Optical Properties Modeling of Superconducting Photonic Crystals Using COMSOL Multiphysics

(以COMOSL Multiphysics模擬超導光子晶體之光學性質)

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Outline

- Papers simulated by RF module of COMSOL Multiphysics 3.5a
- Introduction of photonic crystals
- Finite element method
- Maxwell's equation in two-dimensional photonic crystal system
- Two-fluid model
- Modeling examples of superconducting photonic crystals
  - Transmittance spectra in one-dimensional superconductor-dielectric photonic crystal (1D case)
  - Tunable optical properties of a superconducting Bragg reflector (2D case)
  - Tunable resonant spectra through nanometer niobium grating on silicon nitride membrane (3D case)
- Summary



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歡迎光臨奈米元件暨低溫實驗室！
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主要研究項目：
探究低維度系統的物理現象，透過奈米微影術配合後續的製程，實驗室針對半導體、磁性材料、光子晶體、相變化材料等之微米及奈米元件，進行製作及特性分析，從而研究其相關的物理機制，進而開展其相關之實際應用。
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5. MNE 2010 (09/19)
6. ISAMMA 2010(07/12)
7. EIPBN (06/01)
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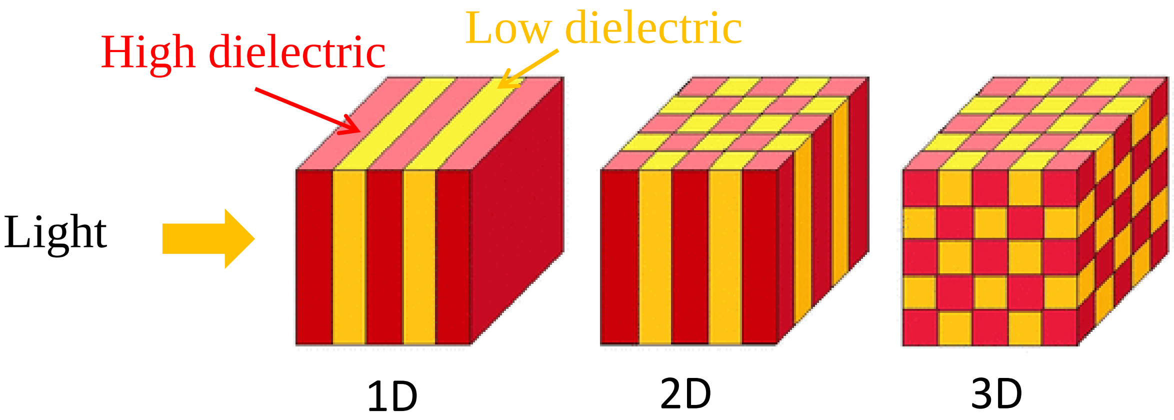
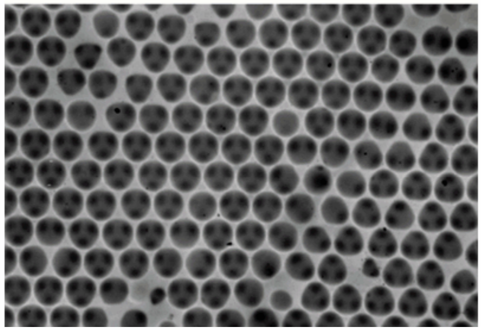
Papers simulated by RF module of COMSOL Multiphysics 3.5a

1. **Huang-Ming Lee,** Jia-Hong Shyu, Lance Horng, Jong-Ching Wu, "Tunable optical properties of a superconducting Bragg reflector", submitted to Thin Solid Films **(2010)** **(Invited talk at ICAUMS 2010, December 5-8, Jeju Island, Korea)**
2. **Huang-Ming Lee**, Lance Horng, Jong-Ching Wu, "Magnetic-field tunable transmittance in a ferrofluid-filled silicon nitride photonic crystal slab", Journal of Physics D: Applied Physics **(2010)** (SCI, IF= 2.083) **in Press** **(Invited talk at ISAMMA 2010, July 12-16, Sendai, Japan)**
3. Jia-Hong Shyu, Jui-Hsing Chien, **Huang-Ming Lee**, Jong-Ching Wu, "The waveguide-plasmon resonance in the gold-capped silicon nitride rods photonic crystal slab", Journal of Vacuum Science and Technology B **(2010)** (SCI, IF = 1.46) **in Press**
4. **Huang-Ming Lee**, Kartika Chandra Sahoo, Yiming Li, Jong-Ching Wu, Edward Yi Chang, "Finite element analysis of antireflective silicon nitride sub-wavelength structures for solar cell applications", Thin Solid Films 518, 7204-7208 **(2010)** (SCI, IF = 1.727)
5. **Huang-Ming Lee**, Chu-Ying Lin, Lance Horng, Jong-Ching Wu, "Tunable resonant spectra through nanometer niobium grating on silicon nitride membrane", Journal of Applied Physics 107, 09E119 **(2010)** (SCI, IF = 2.072)
6. **Huang-Ming Lee**, Jong-Ching Wu, "Transmittance spectra in one-dimensional superconductor-dielectric photonic crystal", Journal of Applied Physics 107, 09E149 **(2010)** (SCI, IF = 2.072)
7. Neil Ou, Jia-Hong Shyu, **Huang-Ming Lee**, Jong-Ching Wu, "Diameter-dependent guided resonance of dielectric hole-array membrane", Journal of Vacuum Science and Technology B 27, 3183-3186 **(2009)** (SCI, IF = 1.46)

What are photonic crystals?

Photonic crystals are periodic systems that consist of separate high dielectric and low dielectric regions. The periodicity or spacing determines the relevant light frequencies.





Lattice of air spheres in titaniamatrix
Wijnhoven& Vos, *Science*, 1998)

Ho,Chan,Soukoulis, (1990) –predicted dielectric spheres in diamond structure should have a band gap.

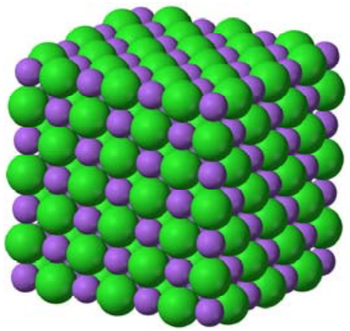
Yablonovitch(1991) First photonic crystal with microwave band gap.

"In the course of four years, my loyal machinist, John Gural, drilled more than 500,000 holes in dielectric plates…It became unnerving as we produced failure after failure."(Yablonovitch, Scientific American, 1991)

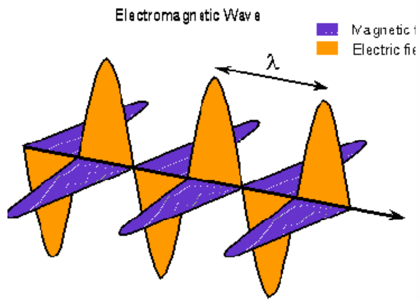
Ref from J. D. Joannapolous et al, Princeton University Press, 1995

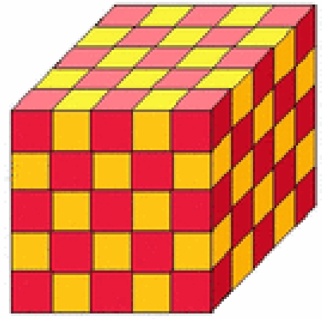
Analog to solid state physics

e^- →



Crystal





Photonic Crystal (Matrix and spheres have different dielectric properties)

- Electrons scatter in the periodic lattice
- **Schrodinger's equation Hψ= Eψ**
- Interacting particles
- Solve approximately–plane waves, Multiple scattering theory,…

Band Diagram -Electron standing waves
Allowed energies (bands)
Forbidden energies (band gaps)

- Photon scatters in periodic lattice
- **Maxwell's equations**
- Non-interacting particles
- Solve exactly-plane waves, Multiple scattering theory,…

Band Diagram -standing waves
Allowed frequencies (bands)
Forbidden frequencies (band gaps)

What can you do with a photonic crystal?

- **Trap Light**

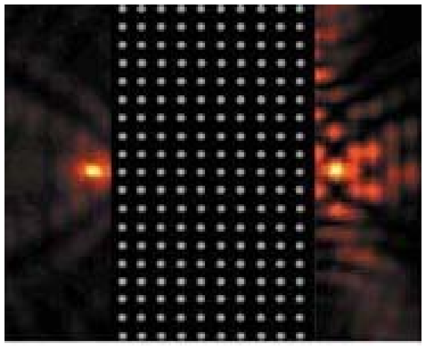
A single defect in a photonic crystals acts like a resonant cavity with a defect level in the band gap.

- **Right turns with photons**

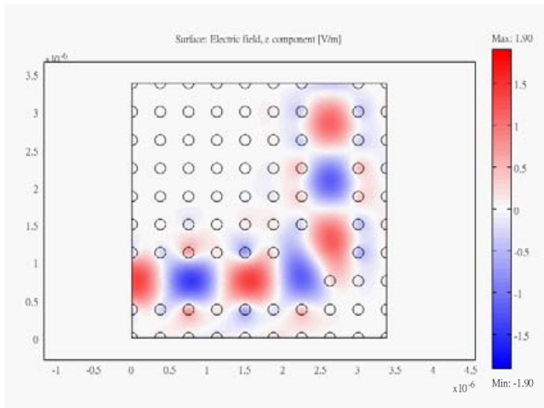
Photonic crystals prevent photons in the band gap from propagating in the material.

