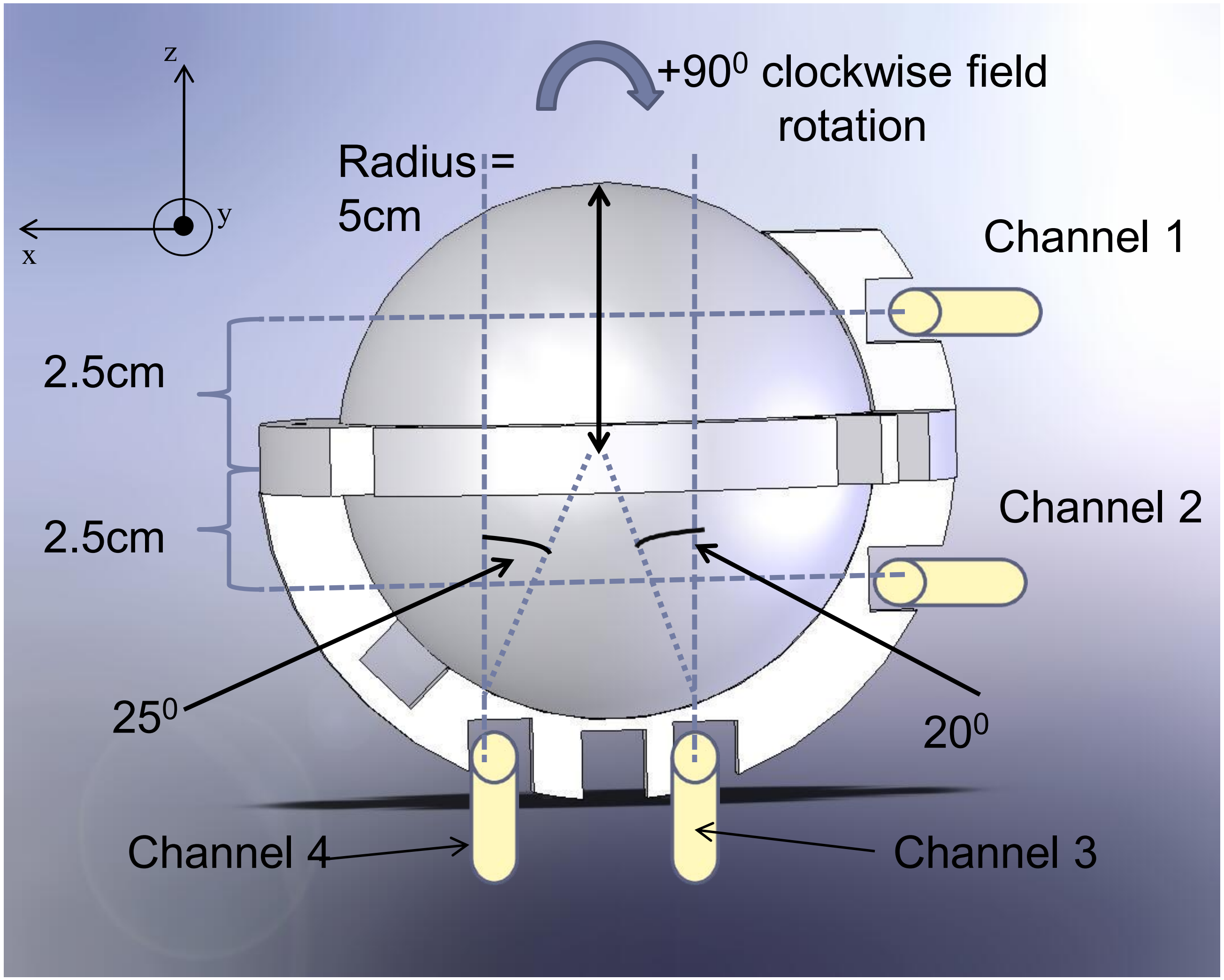


Max Dimensional Velocity ≈ 10^{-8} mm/s

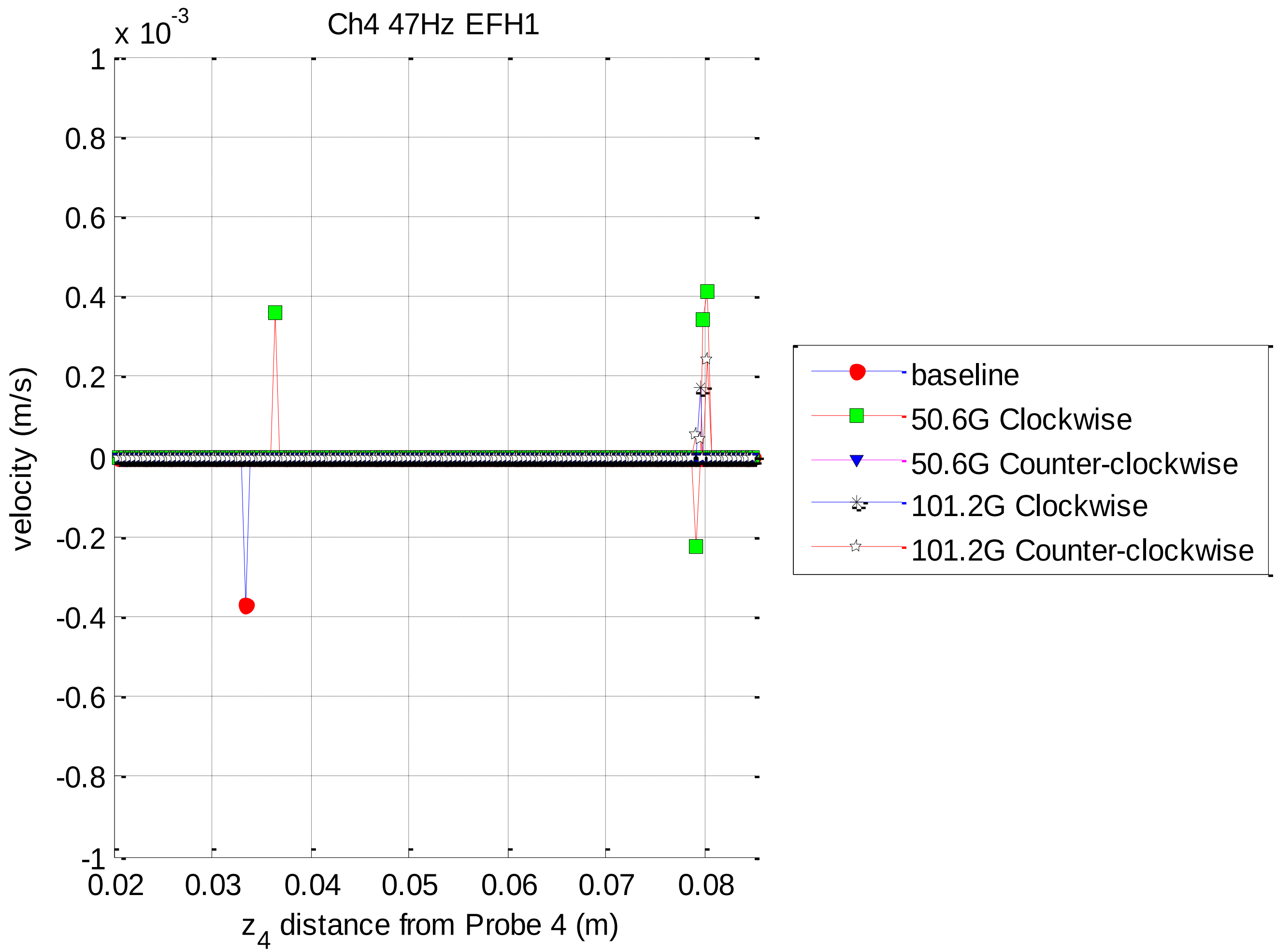
Experiments in Uniform Rotating Magnetic Fields

