

# Simulating Superconductors in AC Environment: Two Complementary COMSOL Models

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**Abstract:** In this paper we present a summary of our work on numerical modeling of superconductors with COMSOL Multiphysics. We discuss the two models we utilized for this purpose: a 2-D model based on solving Maxwell equations and a 1-D model for thin conductors based on solving the integral equation for the current density distribution. The latter is useful for modeling second generation high-temperature superconductor (HTS) tapes, which are extremely thin and can be described as 1-D objects in most situations. The advantages and disadvantages of using each model are pointed out.

**Keywords:** Superconductors, ac losses, partial differential equations, integral equations, non-linearity.

## 1 Introduction

HTS tapes, wires, and devices (cables, coils) have a large potential for different types of applications, ranging from motors and transformers to cables and fault current limiters. One of the most important issues for the commercialization of these materials is the reduction of their energy dissipation when they are operating in an environment with varying currents and magnetic fields; in particular, in power applications characterized by ac currents and fields. Numerical modeling is a useful tool for predicting the ac losses of HTS devices and optimizing their design before committing to manufacturing. The main obstacle for computing the losses is represented by the highly non-linear resistivity of these materials, which makes it impossible to use the ac modules available in COMSOL. This means that the problem has to be simulated as a transient problem.

In the past years we have developed two models for simulating HTS materials. The

first is a 2-D model (where only the cross-section of the HTS tape, perpendicular to the direction of the current, is considered) based on solving Maxwell equations. The model has been successfully utilized for simulating tapes and assemblies of tapes with arbitrary cross-section. Typically, the so called first generation tapes have an elliptical cross-section, with the dimension of the axes in the range of 4 and 0.2 mm, respectively.

At present, the most promising HTS tapes are the so-called second generation tapes (or ReBCO coated conductors), which are much thinner objects. The width of the superconductor film is typically 4-10 mm, but its thickness is around 1  $\mu\text{m}$  only. This results in a cross-section with enormous aspect ratio (width/thickness), and consequently in a very large number of mesh nodes. Things get even worse when assemblies of tapes are considered.

In order to overcome this problem we focused our attention to another type of model, which considers the coated conductors (CCs) as 1-D objects and solves the integral equation for the current density distribution. The tape can be simulated as a 1-D line (with typically 100-200 mesh nodes), which results in a much more affordable problem size and computation time.

In this paper we present a summary of what has been done with these two models, focusing on the recently developed 1-D model, and pointing out when the use of each model is advantageous.

## 2 Governing equations

The superconductor is modeled by means of a non-linear resistivity, derived from a power-law voltage-current characteristics that fits the experimental data:

$$\rho(J) = \frac{E_c}{J_c} \left| \frac{J}{J_c} \right|^{n-1}, \quad (1)$$

where the critical field  $E_c$  is typically equal to  $10^{-4}$  V/m,  $J_c$  is in the range  $10^8$ - $10^{10}$  A/m<sup>2</sup> and  $n=25$ -50. Our models are not restricted to the case of the power-law resistivity of equation (1), and they can in principle take into account any resistivity that can be expressed as a function of the state variables. Of course, the case of constant resistivity can be taken into account as well, and it can be used for validating the model by comparing the results to those of analytical solutions, when they exist.

The ac losses (in J/cycle/m) are computed by integrating  $\rho J^2$  over the superconductor's cross-section and over an ac cycle.

## 2.1 2-D model with edge elements

The 2-D model considers only the transversal cross-section of the tapes, through which the current flows. This approximation is sufficient for most practical configurations, which either involve long straight tapes or present rotational symmetries.

Maxwell equations are written as a function of the two magnetic field components  $H_x$  and  $H_y$  (the plane  $XY$  being the plane of the transversal cross-section). The resistivity depends in general on the current density, which can be expressed as a function of the magnetic field by means of the relation  $\nabla \times \mathbf{H} = \mathbf{J}$ . In 2-D this latter relation is scalar and reduces to

$$J = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}. \quad (2)$$

Typical simulated problems involve the presence of an external applied field and/or a transport current. The external field can be imposed using the appropriate boundary conditions in the domain surrounding the conductors. The transport current can be applied by integral constraints for the current flowing in each conductor. The use of integral constraints allows a great flexibility:

- the transport current and applied field can be managed independently;
- in the case of multiple conductors, parallel and series connections can be imposed very easily by simply modifying the integral constraints for the total current flowing in a given set of conductors.

