Presented at the COMSOL Conference 2010 Boston

Stefan's Problem: Validation of a One-Dimensional Solid-Liquid Phase Change Heat Transfer Process

Wilson Ogoh and Dominic Groulx
Department of Mechanical Engineering
Dalhousie University, Halifax, NS, Canada

Thermal Energy Storage

- **Sensible Heat Storage:**

  A heat storage system that uses a heat storage medium, and where the addition or removal of heat results in a change in temperature

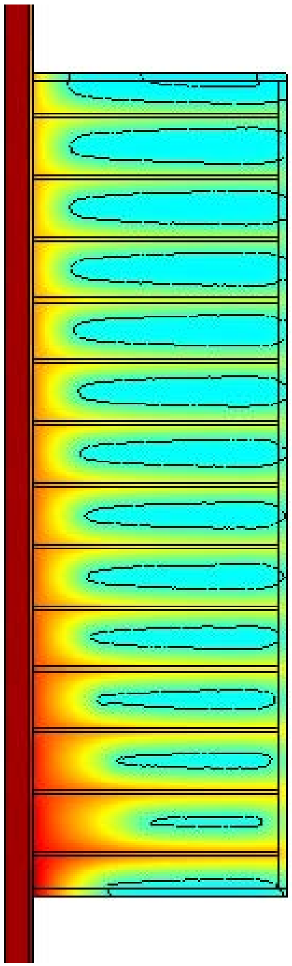
- **Thermochemical Storage:**

  Storage of energy is the result of a chemical reaction

- **Latent Heat Storage:**

  The storage of energy is the result of the phase change (solid-liquid or solid-solid) of a phase change material (PCM). The process happening over a small temperature range.

Validation's Goal



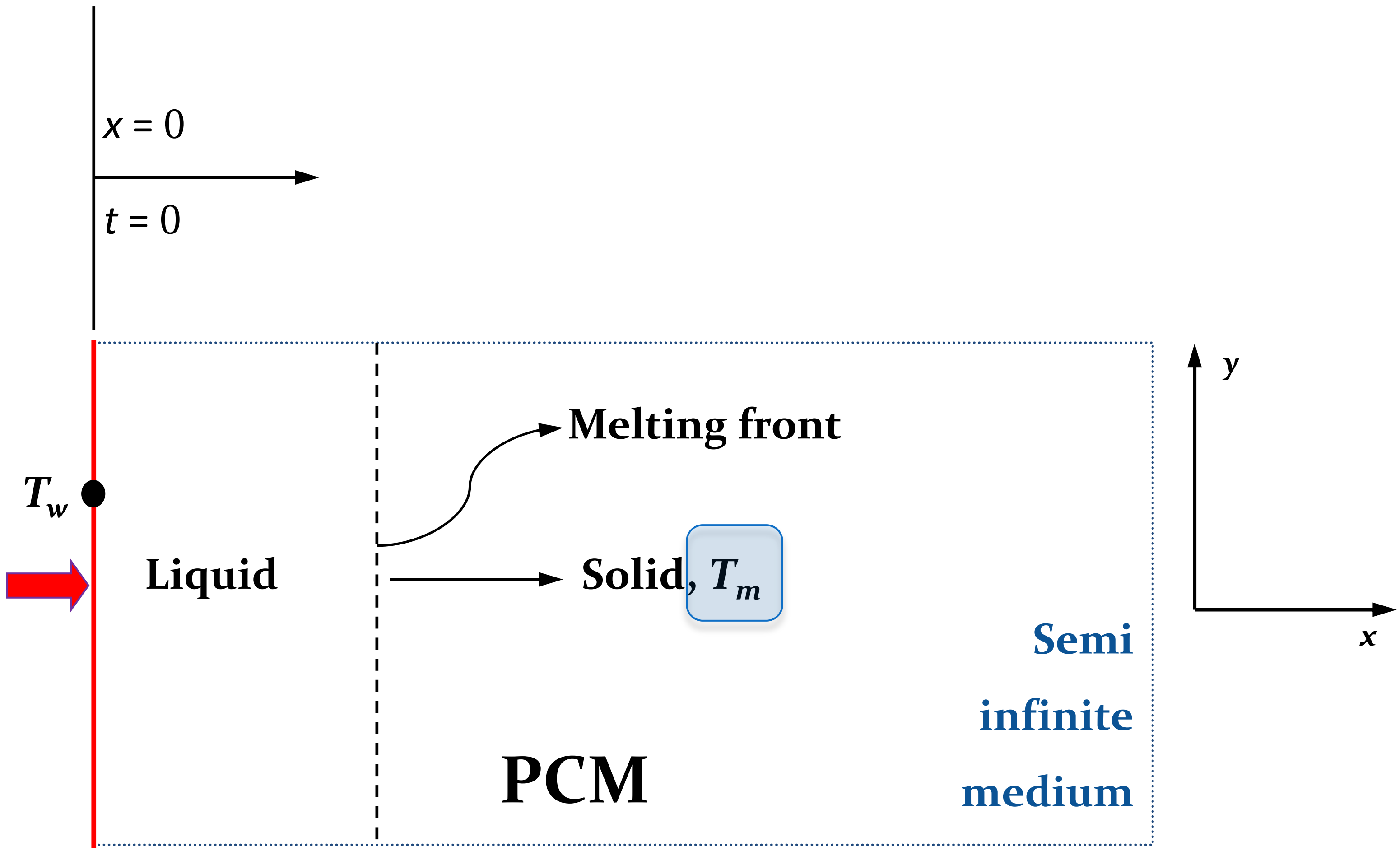
- Finite Elements can be used to help in the design of Latent Heat Energy Storage Systems (LHESS):
  - Determination of the application-dependent size of the LHESS;
  - Choice of geometry;
  - Heat Transfer enhancement (fins for example);
  - Etc ...