Ferrofluid Filled Sphere

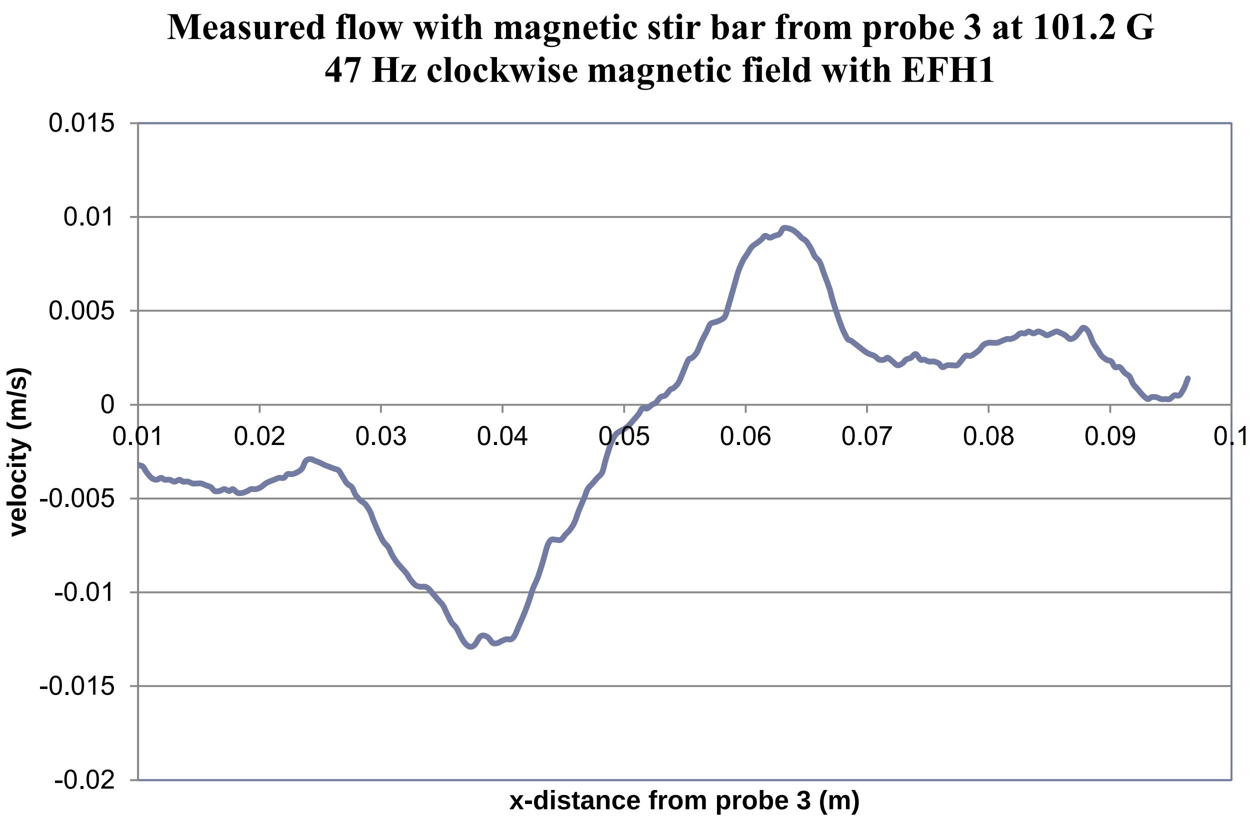
Probe positions



Results with EFH1- no flow



With Magnetic Stir-bar



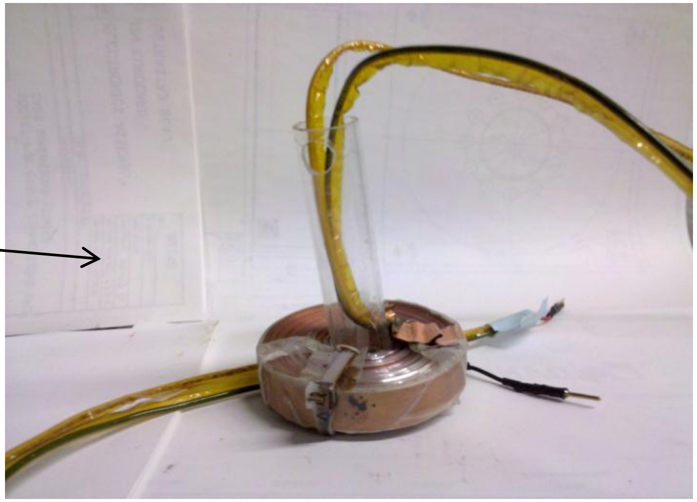
Experiments with Non-uniform Magnetic Fields