If we create a line defect in the structure, it will act like a waveguide.

- **Negative index of refraction –Flat lens   …and much more**



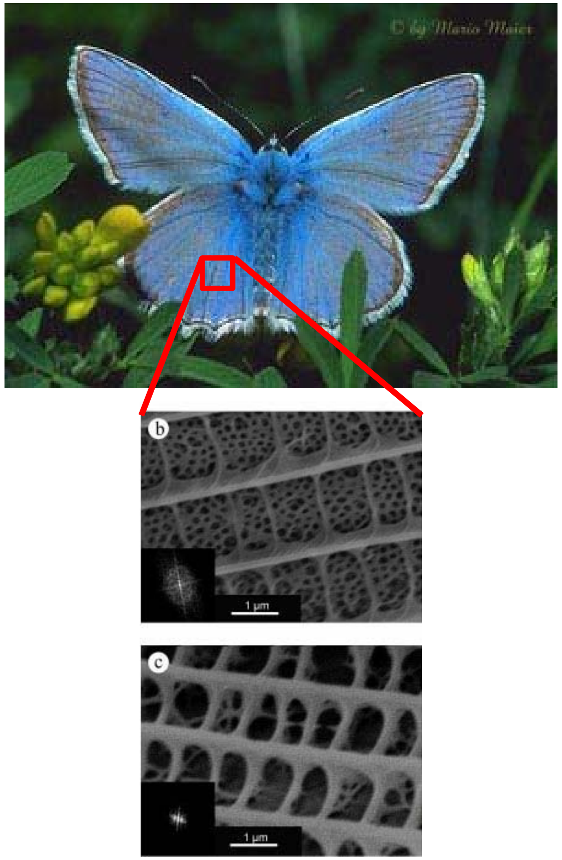
Parimiet al., Nature **426** (2003) 404



COMSOL example

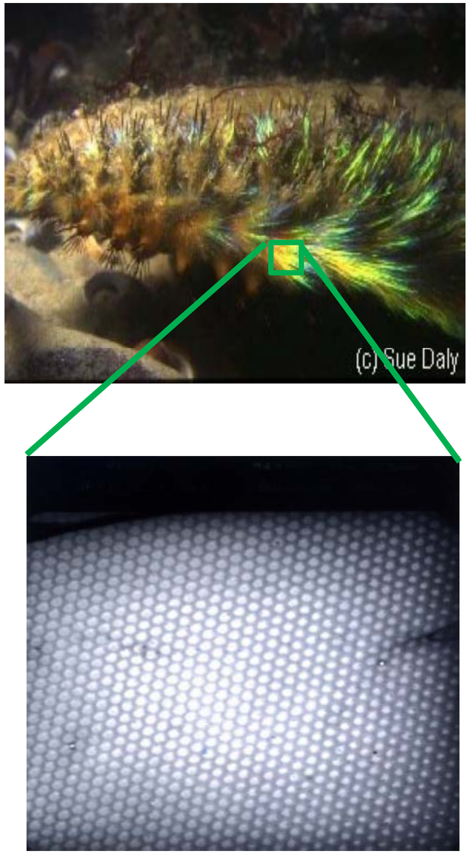
Photonic crystals found in nature

Butterfly



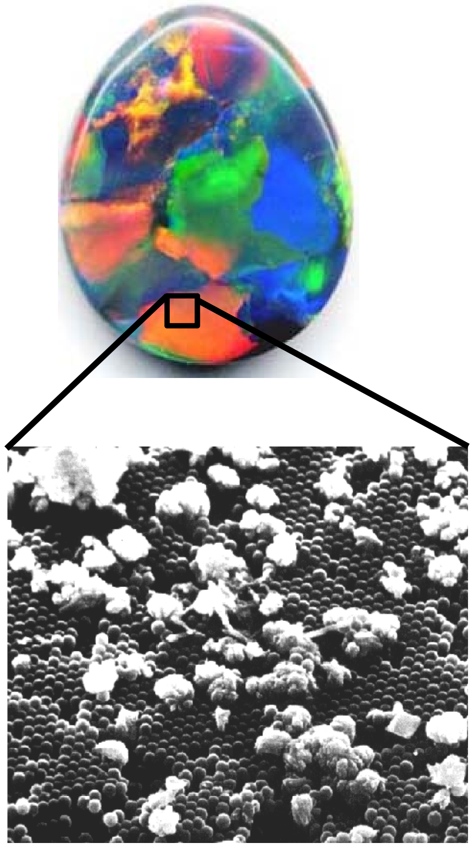
Biro et al., Phys. Rev. E (2003)

Sea mouse



Parker et al., Nature (2001)

Opal



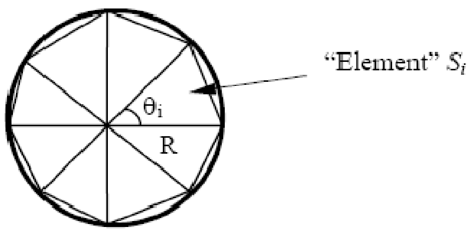
http://www.cmth.ph.ic.ac.uk/photonics

Basic concepts of finite element method

- The finite element method(FEM), or finite element analysis(FEA), is based on the idea of **building a complicated object with simple blocks**, or, **dividing a complicated object into small and manageable pieces**. Application of this simple idea can be found everywhere in everyday life, as well as in engineering.

Examples:

- Lego (kids' play)
- Buildings
- Approximation of the area of a circle:



Area of one triangle: $S_i = \frac{1}{2} R^2 \sin\theta_i$

Area of the circle: $S_N = \sum_{i=1}^{N} S_i = \frac{1}{2} R^2 N \sin(\frac{2\pi}{N}) \rightarrow \pi R^2$ as $N \rightarrow \infty$

Where N = total number of triangles (elements)
Observation: Complicated or smooth objects can be represented by geometrically simple pieces (elements).

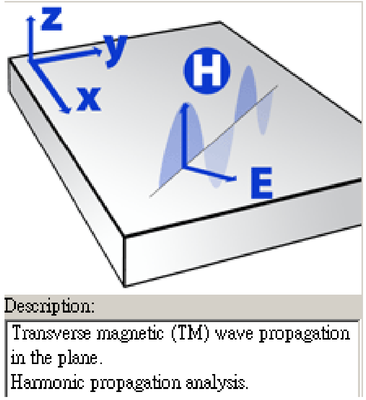
Maxwell's equation in two-dimensional photonic crystal system

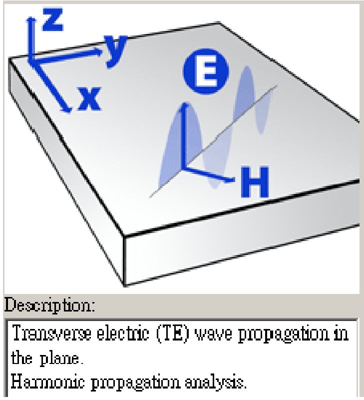
$$\nabla \times (\varepsilon_r^{-1} \nabla \times H_z) = (\frac{\omega}{c})^2 H_z$$

$$\nabla \times (\nabla \times E_z) = \varepsilon_r (\frac{\omega}{c})^2 E_z$$

where ε_r is the relative permittivity and is equal to n^2, in which n is the refractive index.

➢ COMSOL use contrary convention as compared with published papers.





Two-fluid model

◆*Conductivity of Superconductor (normal electron and superconducting electron)*

$$\sigma = \sigma_1 - j\sigma_2$$

◆*Lossless conductivity and relative permittivity of superconductor*

$$\sigma = -j\frac{1}{\omega\mu_0\lambda(T)^2}$$

$$\varepsilon_r = 1 - [\frac{c}{\omega\lambda(T)}]^2$$

$$\sigma_2 = \frac{1}{\omega\mu_0\lambda(T)^2}$$

Where

$$\lambda(T) = \frac{\lambda_0}{\sqrt{[1-G(T)]}}$$

$$G(T) = \left(\frac{T}{T_c}\right)^p$$

P = 2 for high T_c superconductor
P = 4 for low T_c superconductor

M. Tinkham, *Introduction to Superconductivity*, 2nd ed. McGraw-Hill, New York, 1996.

Modeling example 1 (1D case)

JOURNAL OF APPLIED PHYSICS **107**, 09E149 (2010)

Transmittance spectra in one-dimensional superconductor-dielectric photonic crystal

