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On the Numerical Modelling of Elastic Resonant Acoustic Scatterers

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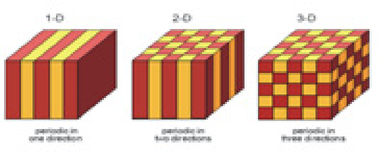
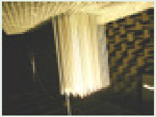
Phononic crystals

Periodic arrangement of elastic materials. 1D, 2D or 3D crystals.

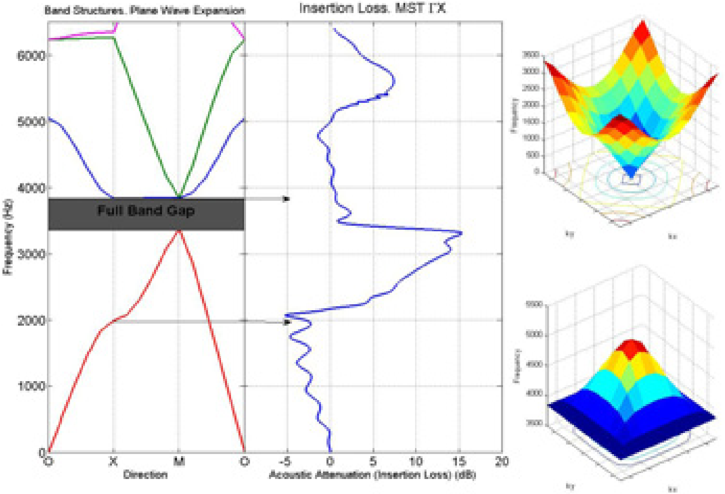


Sonic crystals

Particular case of phononic crystal where at least one elastic material is fluid. Example 2D. Aluminium cylinders in air.

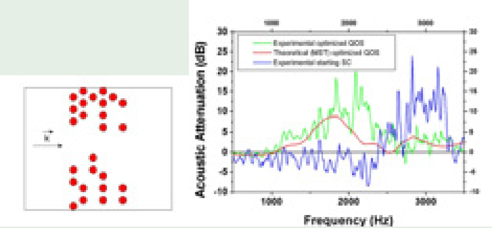


Periodicity $\Rightarrow \omega = \omega(\vec{k})$



Improving the whole Crystal

MST + Genetic Algorithms ⇒ Quasi-Ordered Structures



Romero-García, V. et al. *Hole distribution in phononic crystals: Design and optimization*, J. Acoust. Soc. Am. **125**, 6, (2009)
Hakansson A. et al., *Acoustic lens design by genetic algorithms*, Phys. Rev. B, **70**, 214302, (2004)

Improving the scatterers

Geometrical shape

⟶ Hussein M. et al. *Optimal synthesis of 2D phononic crystals for broadband frequency isolation*
Waves in Random and Complex Media **17**, 4, (2007)

Absorbing materials

⟶ Tourant V. et al. *Multiple scattering of acoustic waves and porous absorbing media*,
Phys. Rev. E, **70**, 026609, (2004)
⟶ Umnova O. et al. *Effects of porous covering on sound attenuation by periodic arrays of cylinders*,
J. Acoust. Soc. Am., **119**, (2006)

Resonant Scatterers

Elastic resonances

⟶ Liu Z. et al. *Locally Resonant Sonic Materials Science*, **289**, 1734, (2000)
⟶ Hirsekorn, M. et al.,*Modelling and simulation of acoustic wave propagation in locally resonant sonic materials*, Ultrasonics, **42**, (2004)
⟶ Fuster-Garcia E. et al.*Targeted band gap creation using mixed sonic crystal arrays including resonators and rigid scatterers*, Appl. Phys. Lett., **90**, 244104, (2007)

Cavity resonances.