Third Coil

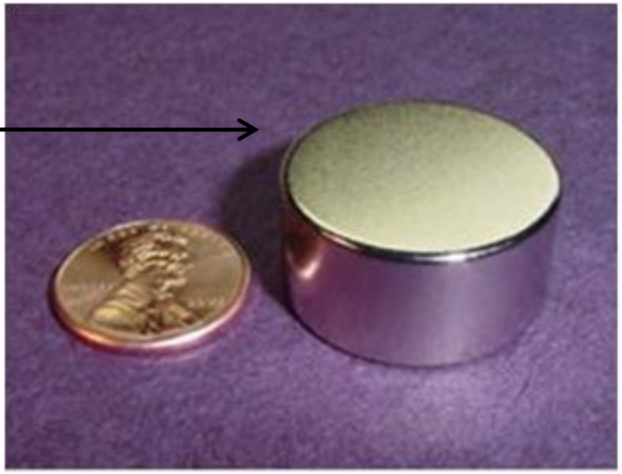
DC Magnet

Non-uniform magnetic field generation

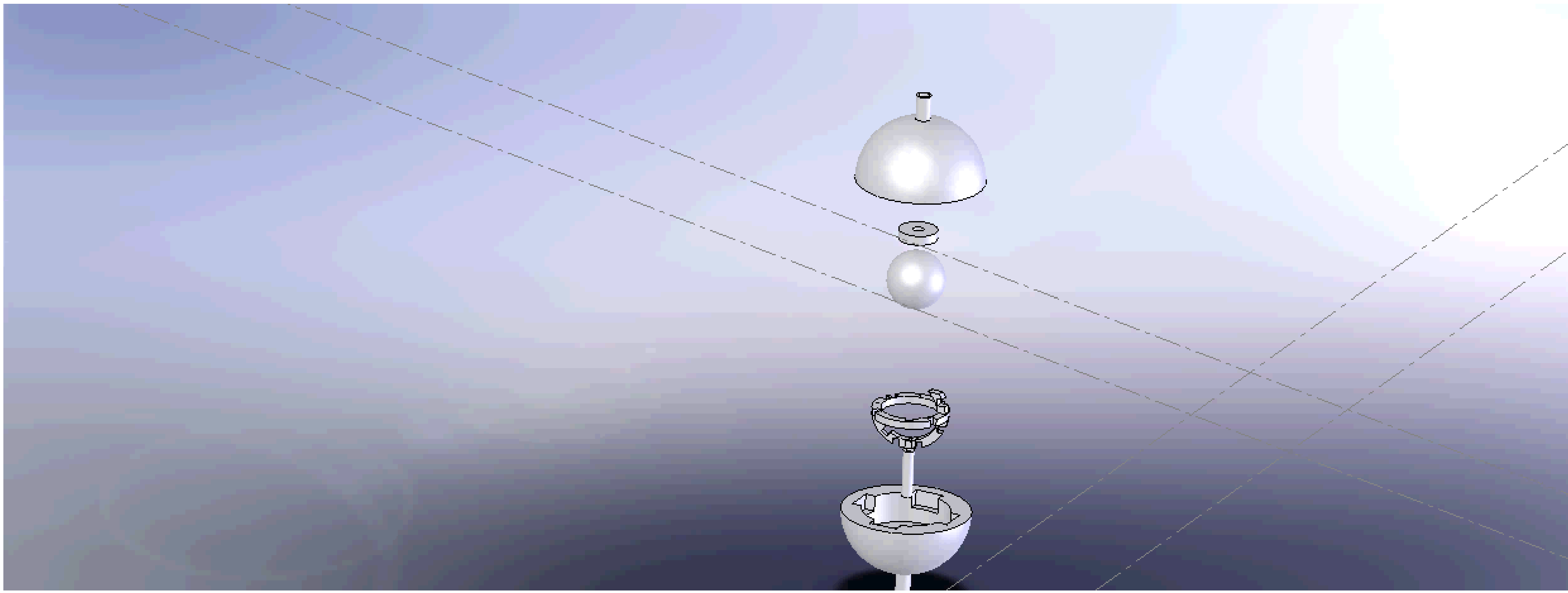
- 150 turn copper foil wound solenoidal coil
  - 0.625” height, 2.61” diameter
  - Inductance 0.7mH
  - Resistance 0.26Ω
  - Can be excited with DC and AC current (42.4 Gauss/IRMS)
  - Max Field (296.8 Gauss AC, 339.2 Gauss DC)



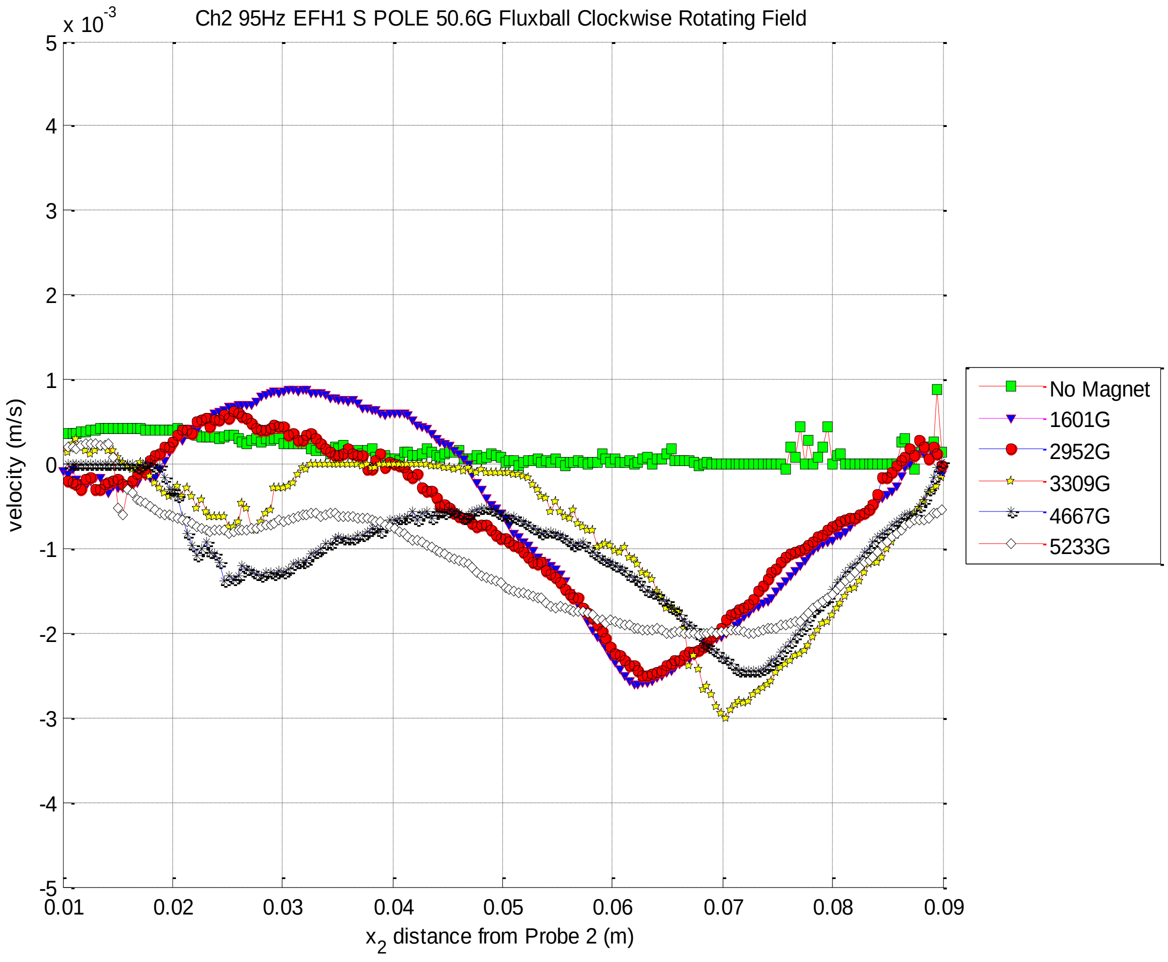
- Permanent Magnets- 0.5’’ radius
  - Surface field strengths
    - 1601G (1/8’’ height)
    - 2952G (1/4’’ height)
    - 3309G (1/4’’ height)
    - 4667G (1/2’’ height)
    - 5233G (1/2’’ height)