- A proper validation of the phase change behavior of the Phase Change Material (PCM) inside the LHESS is necessary to ensure the proper numerical treatment, especially when it comes to accounting for the total amount of energy stored in the system.

Geometry Studied

- 1D Stefan's Problem

Geometry



$x = 0$
$t = 0$
Melting front
T_w
Liquid
Solid, T_m
Semi
infinite
medium
PCM
y
x

Analytical Solution

- 1D Stefan's Problem

Governing Equations

- Heat Conduction in the liquid phase:

$$\left(\rho C_p\right)_l \frac{\partial T_l}{\partial t} = k_l \frac{\partial^2 T_l}{\partial x^2}$$

- Boundary Conditions:

$$T\left(x = 0, t > 0\right) = T_w$$

$$T\left(x > \delta(t), t > 0\right) = \boxed{T_m}$$

with $\delta(t)$ being the solid-liquid interface position.

- Energy balance at the melting interface:

$$-\rho_l L \frac{d\delta(t)}{dt} = k_l \frac{\partial T_l\left(\delta, t\right)}{\partial x} = -\rho_l L \boxed{u_m}$$

Analytical Solution

- Solving the previous equations results in:

$$\frac{T_l(x,t)-T_w}{T_m-T_w}=\frac{erf[x/2\sqrt{\alpha_l t}]}{erf(\beta)}=\frac{erf(\eta)}{erf(\beta)}$$

with $\eta = {x}/{2\sqrt{\alpha_l t}}$

and β determined by solving the following equation:

$$\beta e^{\beta^2} erf(\beta)=\frac{Ste}{\sqrt{\pi}}$$, Ste = $Stefan$ number = $\frac{C_{p,l}(T_w-T_m)}{L}$