Finally, the use of edge elements allows the direct imposition of the continuity of the tangential component of the magnetic field across adjacent elements, with no need to impose the condition separately.

The model is implemented in COMSOL's PDE General Form. More details about this model implementation, its validation, and examples of calculations for cases of practical interest can be found in [1].

## 2.2 1-D model for thin conductors

The 2-D model allows computing the losses in conductors of arbitrary geometry. However, the case of ReBCO coated conductors represents a huge computation burden: due to the very high aspect ratio (width/thickness) of the superconducting film (typically 1,000-10,000), the number of mesh nodes is very large and so is the size of the problem and the computation time. Recently, we have focused on simulating this kind of HTS tapes as 1-D objects. This means that the relevant flux dynamics are expected to occur along the tape width and that the variation of the electromagnetic quantities along the tape's thickness can be neglected. Therefore the dynamics can be described as a function of the *sheet* current density, i.e. the current density integrated over the tape's thickness. In the literature analytical models for thin superconductors have been proposed – see for example [2, 3, 4] for isolated tapes and [5, 6, 7] for stacks and arrays. Their main drawback is three-fold:

- they are developed in the framework of the critical state model, which is a very rough approximation for HTS materials;
- they are limited to the case of isolated tapes or to infinite arrays/stacks, so that end effects cannot be taken into account;
- in the case of externally applied fields, they are limited to uniform fields.

Our model overcomes these three important limitations.

The basic idea behind our model is solving the integral equation for the current density distribution in a thin conductor. The general form of the equation is:

$$\rho J(x, t) = \mu d \left[ \frac{1}{2\pi} \int_{-a}^a \dot{J}(\xi, t) \log |x - \xi| d\xi + \int_{-a}^x \dot{H}_{ey}(\xi, t) d\xi \right] + C(t), \quad (3)$$

where  $\rho$  is the conductor's resistivity (constant for normal materials, but strongly dependent on  $J$  for superconductors),  $J$  is the sheet current density,  $\mu$  is the magnetic permeability,  $a$  is the tape's half-width, and  $H_{ey}$  is the magnetic field component perpendicular to the tape's flat face. The first integral gives the contribution due to the self-field generated by the tape and the second integral gives the contribution coming from the presence of any other external field. The term  $C(t)$  has to be inserted to satisfy the integral constraint

$$\int_{-a}^a J(x, t) dx = I(t) \quad (4)$$

imposed by the connections to current sources and by the presence of external fields.

Since the use of COMSOL to solve integral equation is not well referenced, we give here some details. The possibility of solving integral equation using COMSOL is offered by the availability of an operator, called `dest()`, which allows transforming the integrals in equation (3) into *Boundary Integral Coupling Variables* (BICV) that can be inserted in the equation settings. Generally, if a function  $f(x)$  is defined as an integral on a segment  $(a, b)$  of the  $x$ -axis:

$$f(x) = \int_a^b K(x, y) dy, \quad (5)$$

its equivalent in COMSOL is a BICV  $\mathbf{f}$  defined as  $\mathbf{K}(\mathbf{dest}(\mathbf{x}), \mathbf{x})$  on the corresponding boundary  $[a, b]$  of the  $x$ -axis of the model. Similarly, if  $f(x)$  is defined as a function dependent on the upper limit of integration:

$$f(x) = \int_a^x g(y) dy, \quad (6)$$

its BICV translation is  $\mathbf{f}$  defined as  $\mathbf{g}(\mathbf{x}) * (\mathbf{x} < \mathbf{dest}(\mathbf{x}))$ .

In particular, for writing the integral equation (3) using COMSOL's syntax, we define the following two BICV:

$$\begin{aligned} \mathbf{Q}: & \mathbf{Jt} * \log(\mathbf{abs}(\mathbf{dest}(\mathbf{x}) - \mathbf{x})) / (2 * \mathbf{pi}) \\ \mathbf{K}: & \mathbf{Heyt} * (\mathbf{x} < \mathbf{dest}(\mathbf{x})) \end{aligned}$$

where  $\mathbf{Jt}$  is the time derivative of the sheet current density  $J$  (the state variable of the model), whereas  $\mathbf{Heyt}$  is the time derivative of the external magnetic field (perpendicular to the surface of the conductor) and it is defined as boundary expression.

The model is implemented in COMSOL's PDE 1-D Coefficient Form. In the equation settings all coefficients are set equal to zero except  $\mathbf{a}:(\mu * d)$  and  $\mathbf{f}:(\mathbf{Q} + \mathbf{K})$ . The boundary points in  $-a$  and  $a$  are left free (Neumann condition). The linear Lagrange elements are the most appropriate for a sufficiently dense mesh (typically the segment is uniformly divided into 100 elements, if necessary the mesh is refined near the edges). More details about this model can be found in [8].