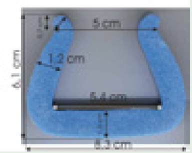
⟶ Movchan A.V. and Guenneau S., *Split-ring resonators and localized modes*,
Pnys. Rev. B., **70**, 125116, (2004)
⟶ Hu X. and Chan, C.T., *Two-dimensional sonic crystals with Helmholtz resonators*,
Phys. Rev. E, **71**, 055601(R), (2005)

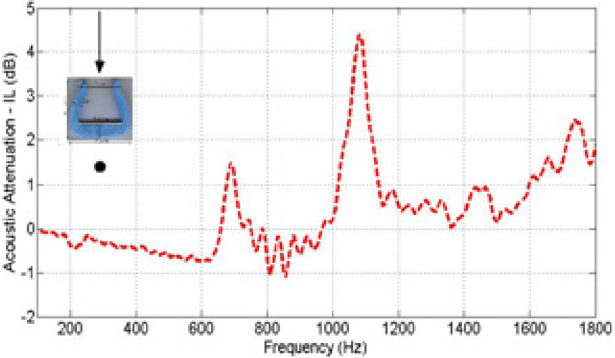
Low Density Polyethylene Foam (LDPE Foam)

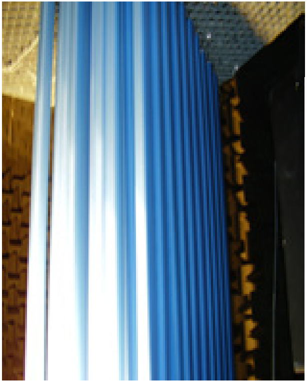
Physical Parameters

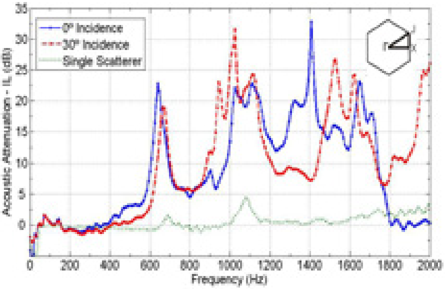
	LDPE Foam
Density (kg/m^3)	50
Young's modulus (10^9 Pa)	0.095
Poisson's ratio	0.32

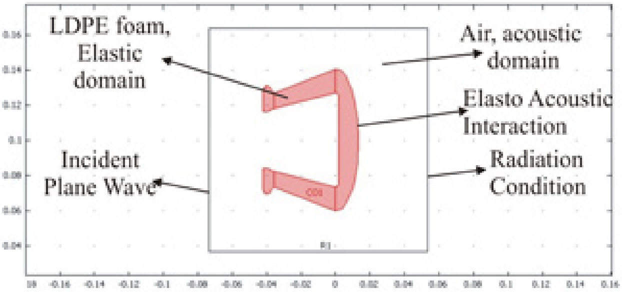
Geometrical Shape




Acoustic Attenuation - IL (dB)
Frequency (Hz)
5
4
3
2
1
0
-1
-2
200
400
600
800
1000
1200
1400
1600
1800




0° Incidence
30° Incidence
Single Scatterer
Acoustic Attenuation - IL (dB)
Frequency (Hz)
0
200
400
600
800
1000
1200
1400
1600
1800
2000
35
30
25
20
15
10
5
0


LDPE foam, Elastic domain
Air, acoustic domain
Elasto Acoustic Interaction
Incident Plane Wave
Radiation Condition

Equation for Elastic waves

$$-\rho_B \omega^2 u_i = \left\{ \frac{\partial \sigma_{ij}}{\partial x_j} \right\}, \tag{1}$$

where

$$\begin{aligned} \sigma_{ij} &= \lambda_B u_{ll} \delta_{ij} + \mu_B u_{ij} \qquad \lambda_B \textit{ and } \mu_b \rightarrow \textit{Lamé Coefficients}, \\ u_{ij} &= \frac{1}{2} \left\{ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right\} \end{aligned}$$

Equation for Acoustic waves

$$-\frac{\omega^2}{c_A^2} p = \nabla \left(\frac{1}{\rho_A} \nabla p \right) \tag{2}$$

Elastic and Acoustic Interaction

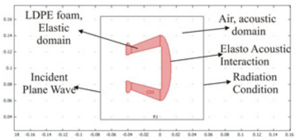
Boundary Conditions

$$\begin{aligned} \frac{\partial p}{\partial n}\Big|_{\partial B} &= \rho_A \omega^2 \vec{u}\vec{n} \\ \sigma_{ij} n_j\Big|_{\partial B} &= -p n_i . \end{aligned}$$