Analytical Solution

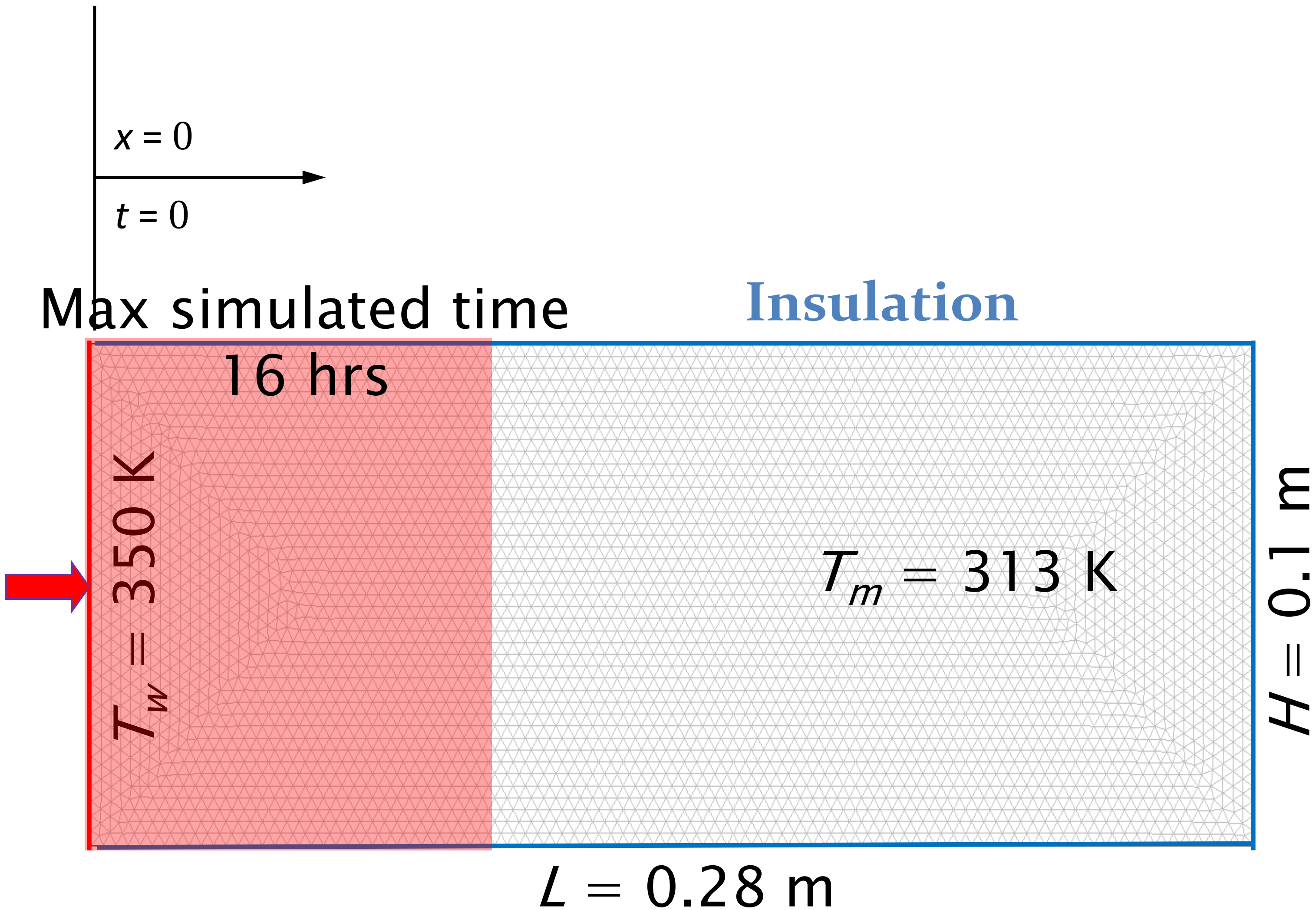
- The following can also be obtained analytically:
  - Melting front position: $\delta(t) = 2\eta\sqrt{\alpha_l t}$
  - Melting front velocity: $u_m(t) = \dfrac{d\delta(t)}{dt} = \dfrac{\eta\sqrt{\alpha_l}}{\sqrt{t}}$
  - Heat transfer rate at the solid-liquid interface:

$$q''[\delta(t)] = k_l \left.\frac{\partial T_l}{\partial x}\right|_{x=\delta(t)} = -\rho_l L \frac{\eta\sqrt{\alpha_l}}{\sqrt{t}}$$

Numerical Modeling

2D Stefan's Problem

Geometry



x = 0
t = 0
Max simulated time
16 hrs
Insulation
$T_w = 350$ K
$T_m = 313$ K
$H = 0.1$ m
$L = 0.28$ m

PCM

The phase change material used in the validation study is Paraffin wax

Thermal Conductivity	0.21 W/m·K
Heat Capacity	2.4 kJ/kg·K
Density	750 kg/m^3
Enthalpy of Fusion	175 kJ/kg
Melting Temperature Range	313 K to 316 K

Modeling in COMSOL

- Problem type: Transient thermal fluid*

- Model used: Heat Transfer in a Solid

  ↳ Transient Analysis

  This model encompasses:
  - Heat transfer by conduction.