### 3 Examples of use of the two models

The 2-D model can be used to simulate conductors of arbitrary shape. As an example, figures 1 and 2 show the typical mesh and current density distribution for a first generation HTS tape, composed of an elliptical superconducting core surrounded by a silver matrix. The main limitation of this model is the size of the problem when second generation thin tapes are considered. To a certain extent, the problem can be avoided by simulating a geometry thicker than the real one; the basis of this is that the thin superconductor is practically a 1-D object. This has proved to be true in the case of isolated tapes: in that case, as long as the aspect ratio is sufficiently large (e.g.  $>100$ ), the tape behaves as a 1-D object independently of the thickness, and current/field profiles and ac losses have been found to be in good agreement with those computed by analytical models for 1-D tapes. However, this is no longer true in the case of interacting tapes, particularly when their mutual distance becomes comparable to the artificially increased thickness.

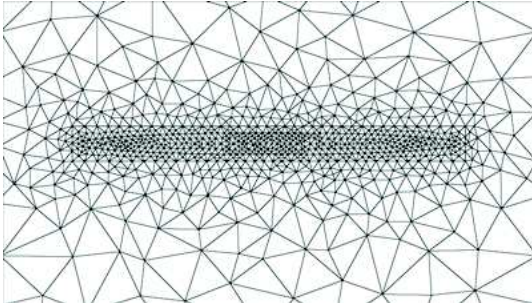


Figure 1: Example of the mesh for a first generation HTS tape. The superconductor cross-section has elliptical shape and is embedded into a rectangular silver matrix.

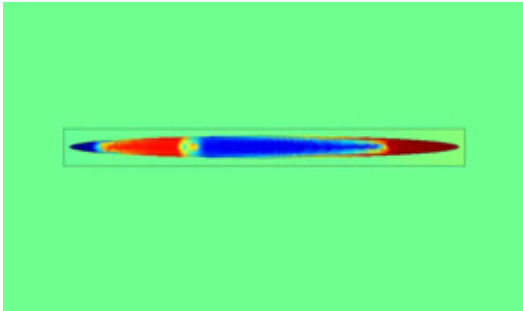


Figure 2: Typical current density distribution in a first generation HTS tape.

The 1-D model proved to be a powerful tool for simulating thin superconductors. Our first step was to develop the integral equation for an individual tape; then we tested the model against cases for which analytical solutions exist, and we extended it to the case of non uniform fields. As an example, figure 3 shows the magnetic flux density lines generated by two current lines in opposite directions in the vicinity of a superconducting thin conductor; figure 4 shows the corresponding profile of the magnetic flux density along the tapes' width.

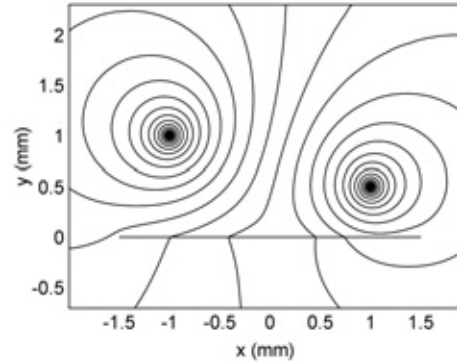


Figure 3: Magnetic flux density lines generated by two current lines in opposite directions in the vicinity of a superconducting thin conductor.

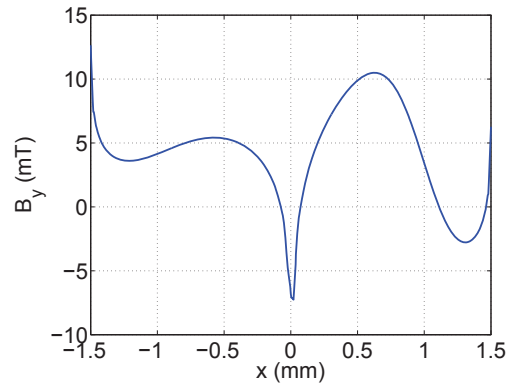


Figure 4: Profile of the magnetic flux density along the thin conductor (at the peak value of the applied currents) for the case considered in figure 3.