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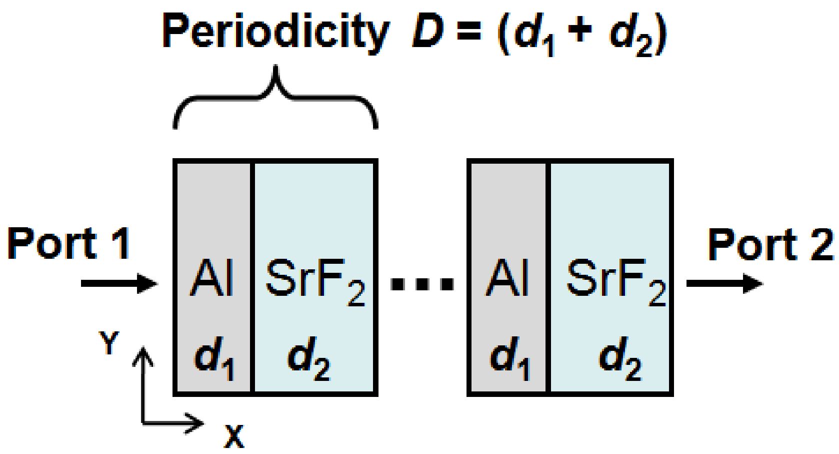
(Presented 19 January 2010; received 27 October 2009; accepted 5 January 2010; published online 12 May 2010)

Transmission characteristics at visible light range in a one-dimensional superconductor-dielectric photonic crystal have been numerically analyzed based on the finite element method using COMSOL RF module. The two-fluid model and wavelength-dependent dispersion formula were adopted to describe the optical response of the low temperature superconducting system. The simulation results clearly reveal a cutoff frequency or a photonic band gap that can be manipulated through the thicknesses of the superconductor and dielectric layers as well as the ambient temperature of the system. It is observed that the shift of cutoff frequency becomes more noticeable by adjusting the thickness of the superconductor layer than that of the dielectric one. Furthermore, the cutoff frequency becomes very sensitive when the system temperature is tuned to close vicinity of the critical temperature of the superconductor.
[doi:10.1063/1.3362935]

Modeling structure



A schematic drawing of superconductor (Al)-dielectric (SrF_2) photonic crystal structure. The thicknesses of Al and SrF2 are denoted as d_1 and d_2, respectively, and the periodicity $D = d_1 + d_2$.

Al:

T_c = 1.18 K, λ_0 = 51.5 nm

SrF_2:

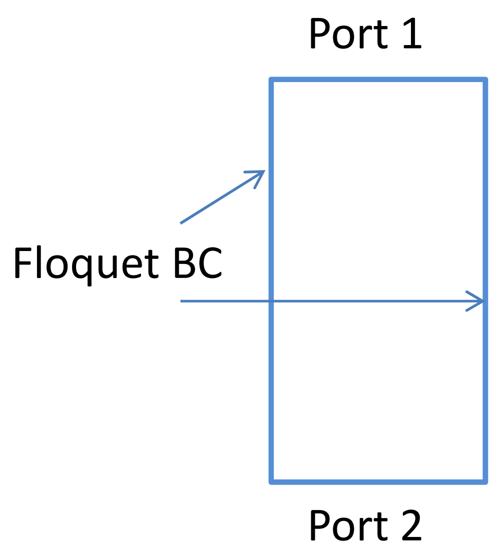
$$n = \sqrt{1 + C_1\lambda^2/(\lambda^2 - C_2^2) + C_3\lambda^2/(\lambda^2 - C_4^2) + C_5\lambda^2/(\lambda^2 - C_6^2)}$$

where C_1 = 0.678, C_2 = 0.056, C_3 = 0.371, C_4 = 0.108, C_5 = 3.848, C_6 = 34.649, respectively.

Boundary conditions

◆ ***Transmission Coefficient (Port BC):***

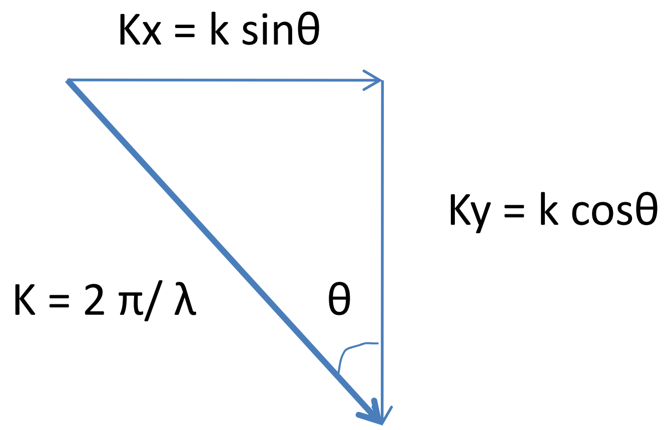
$$S_{21} = \sqrt{\frac{Power\ delivered\ to\ port\ 2}{Power\ incident\ on\ port\ 1}}$$

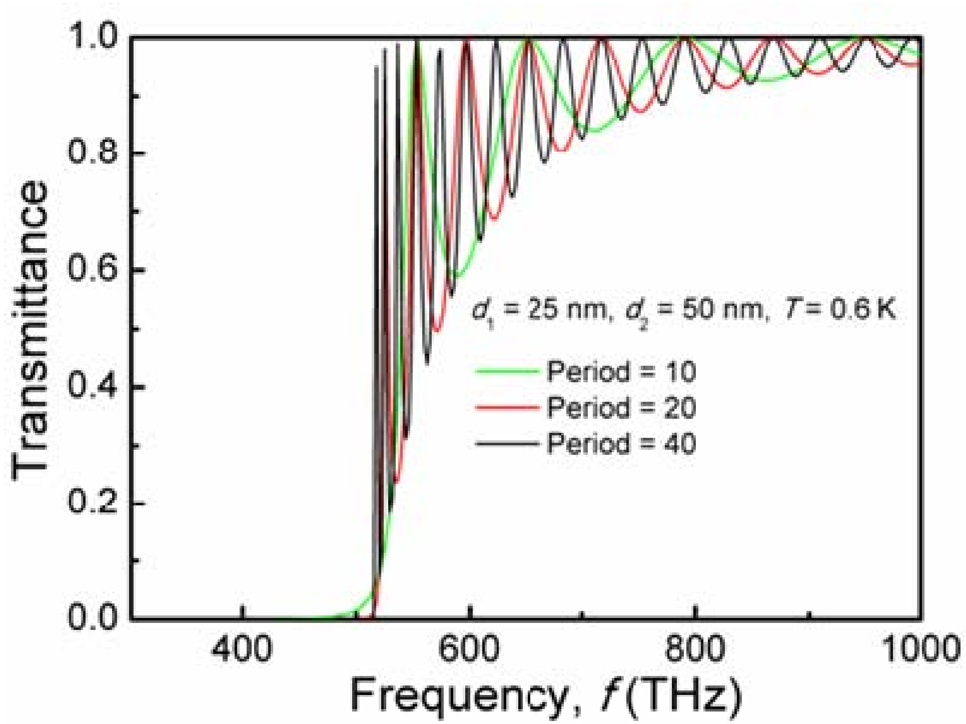


◆ ***Periodic Boundary Condition: Floquet BC***

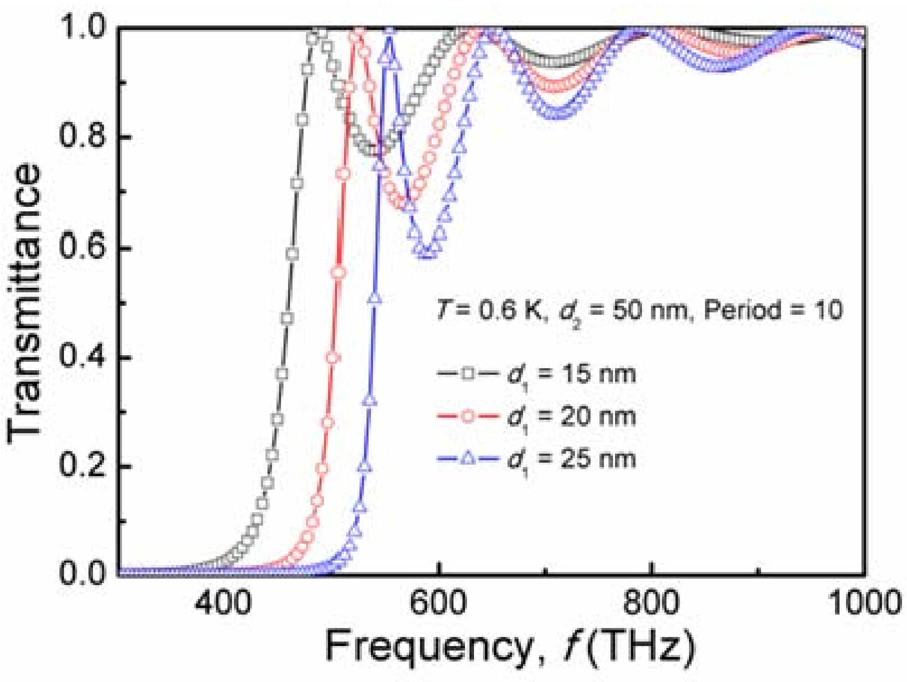
$$E_{dest} = E_{source}\exp[-ik\cdot(r_{dest} - r_{source})]$$

Floquet BC ensures that a wave, when reaching the source BC, is transposed to the destination BC with the appropriate phase shift.