    ↳ In Stefan's Problem, convection is neglected in the liquid PCM
  - Modified using the Effective Heat Capacity Method.

- Geometry is considered 2D

---

* The treatment of phase change renders the problem non-linear as well.

Modified C_p Method

$$C_p = \begin{cases} C_{p,s} & T < 313\ K \\ C_{p,eff} & 313\ K < T < 316\ K \\ C_{p,l} & T > 316\ K \end{cases}$$

$C_{p,eff}$ = Effective C_P
= 60.5 kJ/kg

$C_{p,s}$ = Solid phase C_P
= 2.4 kJ/kg

$C_{p,l}$ = Liquid phase C_P
= 2.4 kJ/kg

L = Latent heat of fusion
= 175 kJ/kg

ΔT_m = Melting Temperature range

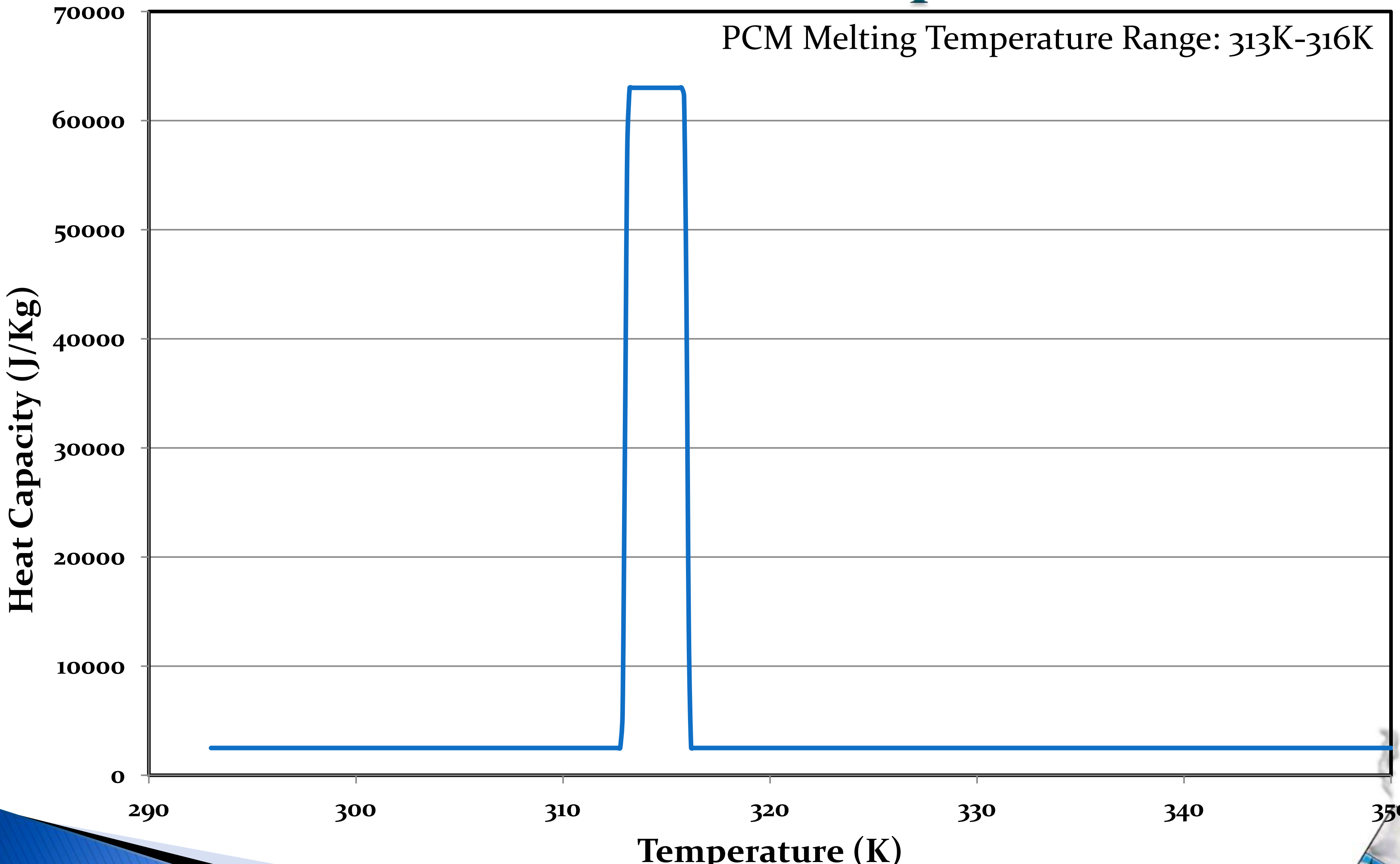
Where

$$C_{p,m} = \frac{L}{(\Delta T_m)} + \frac{(C_{p,s} + C_{p,l})}{2}$$

Numerically

$$C_p = (2.5 + 60.5 * (313 < T) - 60.5 * (T > 316))$$

Modified C_p

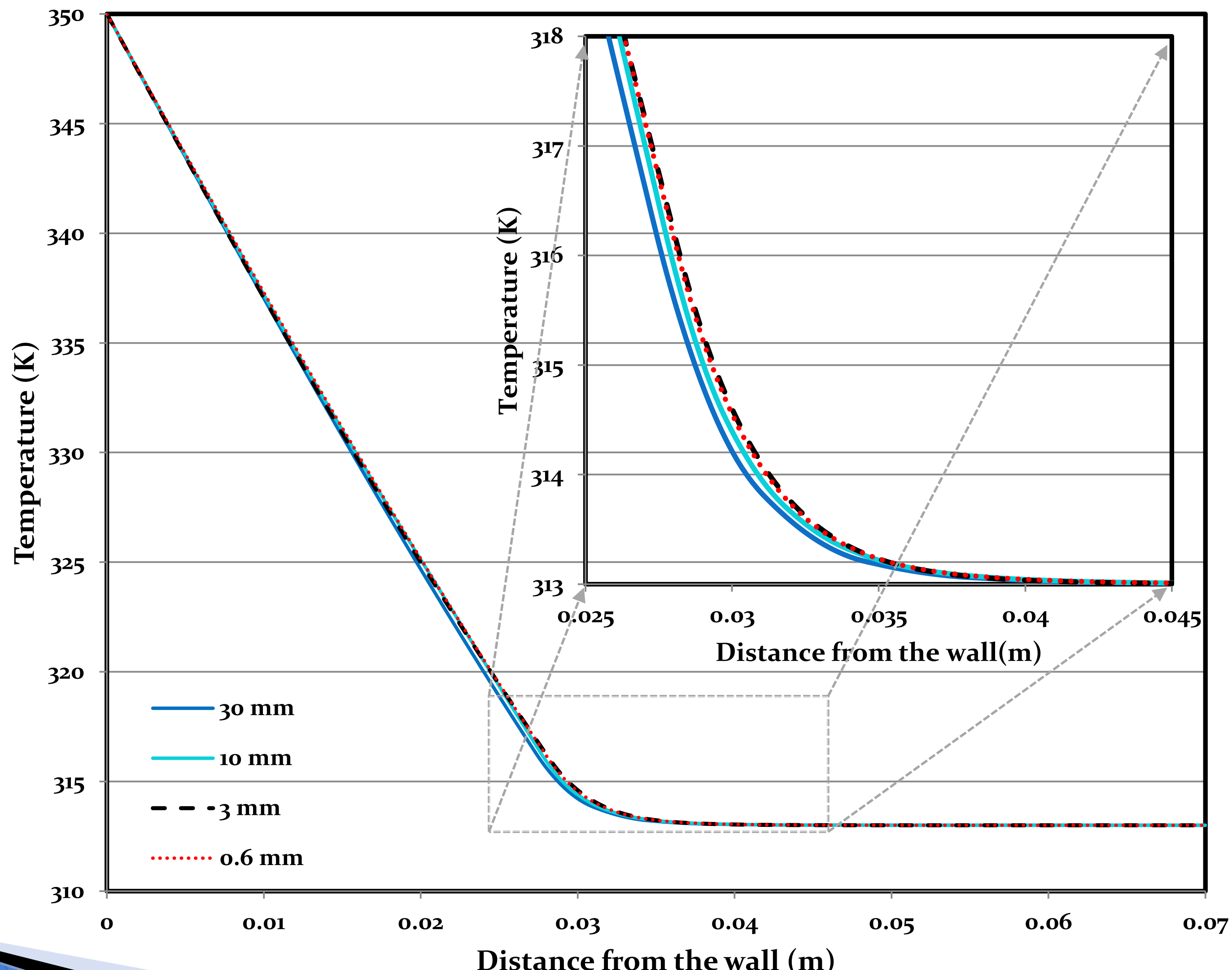


PCM Melting Temperature Range: 313K-316K
70000
60000
50000
40000
30000
20000
10000
0
Heat Capacity (J/Kg)
290
300
310
320
330
340
350
Temperature (K)