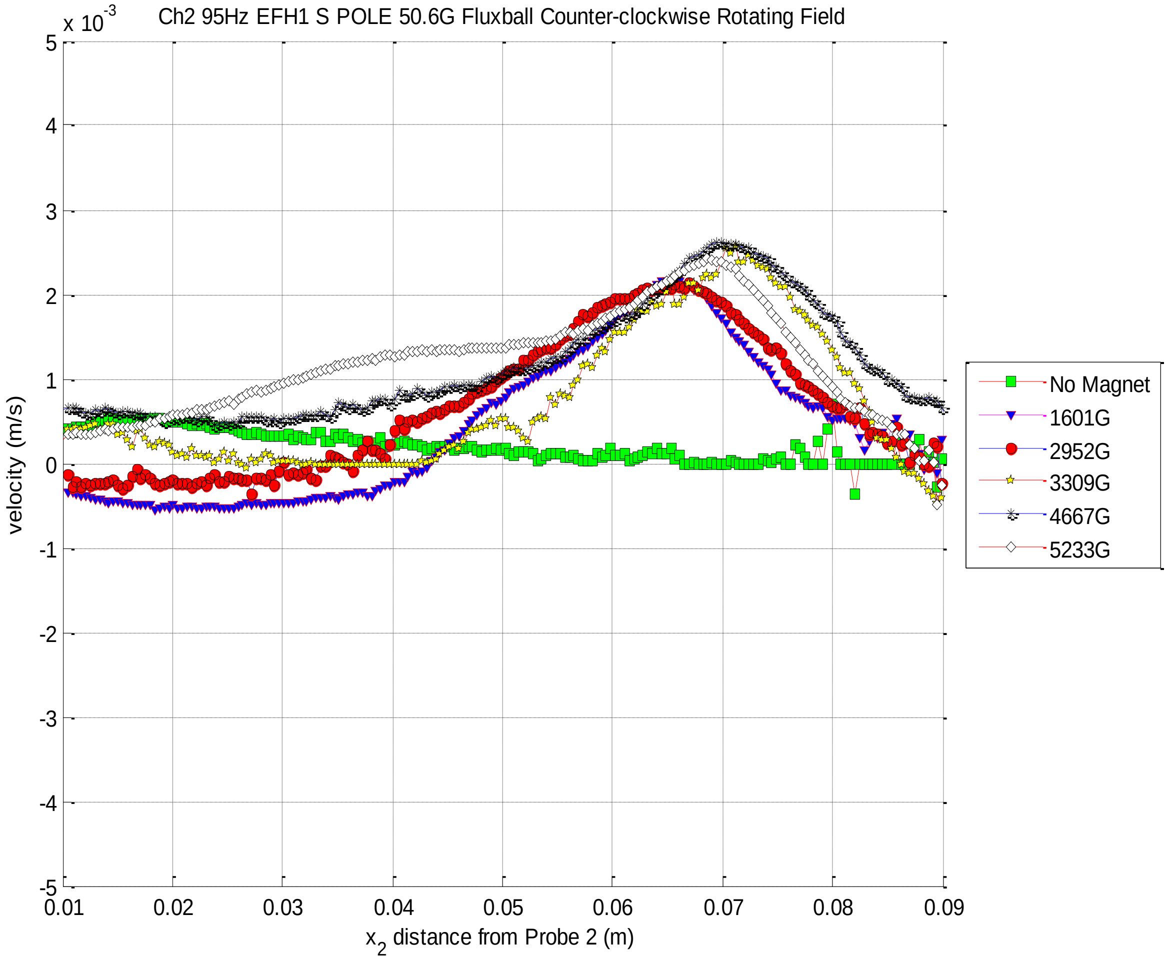


Experimental Setup

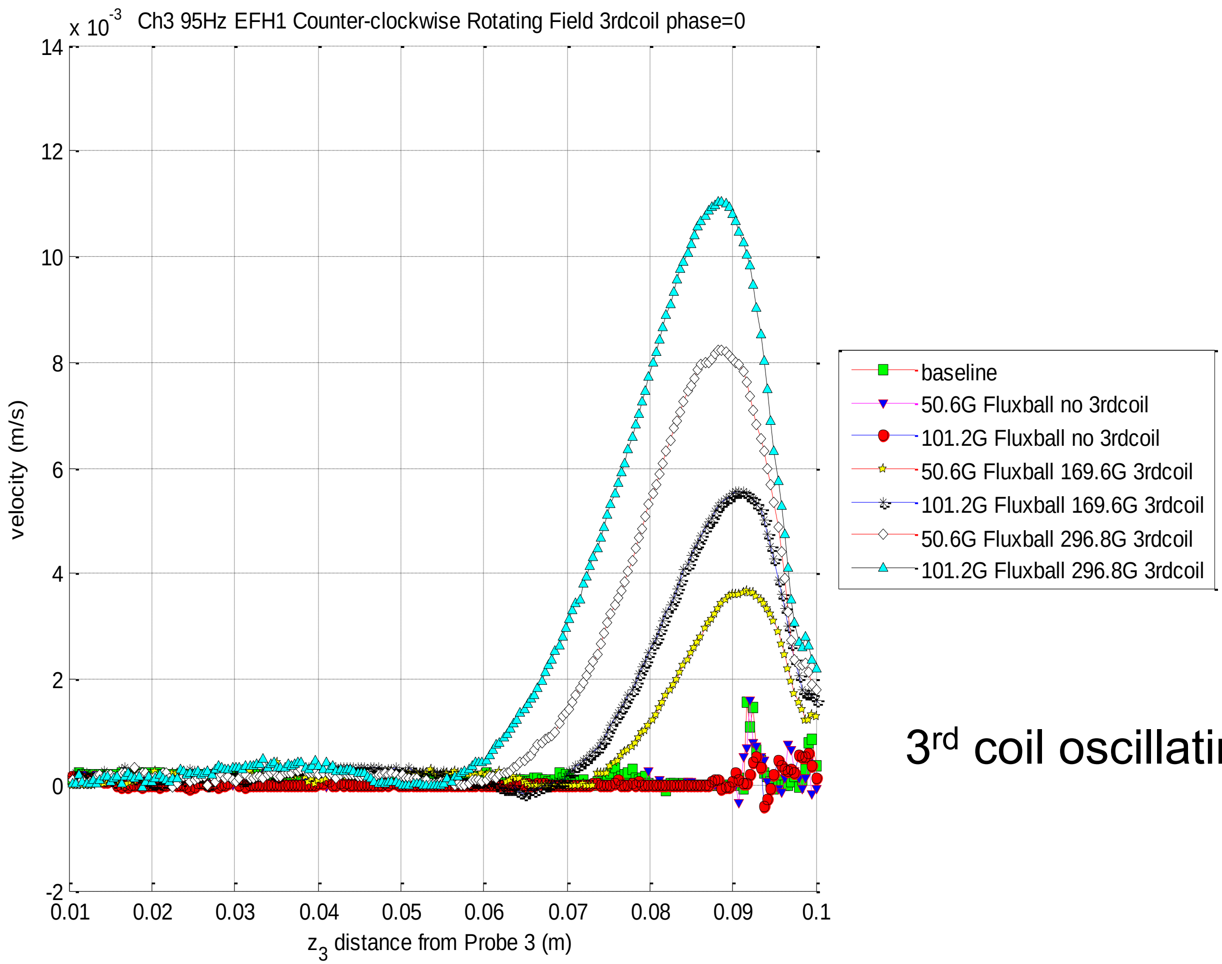


Effect of Rotating Field Direction




Ch2 95Hz EFH1 S POLE 50.6G Fluxball Counter-clockwise Rotating Field
x 10^-3
5
4
3
2
1
0
-1
-2
-3
-4
-5
velocity (m/s)