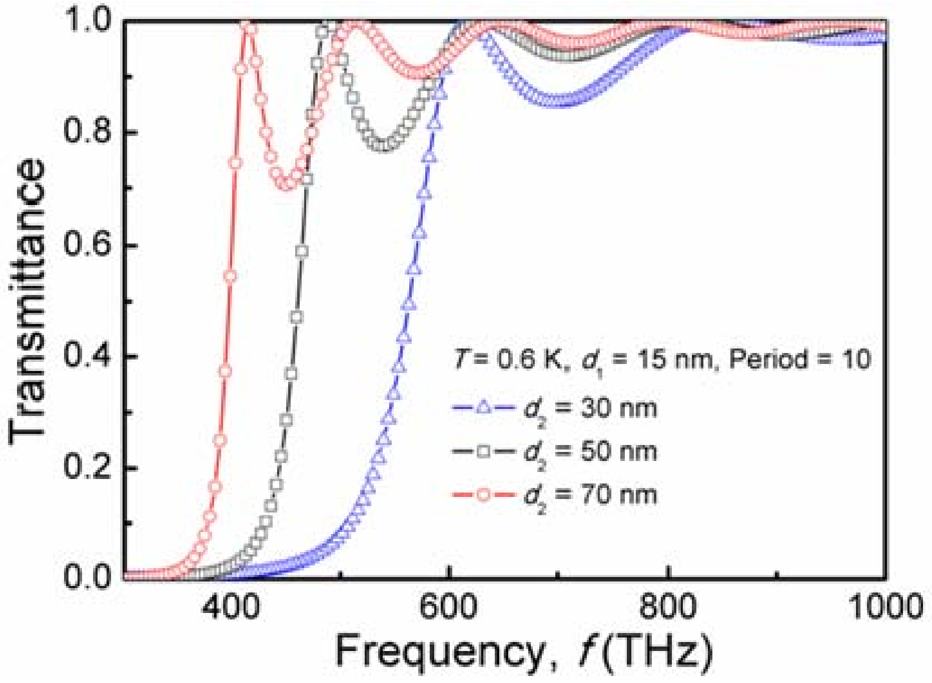




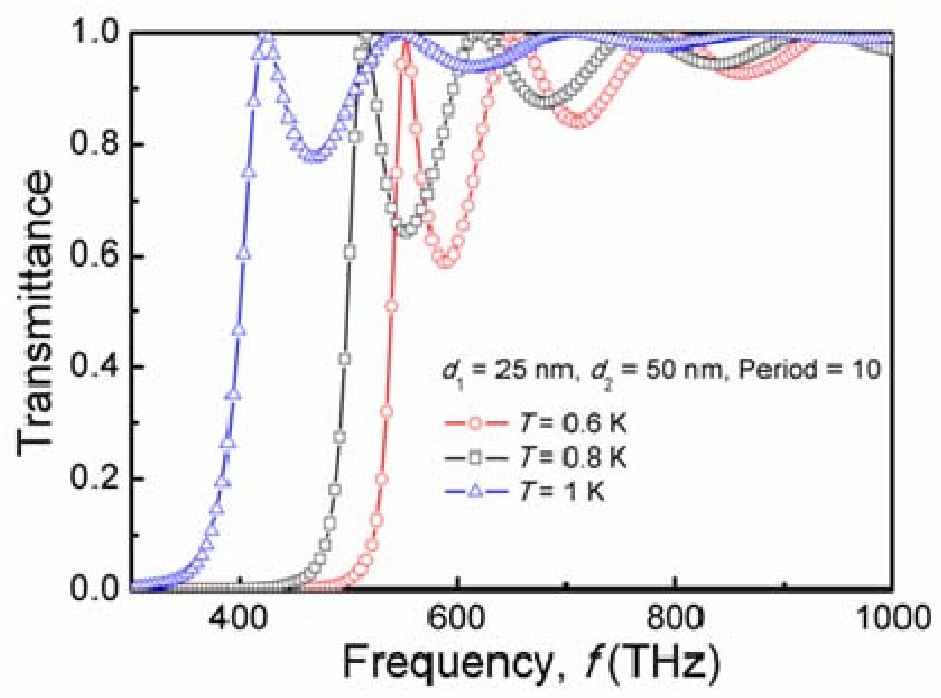
Transmittance spectra simulated on the designed 1D photonic crystal structure with d_1, d_2, and T fixed at 25 nm, 50 nm and 0.6 K, respectively while varying the number of periods from 10 to 40.



Transmittance spectra simulated on the designed 1D PhC structure with d_1 varied to be 15, 20, and 25 nm while d_2 fixed to be 50 nm. The temperature and the number of periods are 0.6 K and 10, respectively.



Transmittance spectra simulated on the designed 1D PhC structure with d_2 varied to be 30, 50, and 70 nm while d_1 fixed to be 15 nm.



Transmittance spectra simulated on the designed 1D PhC structure with temperature T set to be 0.6, 0.8, and 1 K, respectively while d_1 and d_2 fixed to be 25 and 50 nm. The number of periods is 10.

Modeling example 2 (2D case)

Tunable Optical Properties of a Superconducting Bragg Reflector

Huang-Ming Lee, Jia-Hong Shyu, Lance Horng and Jong-Ching Wu*

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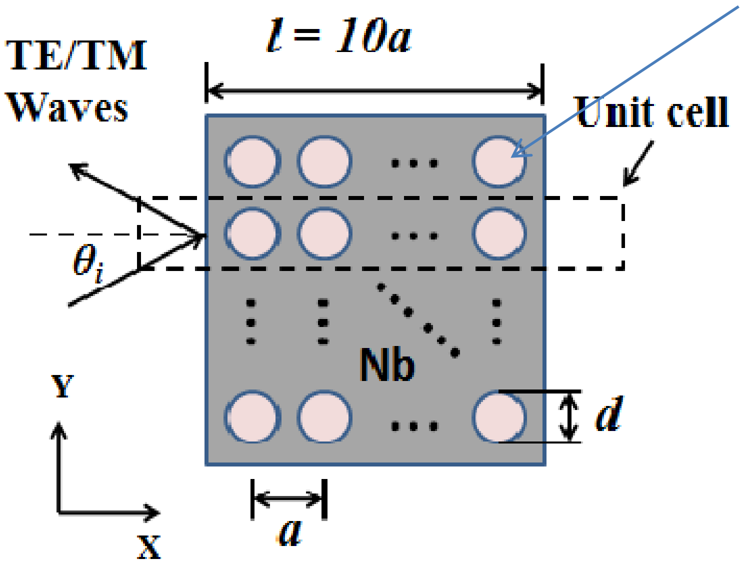
Changhua 500, Taiwan

*phjcwu@cc.ncue.edu.tw

Optical properties of a two-dimensional superconducting Bragg reflector (SBR) are numerically analyzed using the finite element method in conjunction with a two-fluid model. It is found that the wavelength-dependent reflectance spectra of the proposed SBR are strongly dependent on the polarizations of incident light and can be parametrically tuned by the system temperature and the geometric parameters of embedded dielectric rods. Taking advantage of the dispersive superconductor with its zero-refractive index characteristic and the structural periodicity perfectly impedance-matched to the background environment, sharp narrow passband filters can be generated near the threshold wavelength. Furthermore, the narrow passband features and transmittance intensity of the SBR are found to be insensitive to the angle of incidence. This angle-independent property implies that the proposed SBR may be applied to the design of an omnidirectional narrowband transmission filter.

Keywords: Superconductor, Bragg reflector, Filter

Modeling structure



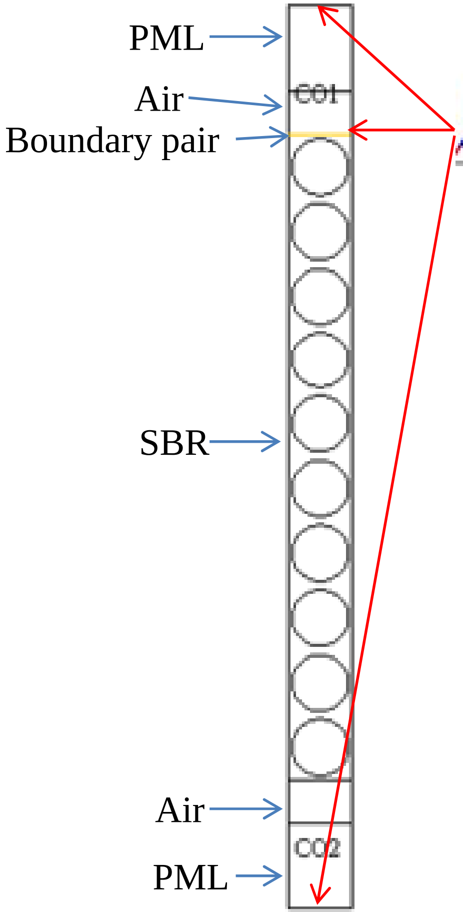
Nb:

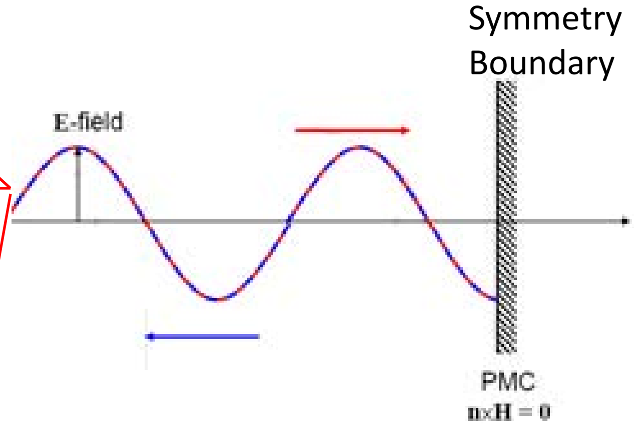
T_c = 9.2 K, λ_0 = 83.4 nm

Dielectric rod (ε_r = 10)

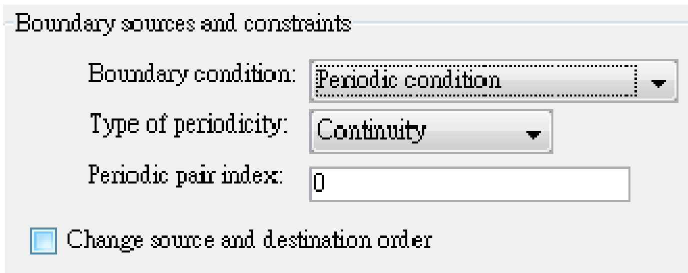
Schematic drawing of the proposed superconducting Bragg reflector (SBR). The diameter and lattice constant of the dielectric rod are denoted as *d* and *a*, respectively. The length of the SBR is set to be *l* = 10*a*. The plane waves with transverse electric (TE) and TM polarizations are incident with an angle of incidence θi. Modeling unit cell is indicated as the dash box.