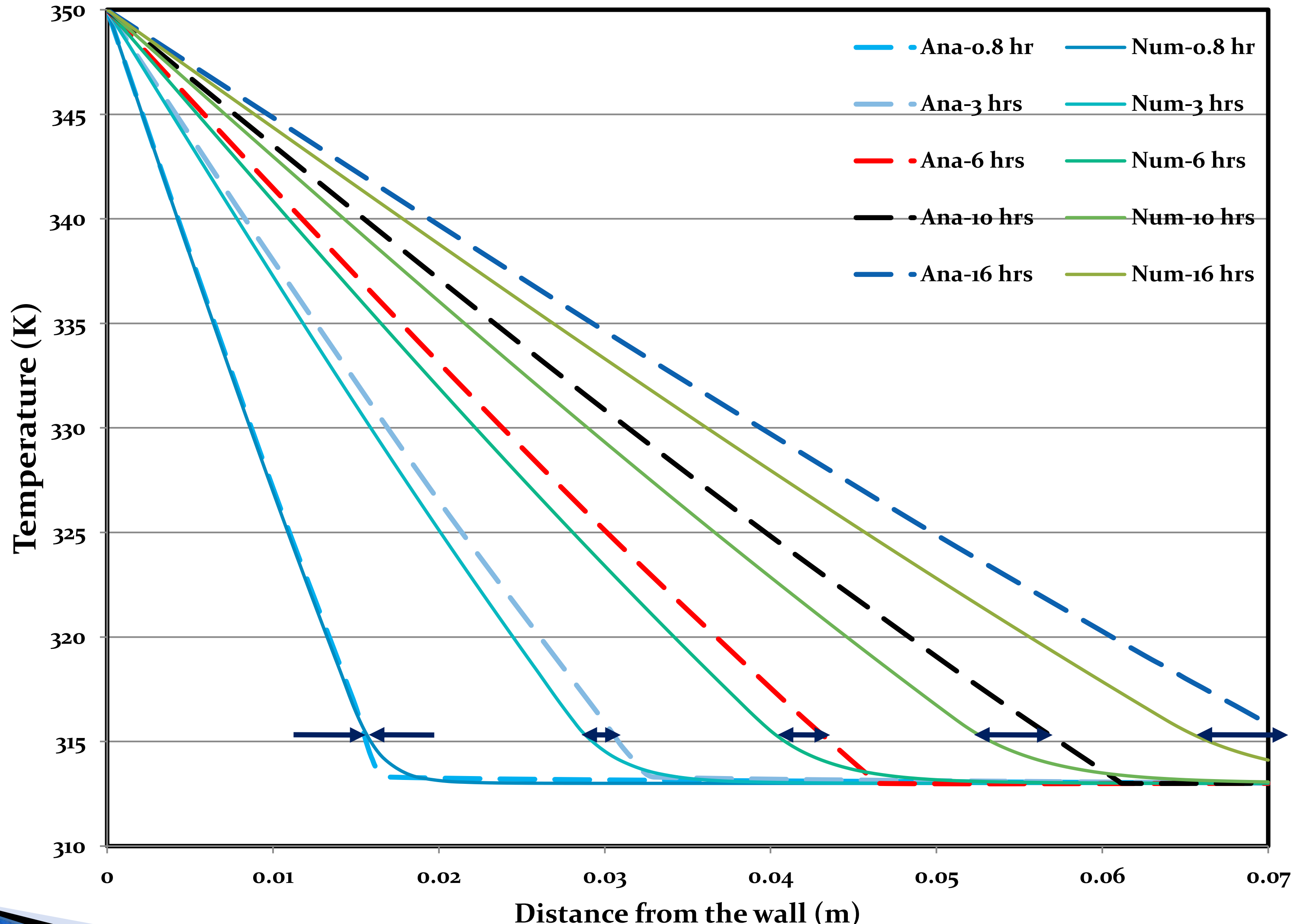
Validation

- Stefan's Problem

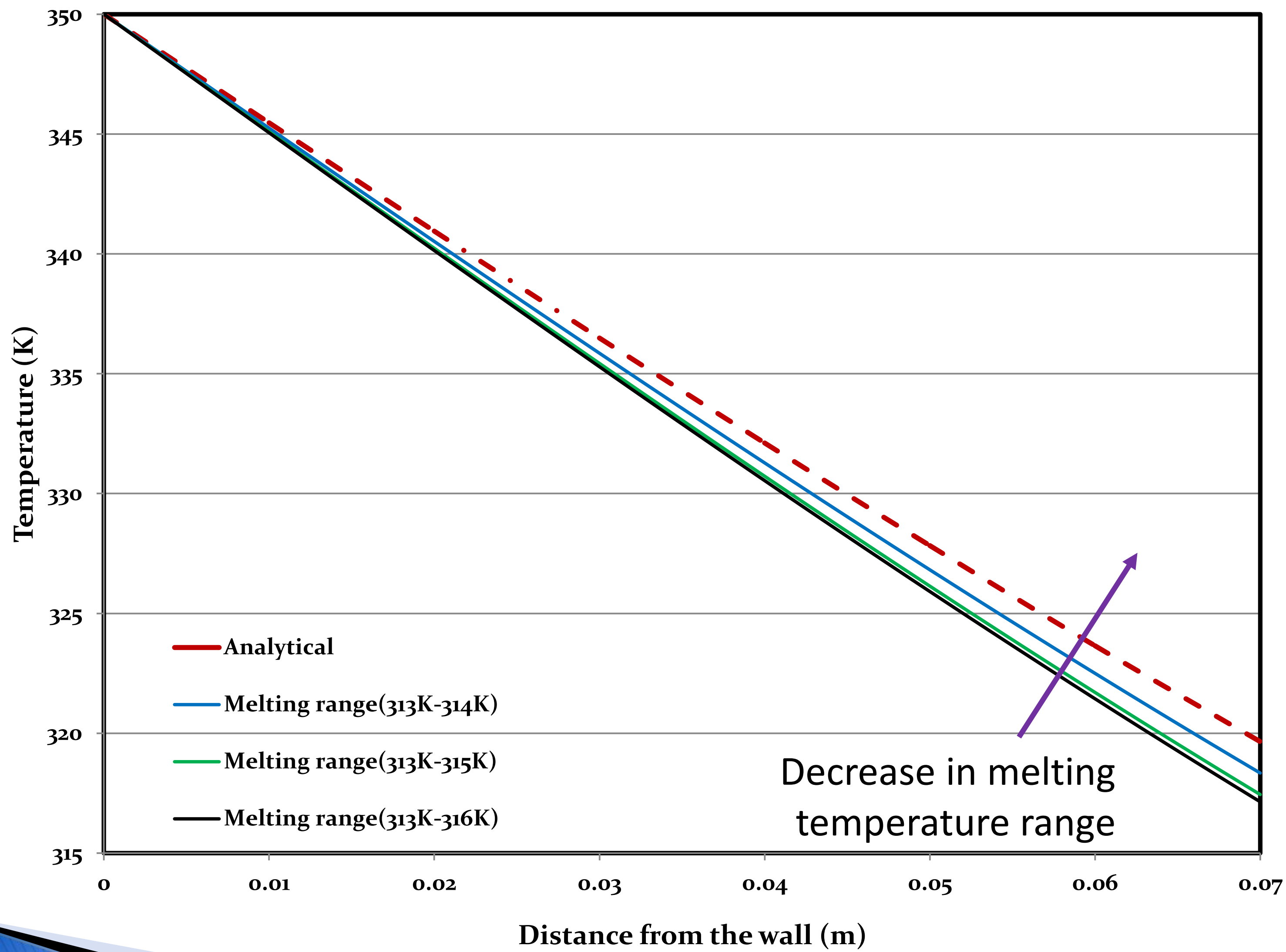
Element Size



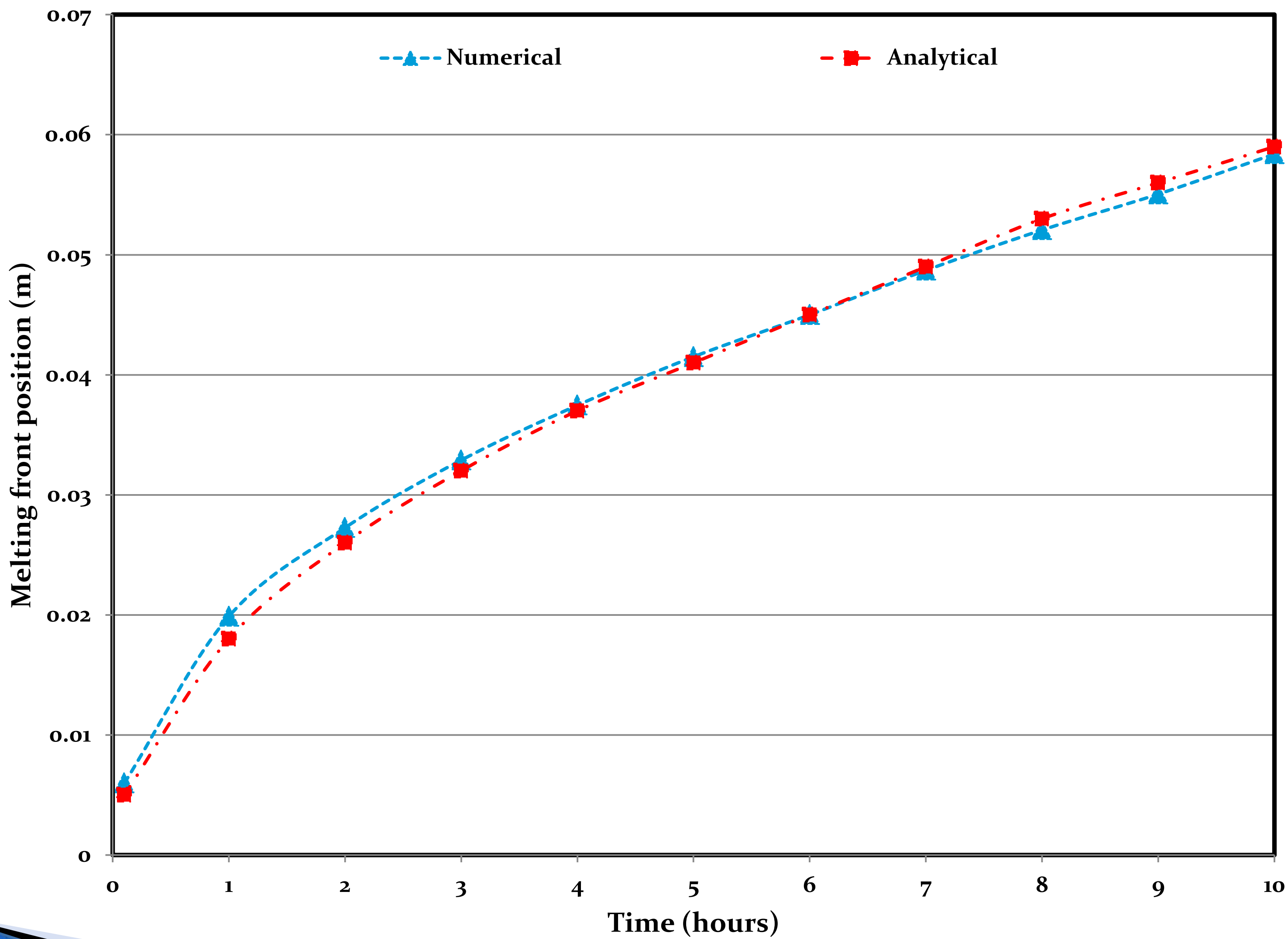
Numerical vs Analytical



Effect of Melting Temperature Range



Melting Front Position



Conclusion

- The physical processes encountered during **transient phase change heat transfer**, coupled with conduction, in a PCM can be modeled numerically using COMSOL Multiphysics;

- The appearance and the behavior of the melting front can be simulated by **modifying the specific heat of the PCM** to account for the increased amount of energy, in the form of latent heat of fusion, needed to melt the PCM over its melting temperature range.

- The validation showed the effect incorporating a **mushy region** in the physical modeling of the PCM had on the temperature profile in the liquid PCM and the melting front behavior.