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
x2 distance from Probe 2 (m)
No Magnet
1601G
2952G
3309G
4667G
5233G

Coil cases



3rd coil oscillating at 95Hz

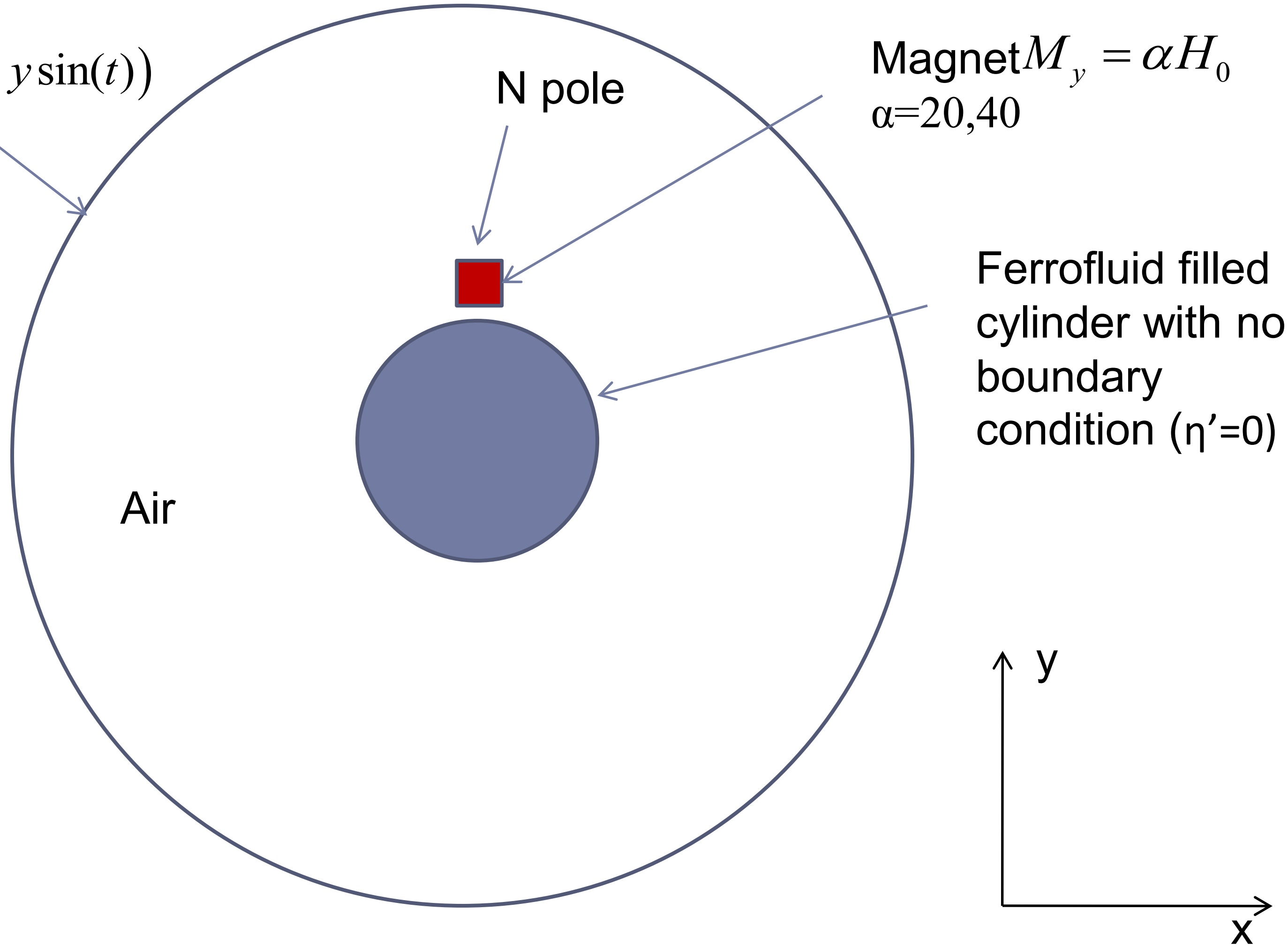
Simulations of Flow with Non-uniform Magnetic Fields in Infinitely Long Cylinder and Magnet