Model setting

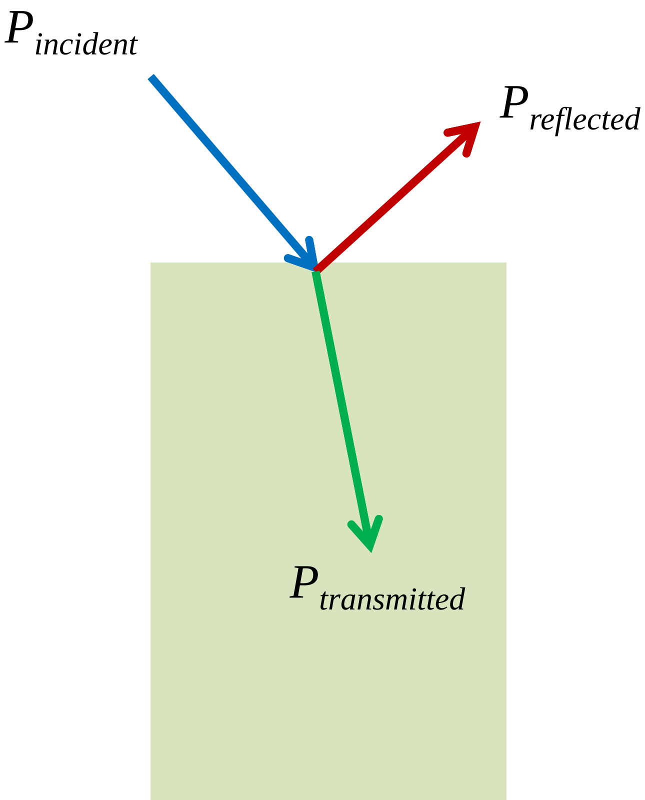




Side boundaries:



Transmittance and reflectance calculations

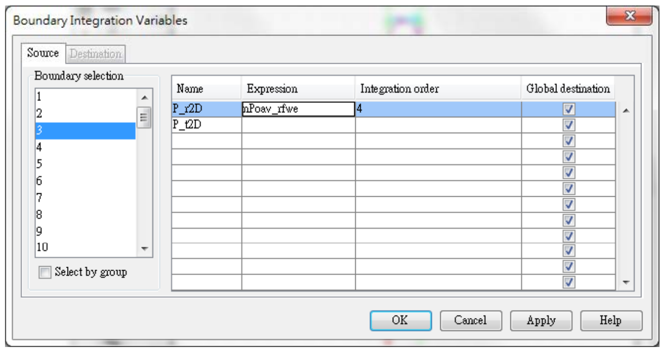


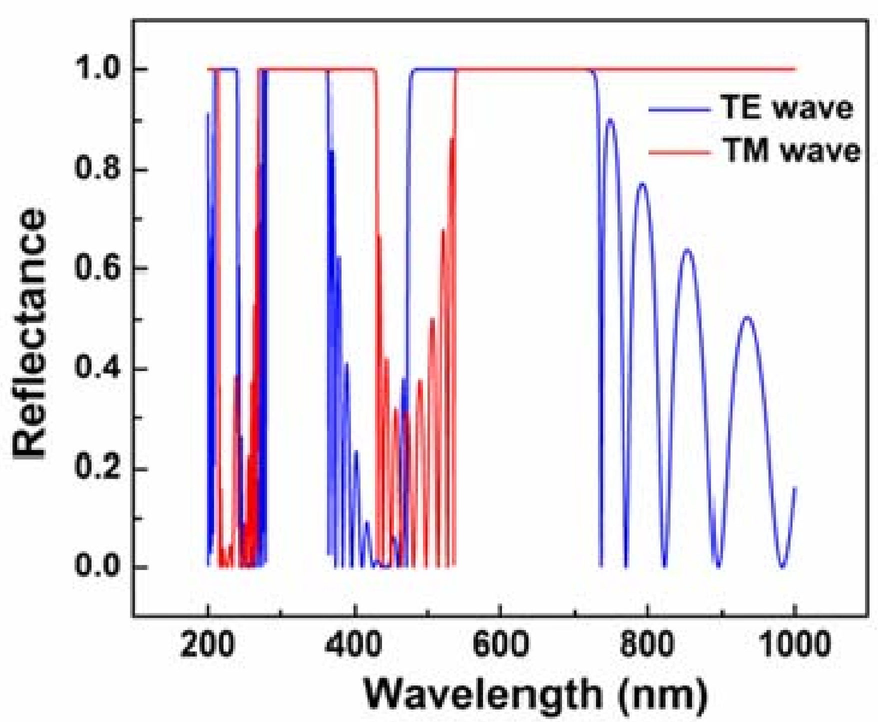
$$P = \int_{boundary} \tfrac{1}{2}\mathbf{n} \cdot \mathrm{Re}\left(\mathbf{E} \times \mathbf{H}^*\right) \partial\Gamma$$

$$R = \frac{P_{reflected}}{P_{incident}}$$

$$T = \frac{P_{transmitted}}{P_{incident}}$$

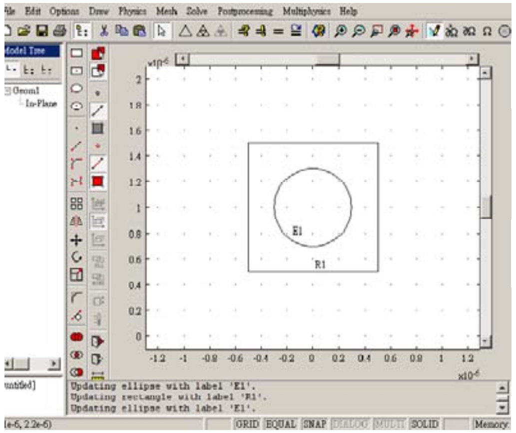
$$R + T = 1$$





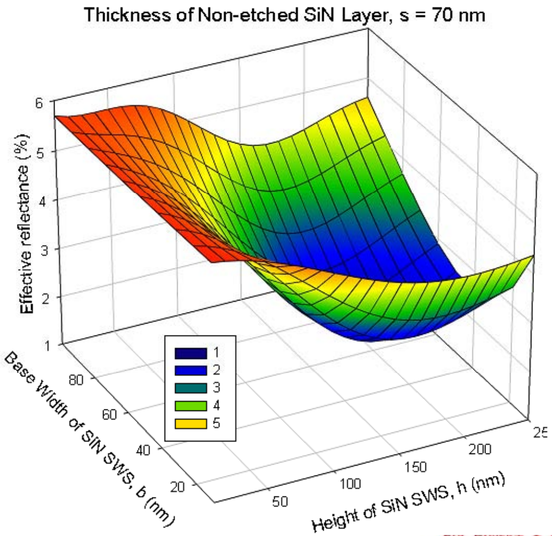
Reflectance spectra of the SBR with plane waves of TE/TM polarizations at T of 4K. The diameter and lattice constant of the dielectric rods are set be d = 100 nm and a = 150 nm, respectively.