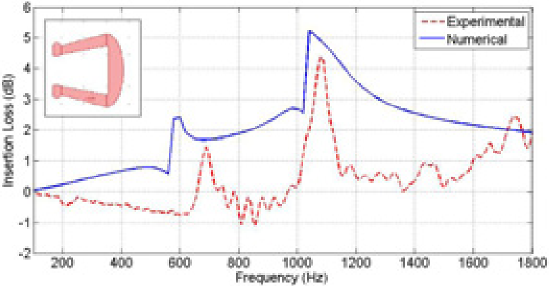
∂B is the boundary of the medium B.
n is the normal unitary vector of the surface

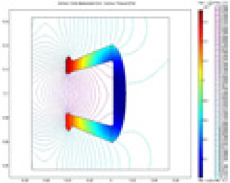
⇒ COMSOL Multiphysics 3.3
⇒ 2D Multiphysics Model of Acoustic and Plain Strain model.

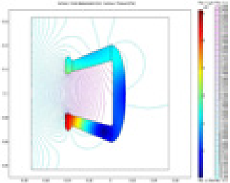
Boundary conditions

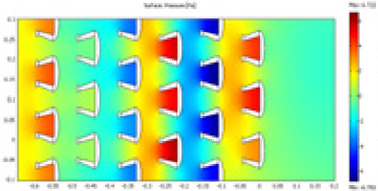


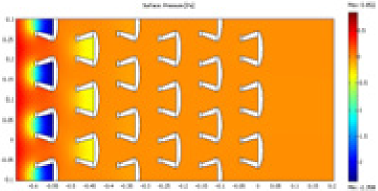
⇒ In ∂B we implement the previous boundary conditions
⇒ Plane wave from left to right: Radiation condition $p_0 = 1$
⇒ Exterior boundaries: radiation condition, $p_0 = 0$

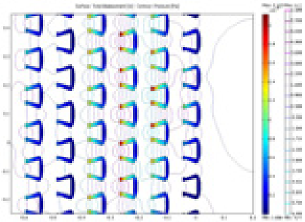

Experimental
Numerical
Insertion Loss (dB)
Frequency (Hz)
6
5
4
3
2
1
0
-1
-2
200
400
600
800
1000
1200
1400
1600
1800

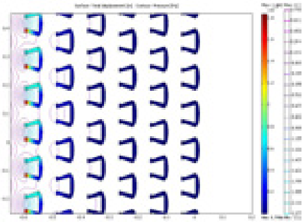


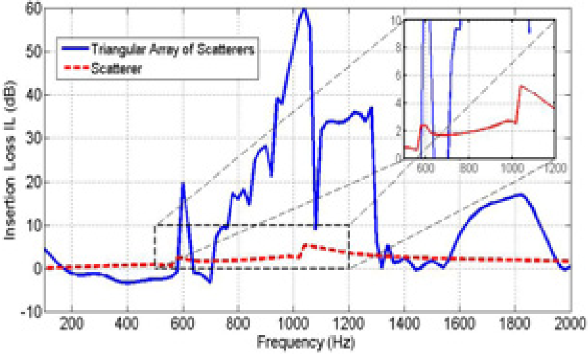











Triangular Array of Scatterers
Scatterer
Insertion Loss IL (dB)
Frequency (Hz)
60
50
40
30
20
10
0
-10
200
400
600
800
1000
1200
1400
1600
1800
2000

Conclusions

1. Soft scatterers made with LDPE foam with a cavity shows mechanical and acoustical properties which can improve the acoustical attenuation capability of SC at low frequencies.
2. Numerical results obtained by COMSOL Multiphysics are in good agreement with the experimental data.
3. These results open new perspective in the design of SC. Several combinations of the LDPE scatterers with different cavities and shapes could be used to attenuate a wide range of frequencies.
4. Several engineering application, such as constructing effective acoustic barriers.

Future work

1. Change the boundary condition of the exterior boundaries by Perfectly Matched Layers.
2. Analyze simple geometries to demonstrate the resonances.
3. Measure experimentally the vibration of the walls of the scatterer.
4. Design scatterers with different cavities and shapes to attenuate a wide range of frequencies.

Thank you very much for your attention!