2D Problem Setup

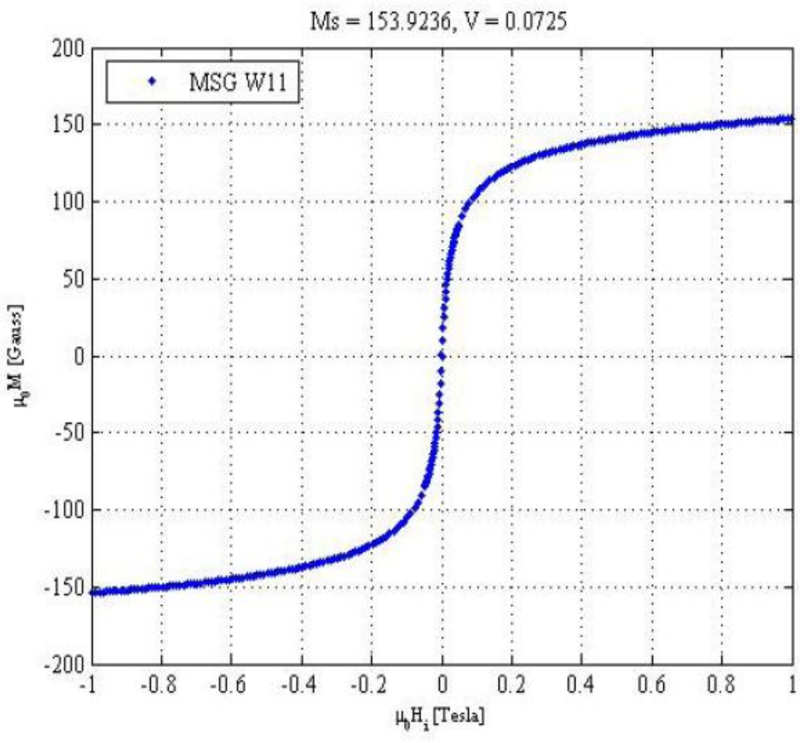
$$\Psi = H_0 \left(x \cos(t) + y \sin(t) \right)$$

Rotating field in terms of magnetic scalar potential far away

$$H = -\nabla \Psi$$



Magnetic nanoparticle in rotating magnetic field



Magnetic Relaxation Equation

$$\frac{\partial \boldsymbol{M}}{\partial t} + \mathbf{v} \bullet \nabla \boldsymbol{M} - \boldsymbol{\omega} \times \boldsymbol{M} + \frac{1}{\tau_{eff}}(\boldsymbol{M} - \boldsymbol{M}_0) = 0$$

Langevin Equation

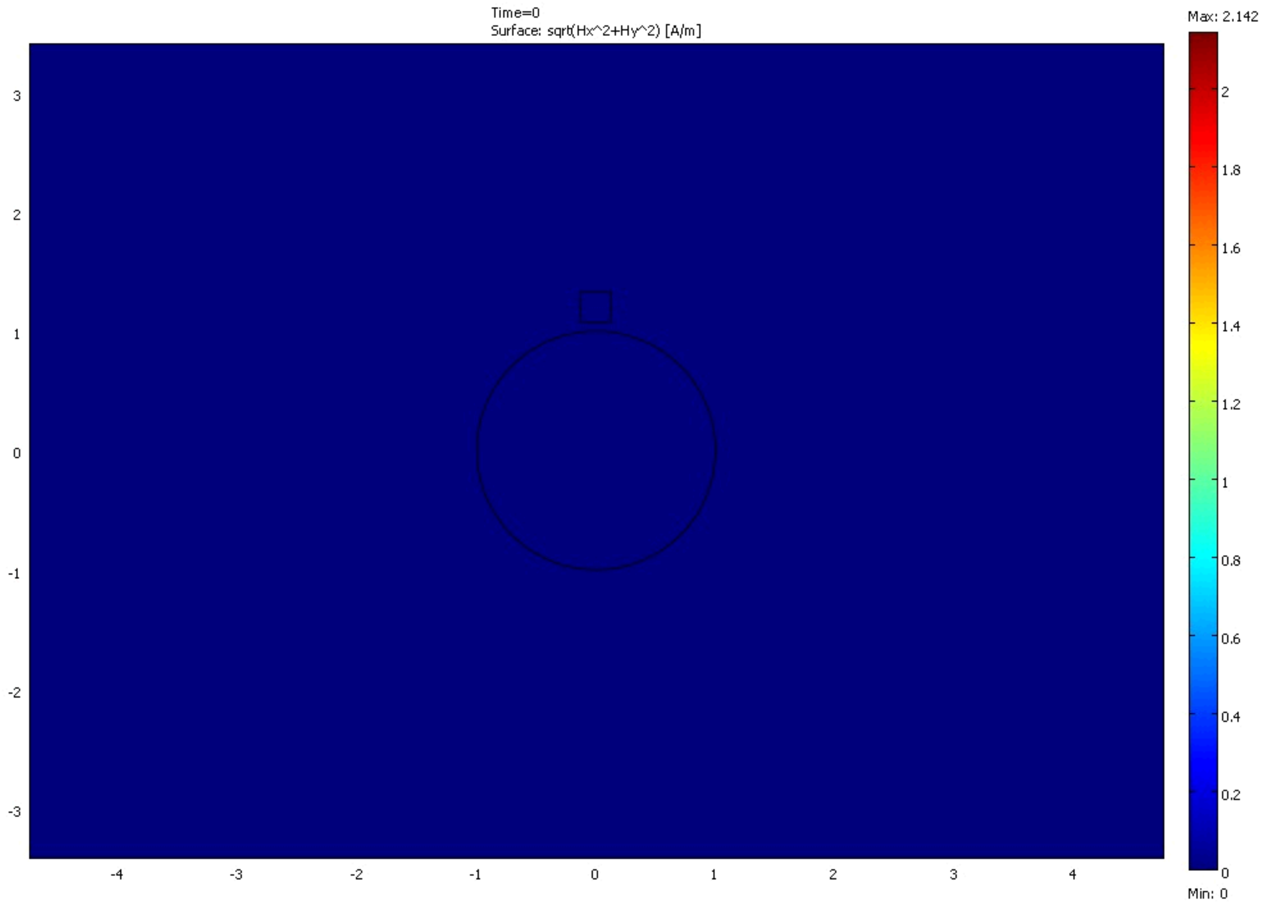
$$\boldsymbol{M}_0 = \boldsymbol{M}_s[\coth(a) - \frac{1}{a}], a = \frac{\mu_0 H_0 M_d V_p}{kT}$$

$$\frac{1}{\tau_{eff}} = \frac{1}{\tau_B} + \frac{1}{\tau_N} \quad \tau_B = 3V_h \frac{\eta_0}{kT}, \tau_N = \frac{1}{f_0} exp\left(\frac{K_a V_p}{kT}\right)$$