Export GUI model to m-file

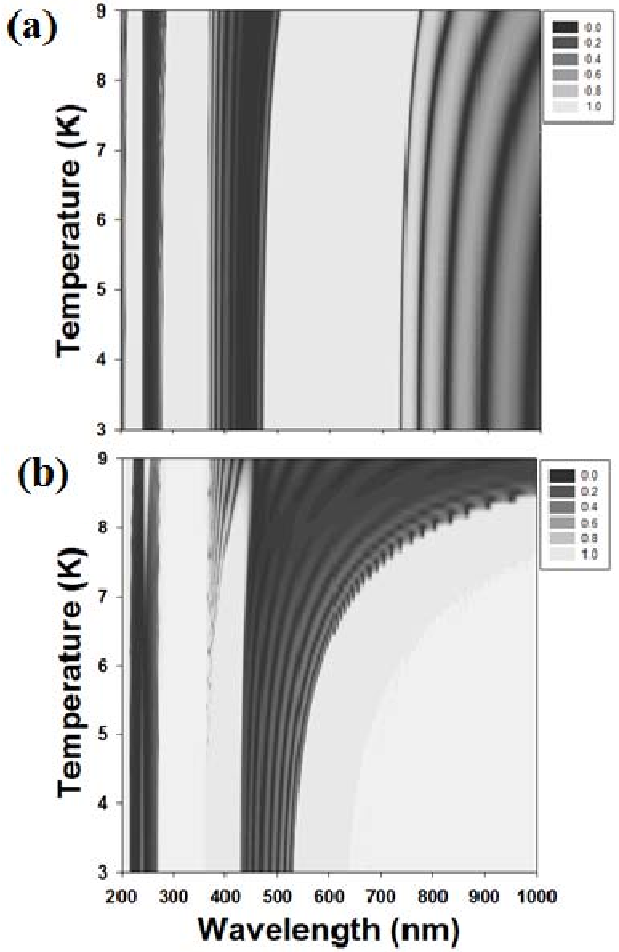



```
% COMSOL Multiphysics Model M-file
% Generated by COMSOL 3.5a (COMSOL 3.5.0.608, $Date: 2009/05/11 07:38:49 $)

flclear fem

% COMSOL version
clear vrsn
vrsn.name = 'COMSOL 3.5';
vrsn.ext = 'a';
vrsn.major = 0;
vrsn.build = 608;
vrsn.rcs = '$Name: v35ap $';
vrsn.date = '$Date: 2009/05/11 07:38:49 $';
fem.version = vrsn;

% Constants
fem.const = {'theta','0', ...
  'T','4[K]', ...
  'Lambda','1e-6[m]'};

% Geometry
g1=rect2(0.2,0.4,'base','corner','pos',[0,0]);
g2=rect2('150e-9','200e-9','base','corner','pos',{'0','0'},'rot','0');
g3=rect2(1.5E-7,1.0E-7,'base','corner','pos',[0,2.0E-7]);
g4=rect2(1.5E-7,1.375E-6,'base','corner','pos',[0,3.0E-7]);
g5=rect2(1.5E-7,1.5E-6,'base','corner','pos',[0,3.0E-7]);
g6=rect2(1.5E-7,1.0E-7,'base','corner','pos',[0,1.8E-6]);
g7=rect2(1.5E-7,2.0E-7,'base','corner','pos',[0,1.9E-6]);
```

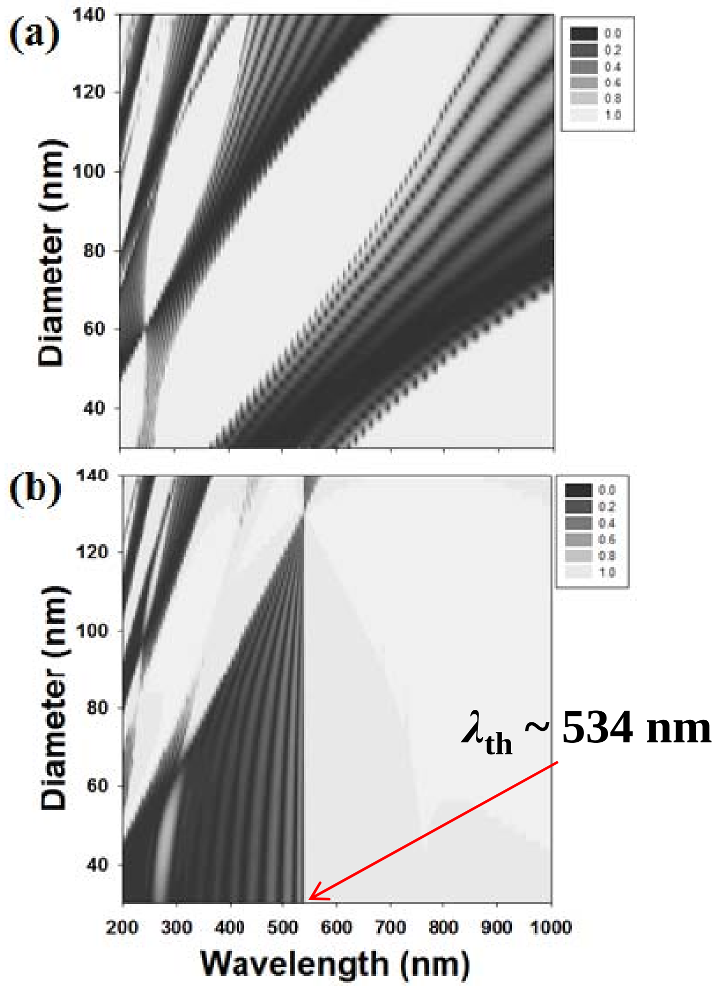
Use m-file to analyze model with more than one parameters.



Contour plot



Contour plots of the TE(a)/TM(b)-polarized wavelength-dependent reflectance spectra of the SBR at different *T* ranging from 3 K to 9 K.



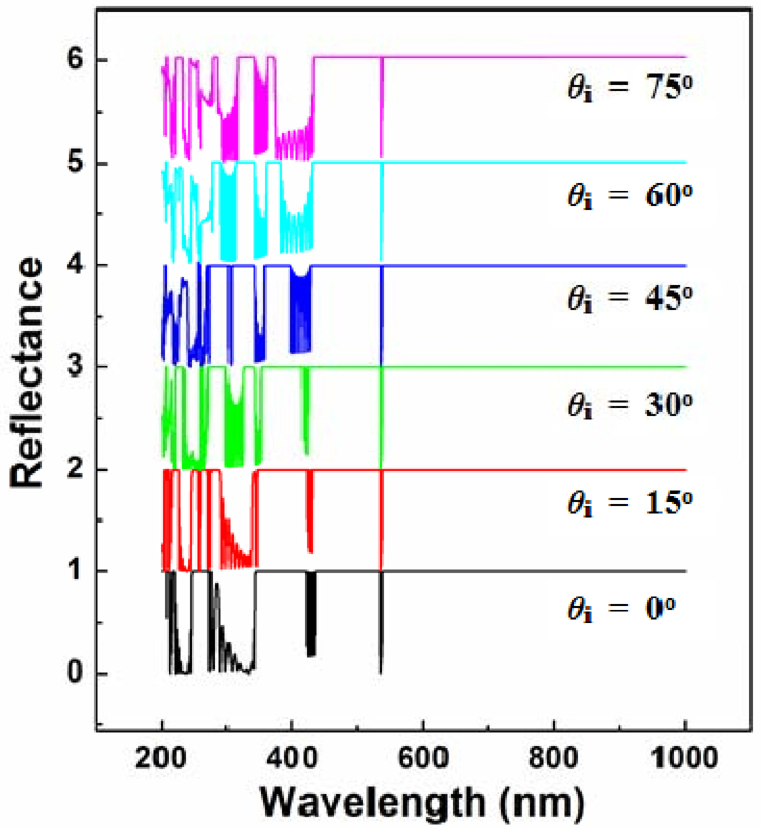
$$\varepsilon_r = 1 - [\frac{c}{\omega\lambda(T)}]^2$$

$$n_{SC} = \sqrt{1 - \frac{c^2[1-(T/T_c)^4]}{\omega^2\lambda_0^2}}$$

If n_{SC} is equal to zero,
we obtain the threshold wavelength

$$\lambda_{th} = \frac{2\pi\lambda_0}{\sqrt{1-(T/T_c)^4}}$$

Contour plots of the TE(a)/TM(b)-polarized wavelength-dependent reflectance spectra of the SBR at different *d* ranging from 30 nm to 140 nm while *T* fixed at 4 K.



TM-polarized wavelength-dependent reflectance spectra of the SBR at different θ_i from 0^o to 75^o. The diameter and lattice constant of the dielectric rods are set to d = 130 nm, and a = 150 nm, respectively while T is fixed at 4 K.