The second step was to extend the model to the case of interacting tapes. In this case the magnetic field generated by one tape acts as “external” field on the other tapes. This electromagnetic coupling was performed by means of auxiliary 2-D magnetostatic models, where the geometry of the tape assembly (tape width, mutual distance) is taken into account. The interaction is schematically shown in figure 5 for the simple case of two tapes. The current density in each tape calculated by means of two separate 1-D models is transferred to the line representing each tape in the 2-D magnetostatic model, which then transfers back the magnetic field generated by the other tape. This procedure deserves a few remarks:

- the coupling is computed at each time

step of the ac cycle, which is why a magnetostatic model is sufficient;

- the transfer of the variables is performed by means of COMSOL's *Extrusion Coupling Variables*;
- in the 2-D geometry, the thin tapes are represented by lines, which allows to keep the number of mesh nodes at a reasonable value;
- the transfer of the magnetic field to the 1-D models is done by means of the electromagnetic vector potential  $A$ , which is the state variable in the magnetostatic 2-D model.

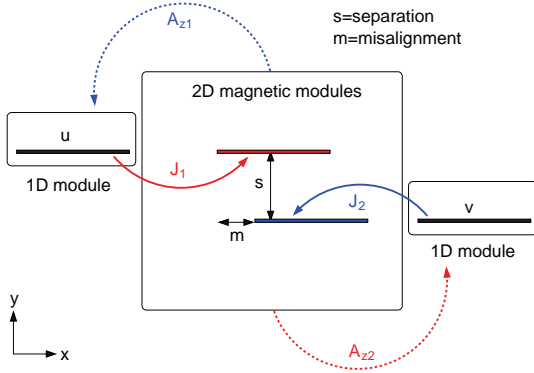


Figure 5: Coupling between two 1-D modules, each representing one tape, by means of two 2-D magnetic modules, which use the magnetic vector potentials  $A_{z1}$  and  $A_{z2}$  as state variables.

In the 1-D modules,  $u$  and  $v$  represent the current density distributions in the two tapes.

The main drawback of this method is that a 2-D magnetostatic model is needed for representing each electromagnetic coupling, which makes the size of the problem rapidly increase with the number of interacting tapes. Details about this method can be found in [9], together with a comparison with experimental results.

The third step was to use the integral equation method to simulate infinite arrays or stacks of tapes. By using the natural symmetries of these geometries the interaction between the tapes can be automatically included in the integral equation, so that only one tape need to be simulated. Here below we summarize the integral equations for the most interesting cases.

For an infinite  $z$ -stack of superposed tapes separated by a distance  $S$  the integral equation to be solved is:

$$\rho J(x, t) = \frac{\mu d}{2\pi} \int_{-a}^a \dot{J}(\xi, t) \log \sinh \frac{\pi|x - \xi|}{S} d\xi + C(t). \quad (7)$$

An interesting variation of the  $z$ -stack, more useful for practical applications (e.g. for non-inductive coils), is the bifilar stack, where adjacent tapes in the stack carry opposite current. In this case the integral equation is:

$$\rho J(x, t) = \frac{\mu d}{2\pi} \int_{-a}^a \dot{J}(\xi, t) \log \tanh \frac{\pi|x - \xi|}{2S} d\xi + C(t). \quad (8)$$

The  $x$ -array is the configuration that describes an infinite solenoid. The integral equation is:

$$\begin{aligned} \rho J(x, t) &= \frac{\mu d}{2\pi} \int_{-a}^a \dot{J}(\xi, t) \log \sin \frac{\pi|x - \xi|}{L} d\xi + C(t) \\ &= \mu d K_X(x, t) + C(t), \end{aligned} \quad (9)$$

where  $L$  is the size of the periodic cell, i.e. the sum of the tape width and the lateral spacing. Again, for practical applications, a configuration with opposite currents reducing the inductance of the system is preferable. Therefore, we derived the integral equation for a two-layer solenoid, which can be simulated as two superposed infinite  $x$ -arrays, each carrying the same current in opposite directions. The integral equation has two contributions and can be written in the following compact form:

$$\rho J(x, t) = \mu d [K_X(x, t) \pm K_{2P}(x, t)] + C(t) \quad (10)$$

where  $K_X$ , defined in equation (9), is the contribution of the layer the tape belongs to, whereas  $K_{2P}$  is the contribution of the other layer and has the following form:

$$\begin{aligned} K_{2P}(x) &= \frac{1}{4\pi} \int_{-a}^a \dot{J}(\xi) \log \left[ \cosh(2\pi S/L) - \right. \\ &\quad \left. \cos(2\pi(x - \xi)/L) \right] d\xi. \end{aligned} \quad (11)$$

In equation (10), the plus sign refers to inductive winding (the current direction in the two layers is the same) and the minus sign refers



to anti-inductive winding (the current direction in the two layers is opposite).