$\mathbf{M_s}$ [Amps/m] represents the saturation magnetization of the material, $\mathbf{M_d}$ [Amps/m] is the domain magnetization (446kA/m for magnetite), V_h is the hydrodynamic volume of the particle, V_p is the magnetic core volume per particle, T is the absolute temperature in Kelvin, k = 1.38 × 10^{-23} [J/K] is Boltzmann's constant, f_0 [1/s] is the characteristic frequency of the material and K_a is the anisotropy constant of the magnetic domains

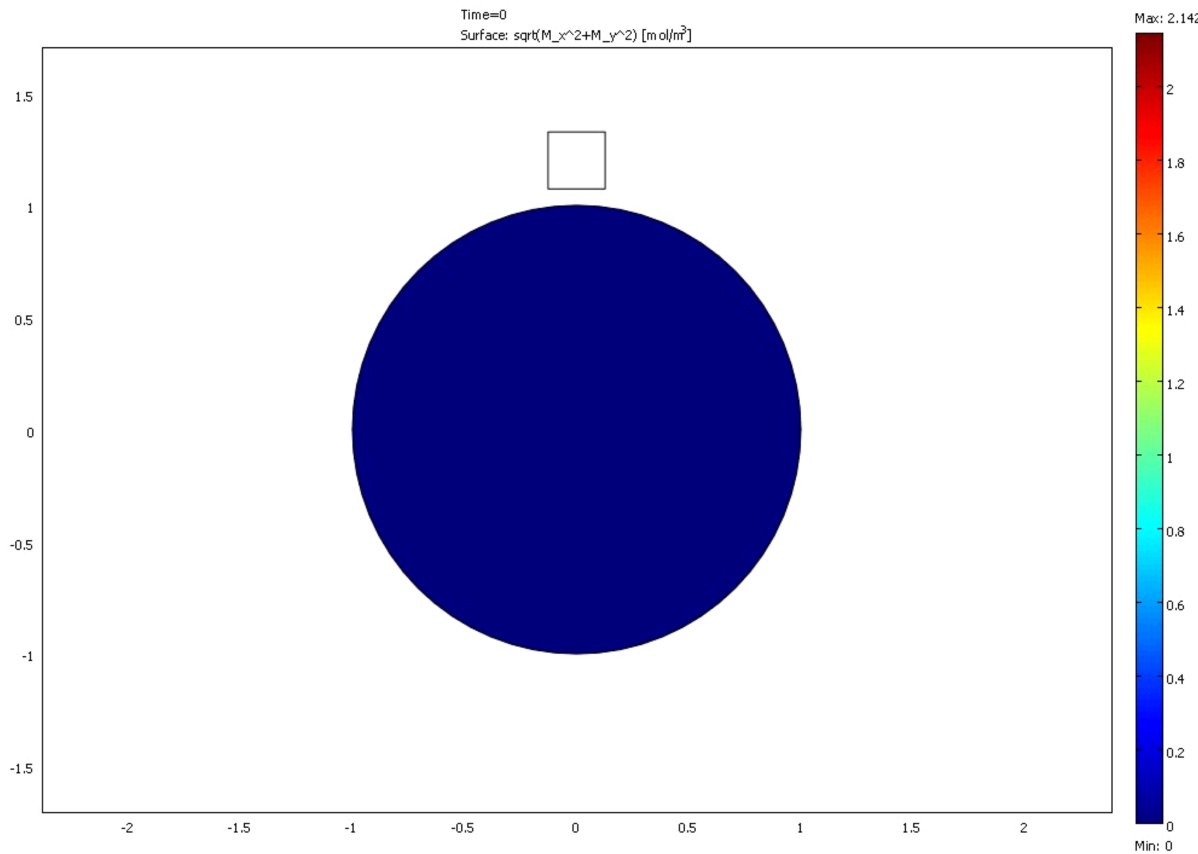
M. I. Shliomis, *Effective viscosity of magnetic suspensions*, Soviet Physics JETP 34 (1972), 1291-1294.
S. Elborai, "Ferrofluid surface and volume flows in uniform rotating magnetic fields," Ph.D thesis, Massachusetts Institute of Technology, 2006.
P. J. Cantillon-Murphy, "On the dynamics of magnetic fluids in magnetic resonance imaging," Ph. D. thesis, Massachusetts Institute of Technology, 2008.