Modeling example 3 (3D case)

JOURNAL OF APPLIED PHYSICS **107**, 09E119 (2010)

Tunable resonant spectra through nanometer niobium grating on silicon nitride membrane

Huang-Ming Lee, Chu-Ying Lin, Lance Horng, and Jong-Ching Wu[a)]
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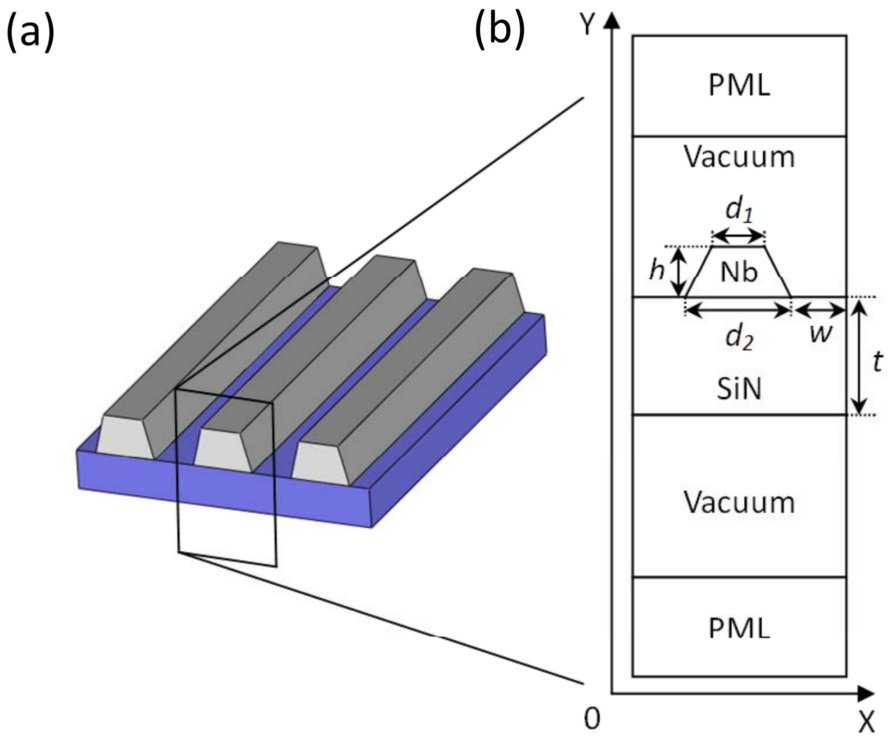
Transmission characteristics at visible light range in a designed superconducting niobium grating on the silicon nitride membrane have been numerically analyzed based on the finite element method in conjunction with a two-fluid model. The niobium strips are premeditated to possess a trapezoid cross section, giving rise to an extra tuning parameter of top/bottom width. The simulation results clearly reveal that the resonant features of transmittance spectra of the superconducting system can be altered by the spacing, the geometry parameters of the superconducting grating, and the ambient temperature of the system. It is found that the positions of the resonant peaks can be manipulated either by the spacing of the grating or the bottom width of the trapezoid cross section of the superconducting strip or their combinations. In addition, the transmission resonances possess higher quality factors when either decreasing the height and the top width of the trapezoid cross section of the superconducting strip or increasing the temperature close to the critical temperature of the superconductor. © *2010 American Institute of Physics*. [doi:10.1063/1.3365617]

Modeling structure

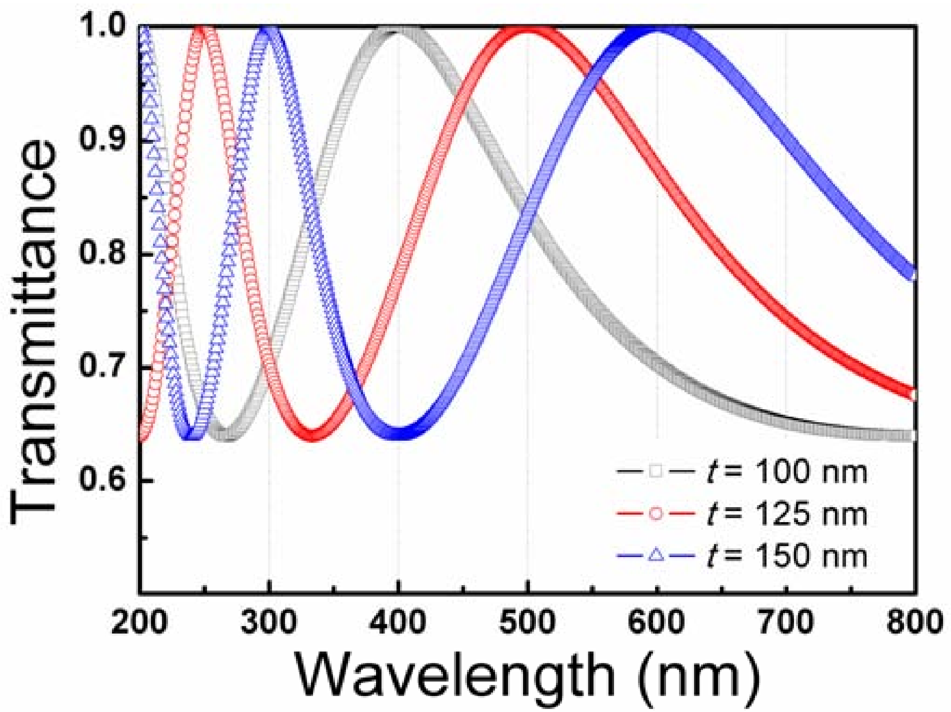


Nb:

T_c = 9.2 K, λ_0 = 83.4 nm

Dielectric rod (ε_r = 10)

The 3D superconducting PhC structure (a) and the magnified cross-section of the 2D unit cell (b). The cross-section of the Nb grating is assumed to be trapezoid with height *h*, top width d_1, and bottom width d_2, respectively. The thickness of the SiN membrane is denoted as *t*. The spacing *s* between two adjacent Nb strips is defined as 2*w*. Two perfect matched layers (PML) are used to prevent reflection particularly.

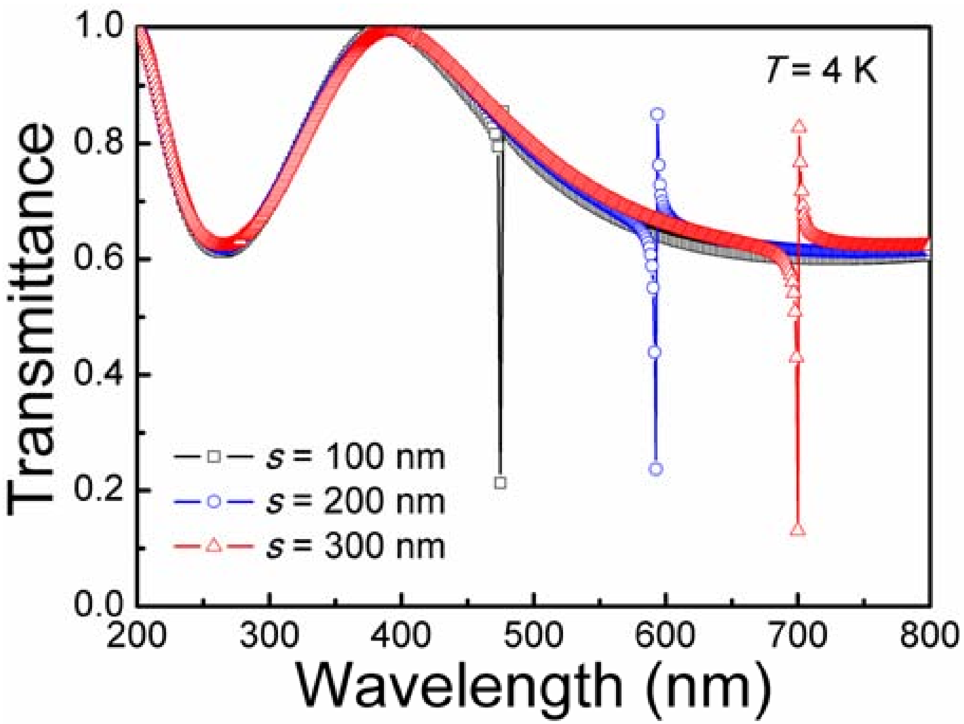


Fabry-Perot Cavity

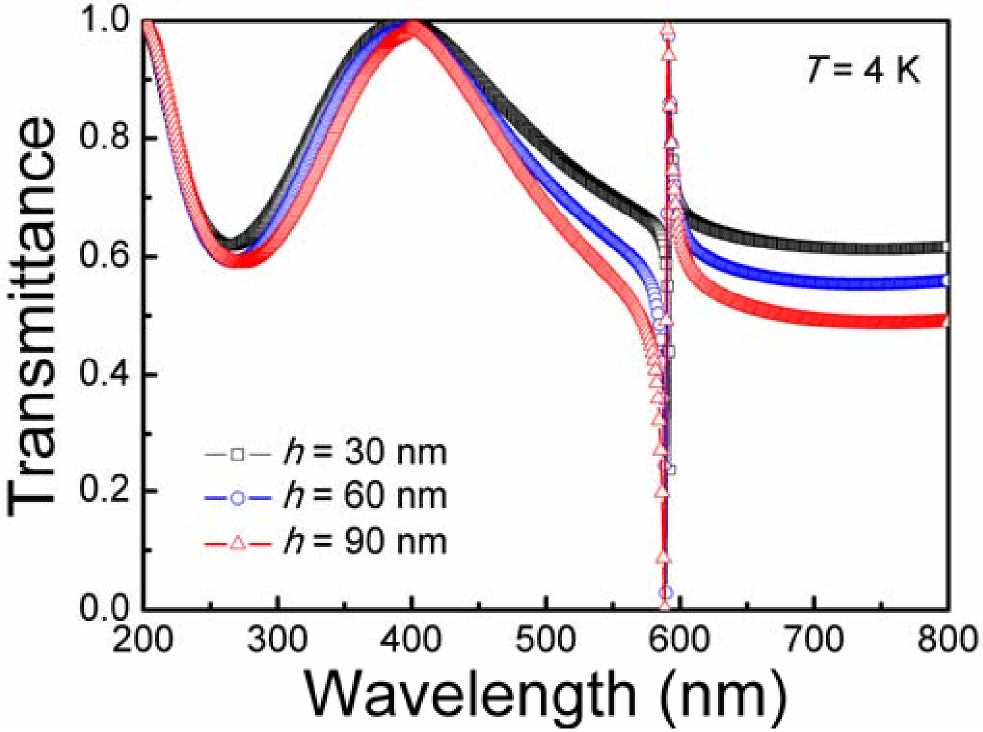
$$2nt = m\lambda$$

n : refractive index of SiN
m: integer

Transmittance spectra of the SiN membrane with thickness t = 100, 125, and 150 nm, respectively.