As an example of the influence of the tape interaction on the current density and magnetic field profiles, figures 6 and 7 display the current density and field profiles at the peak current for an isolated tape, a tape in a  $z$ -stack, and a tape in a  $x$ -stack. The amplitude of the transport current is equal to  $I_c$ .

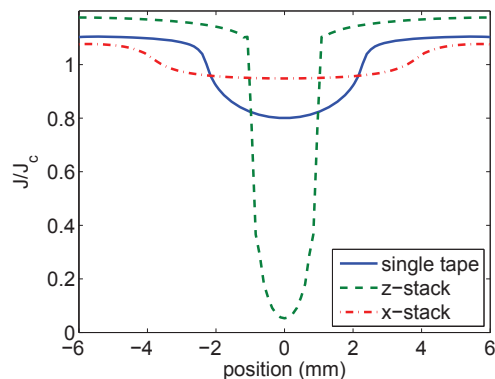


Figure 6: Current density distribution along the tape's width for an isolated tape, a tape in a  $z$ -stack, and a tape in a  $x$ -stack. The applied current is equal to the critical value  $I_c$  and the profiles are taken at the current peak.

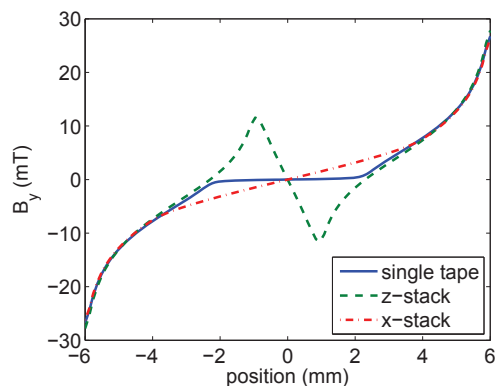


Figure 7: Magnetic flux density distribution along the tape's width for an isolated tape, a tape in a  $z$ -stack, and a tape in a  $x$ -stack. The applied current is equal to the critical value  $I_c$  and the profiles are taken at the current peak.

In summary, in all cases shown here involving an infinite number of tapes, the mutual coupling between the conductors is accounted directly in the integral equation by means of

the term

$$K(x, t) = \int_{-a}^a k(x - \xi) \dot{J}(\xi) d\xi,$$

i.e. the finite space convolution of the time derivative of the sheet current  $J(x, t)$  with a kernel  $k(x)$  of logarithmic type, which varies according to the considered configuration (stack, array, etc.). Since no auxiliary 2-D field computations are necessary for the magnetic coupling and since the tapes are modeled as unidimensional domains, the major result is the very short computing times required as compared to previous formulations. Indeed, the numerical solution of all integral equations we derived here rarely lasts more than tens of seconds on standard desktop computers.

This kind of integral equations cannot be solved analytically. In the case of constant resistivity the equations can be solved by standard numerical methods (for example Nyström method), which however fail when the resistivity is non-linear. On the contrary, the solution based on finite elements is very effective also in these challenging cases.

Details about the derivation of the various equation can be found in [10], together with a comparison of the results with those obtained with analytical formulae developed using the critical state model, when they exist.

The main drawback of the 1-D model is that, by its nature, it cannot take into account the variation of electromagnetic quantities across the tape's thickness. In many practical configurations, HTS thin tapes actually behave as 1-D object, and the description given by the 1-D model is accurate. However, in certain particular applications, for example in coils where the tapes are closely packed, there is a substantial magnetic flux penetration from the top and bottom wide faces. This penetration contributes to the losses and in some cases (low applied current, small tape separation) it can become comparable or even exceed the contribution to the losses caused by the flux penetration from the edges. Presently, we are investigating in detail the situations when this occurs.

## 4 Conclusion

In this paper we described the main features of two finite-element models for simulating

the behavior of superconductors in ac environment: a 2-D model based on solving Maxwell equations and a 1-D model based on solving the integral equation for the current density. The 2-D model is suitable for arbitrary cross-sections of superconducting tapes and devices; the 1-D model, complementary to the 2-D one, is proper for thin tapes (e.g. ReBCO coated conductors), whose very large aspect ratio makes the use of the 2-D model impractical. It is worth noticing that both models can be easily coupled with standard COMSOL ac modules in the case of more complex geometries involving other components, such as substrates, insulating layers, shields.

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