4000 Gauss Magnet with MSGW11 – H field



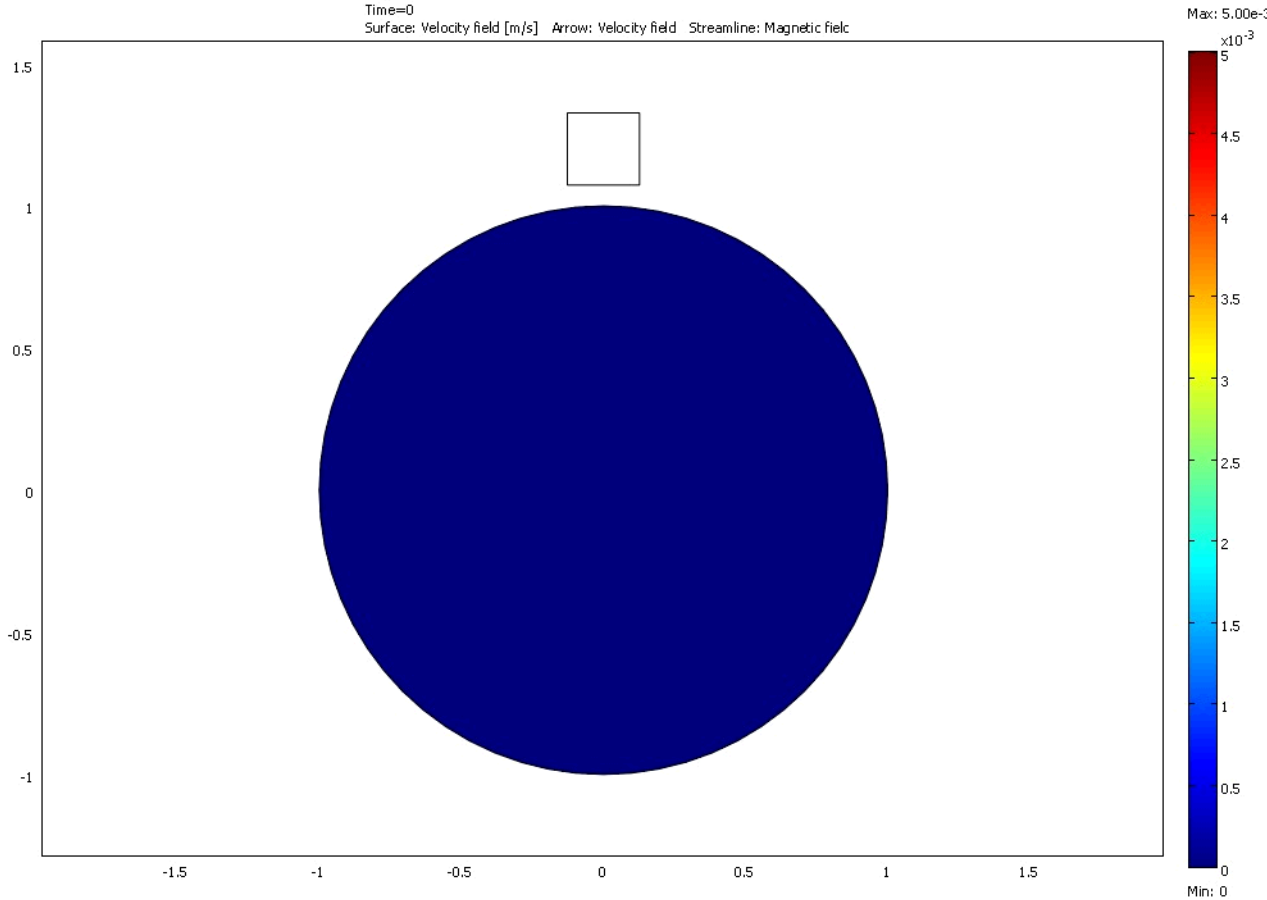
Rotating field strength 100G

4000 Gauss with MSGW11 - Magnetization



Rotating field strength 100G

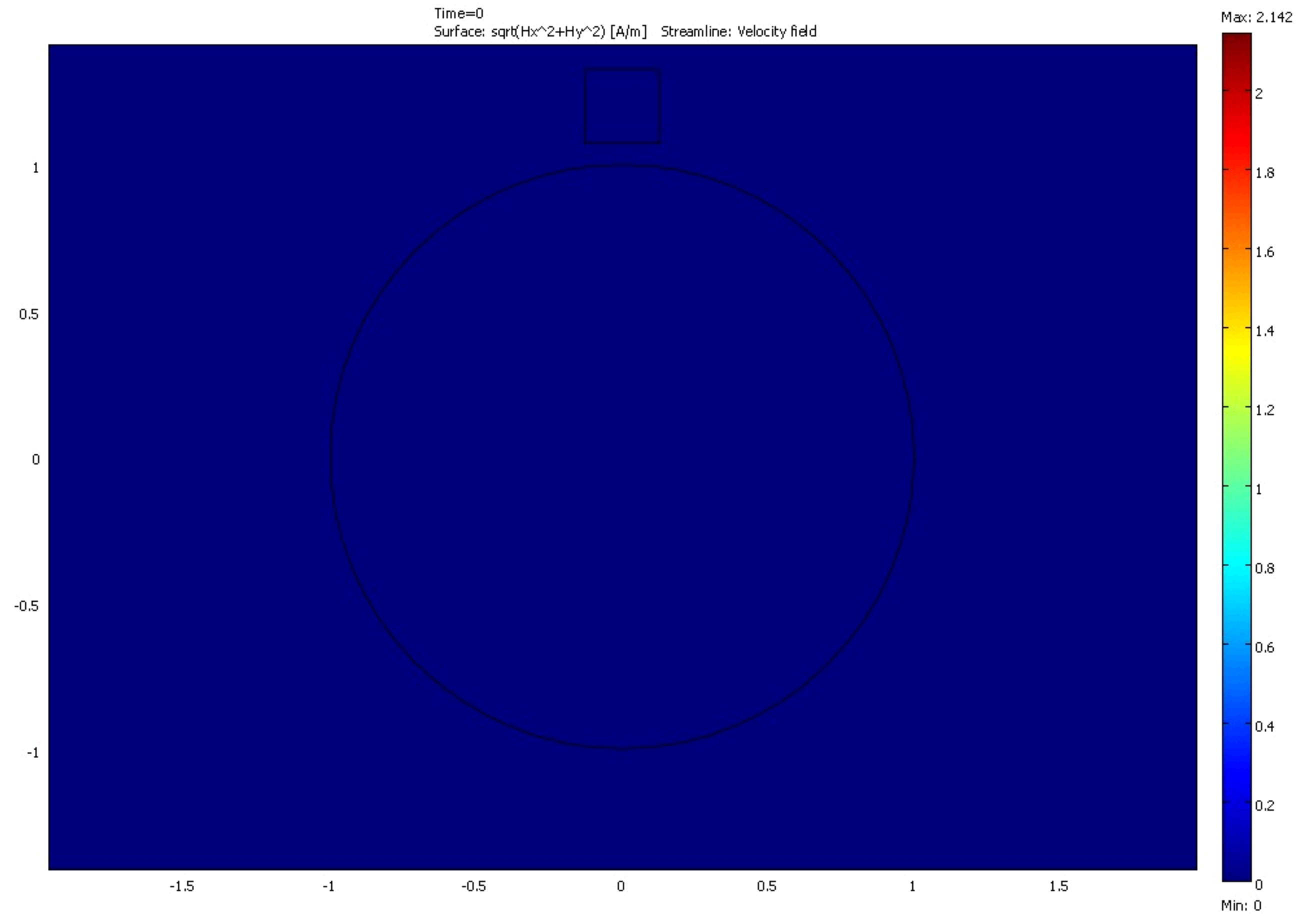
4000 Gauss with MSGW11 - flow



Dimensional velocity magnitude ≈3-30mm/s

Rotating field strength 100G

2000 Gauss with MSGW11 – flow streamlines



Rotating field strength 100G

Conclusions

Conclusions

- Simulations (spherical geometry) show flow exists when $\eta' \neq 0$ and no flow when $\eta'=0$ in uniform fields
- Experiments give no flow in a uniform rotating field (ferrofluid filled sphere)
- Experiments all confirm that flow exists in the presence of a non-uniform field
- Simulations (cylindrical geometry) confirm flow exists in non-uniform field with $\eta'=0$
- Flow profiles are very complicated with vortices with non-uniform fields
- Spin-diffusion theory is a negligible effect
  - Its effect has been overstated by using values of η' that are many orders of magnitude higher than theoretically derived values

Questions