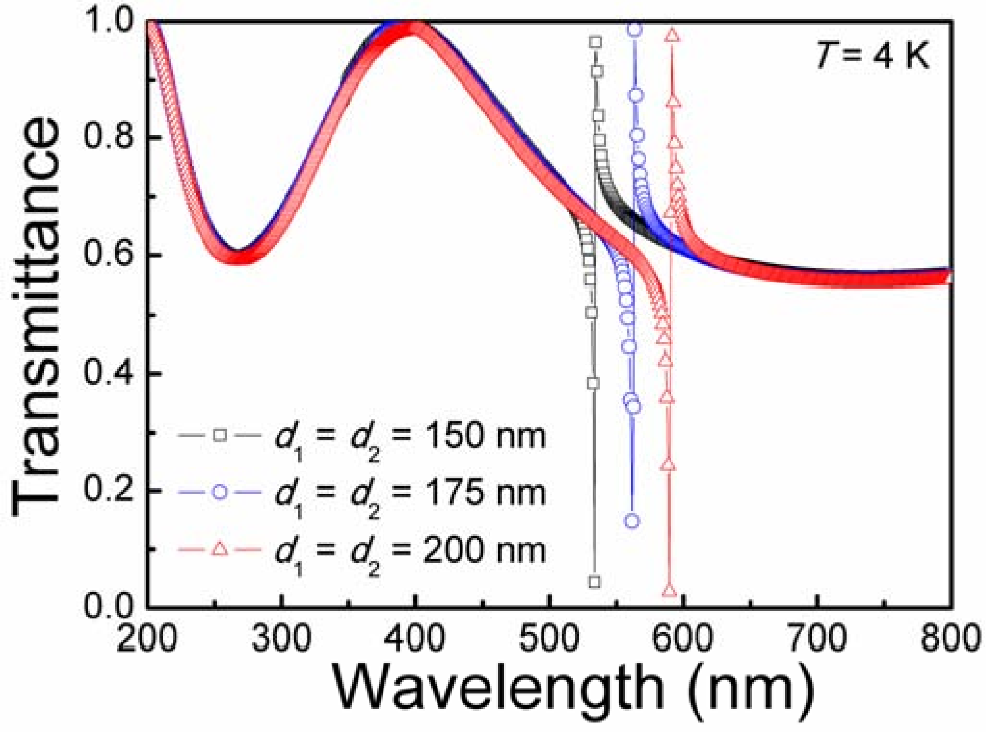
Transmittance spectra of the superconducting structure at T = 4 K with s = 100, 200, and 300 nm, respectively. The geometry parameters of h, d_1, and d_2 of the Nb strip are fixed at 30, 200, and 200 nm, respectively, whereas t is fixed at 100 nm.



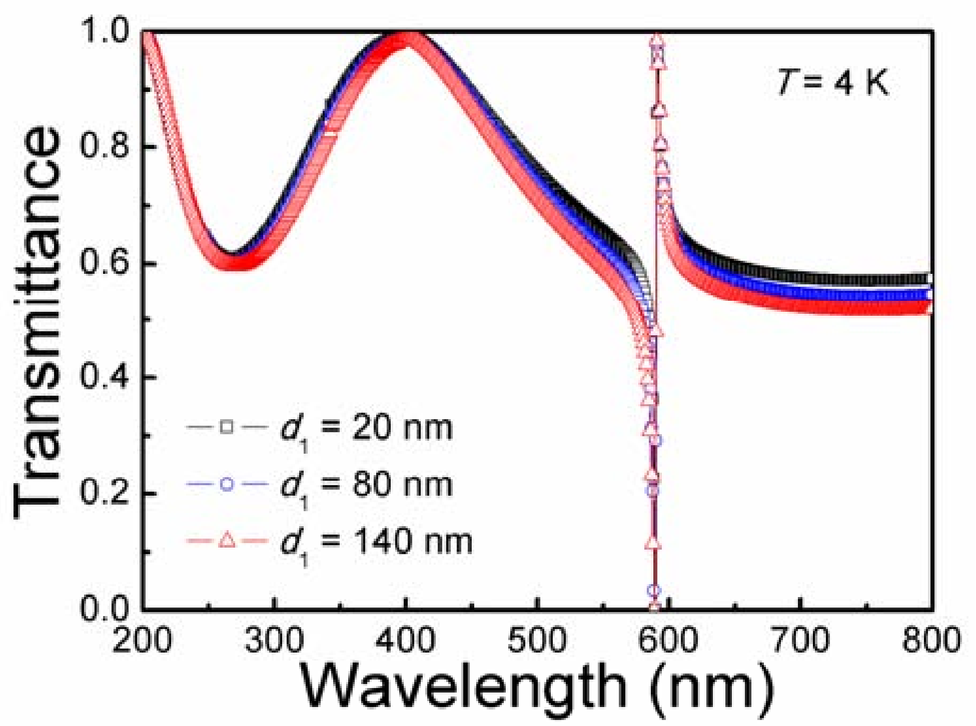
Quality factor

$$Q = \frac{\lambda_c}{d\lambda_c}$$

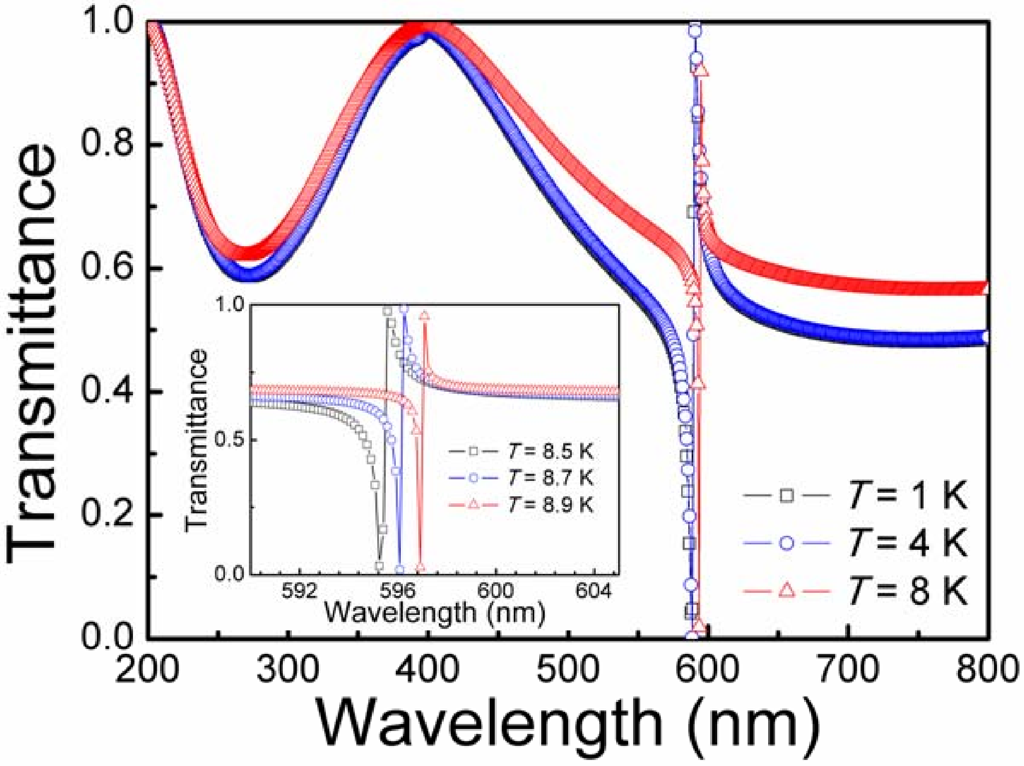
Transmittance spectra of the superconducting structure at T = 4 K with h = 30, 60, 90 nm, respectively, whereas s = 200 nm, t = 100 nm, and $d_1 = d_2$ = 200 nm.



Transmittance spectra of the superconducting structure at T = 4 K with $d_1 = d_2$ = 150, 175, 200 nm, respectively, whereas s = 200 nm, t = 100 nm, and h = 60 nm.



Transmittance spectra of the superconducting structure at T = 4 K with d_1 = 20, 80, 140 nm, respectively, whereas $s = d_2$ = 200 nm, t = 100 nm, and h = 90.



Temperature-dependent transmittance spectra of the superconducting system with T = 1, 4 and 8 K, respectively, whereas h = 90 nm, d_1 = d_2 = 200 nm, t = 100 nm, and s = 200 nm. The inset shows the resonant peaks shift to longer λ as increasing T approaching to T_c from 8.5 K to 8.9 K.

Summary

- We simulate the optical properties of superconducting photonic crystals by finite element method in conjunction with a two-fluid model.
- We found the band gaps of these proposed superconducting structures can be tuned by temperature and structure geometries of the superconductor.
- The superconducting photonic crystal may be applied for high reflection mirrors, band-pass filters, bolometer and lossless optic components.

Thank you for your attention!

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