The Acoustoelastic Effect: EMAT Excitation and Reception of Lamb Waves in Pre-Stressed Metal Sheets

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Abstract: The acoustoelastic effect relates the change in the speed of an acoustic wave travelling in a solid, to the pre-stress of the propagation medium. In this work the possibility of assessing nondestructively the stress status in metal sheets, by using the acoustoelastic effect, is investigated. As the effect turns out to be very small for practical values of applied stress, the proposed technique relies on the use of a kind of non-contact ultrasonic transducers called EMATs (Electro-Magnetic Acoustic Transducer) for precisely measuring its influence on Lamb waves.

Keywords: Non Destructive Testing, ultrasound, Lamb waves, acoustoelastic effect, residual stress, EMAT

1. Introduction

The acoustoelastic effect relates the change in the speed of an acoustic wave travelling in a solid, to the pre-stress of the propagation medium. It is analogous to the well-known photoelastic effect, where a stressed transparent sample exhibits birefringence when crossed by a ray of light. In the literature it has been shown how this effect can be used to relate the change in the speed of sound to the pre-stress of the material, for monitoring and testing purposes [8][9][10][11][12]. In this work the possibility of assessing nondestructively the stress status in metal sheets, by using the acoustoelastic effect and non-contact transducers, is investigated. Non contact transducers are needed as the effect turns out to be very small for practical values of applied stress [3]. Here a kind of non-contact ultrasonic transducers called EMAT (Electro-Magnetic Acoustic Transducer) [1] will be proposed for precisely measuring the acoustoelastic effect on Lamb waves [2]. EMATs use an optional static magnetic field and a time-varying one, generated by a pulse of current flowing through a coil, to excite acoustic waves in conducting and/or magnetic materials by means of Lorentz or magnetostrictive forces

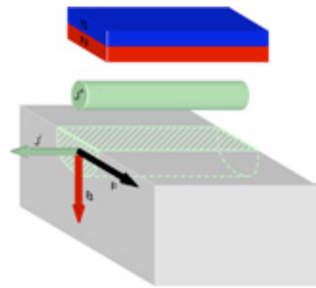

Figure 1. A scheme of a transmitting EMAT. The static field $\boldsymbol{B}$ and the current $\boldsymbol{J}^{i}$, induced by the time varying current $\boldsymbol{J}^{e}$ in the coil, generate a Lorentz force $\boldsymbol{F} = \boldsymbol{J}^{i} \times \boldsymbol{B}$ in the lower conducting medium.

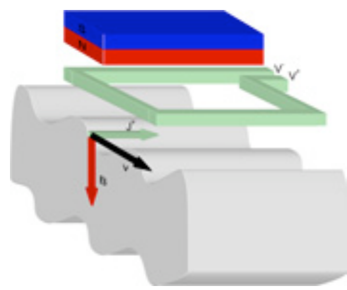

Figure 2. A scheme of a receiving EMAT. The vibration of the lower medium, with conductivity σ and moving at velocity $\boldsymbol{v}$, in presence of the static field B generates a current $\boldsymbol{J}^{v} = \sigma \boldsymbol{v} \times \boldsymbol{B}$. This current induces a voltage in the receiving coil.

(Figure 1). EMATs can be used to receive acoustic waves, too, by employing a fixed magnetic field that induces Foucault currents in conducting media when mechanical vibrations are present. These currents are then sensed by a receiving coil (Figure 2).
Lamb waves are guided acoustic modes propagating in thin structures, such as plates or sheets. Two families of Lamb modes exist, called anti-symmetric and symmetric (Figure 3), sharing the common feature that only their fundamental modes a_0 and s_0 can propagate at any frequency (Figure 4). The use of Lamb waves for assessing the stress status of sheets

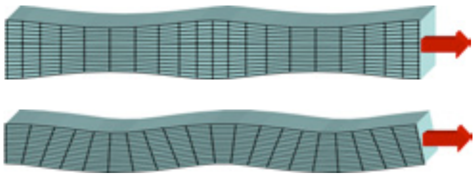

Figure 3. Sections of a sheet of material where a symmetric (upper) and an antisymmetric (lower) Lamb waves propagate.

and foils is promising, and its feasibility was already reported in the literature [7].

The present work will be based on two EMATS arranged in a *pitch-catch* configuration, where the change in wave speed due to the acoustoelastic effect is measured via the change in the acoustic time-of-flight along a lead sheet, 1 mm thick (Figure 5). The model implemented in Comsol Multiphysics will deal with the electromagnetic interaction of the transducer and the lead sheet to account for the ultrasound generation and reception mechanism, and will separately include an elastodynamic application mode for the ultrasound propagation.

After introducing the mathematical equations needed to describe the acoustic wave propagation in a pre-stressed material, the use of Comsol Multiphysics and the obtained results will be reported.

2. The acoustoelastic effect

Usually the linear elastic theory is used when dealing with acoustic waves propagation, as acoustic waves lead to deformations small enough to allow for linear constitutive equations [4]. If the displacement of the points of a solid continuum are described by the vector field ***u***(***x***,*t*), where ***x*** is the position vector, it is possible to define the first order *strain tensor* [ε]:

$$\varepsilon_{ij} \triangleq \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad (1)$$

A *stress tensor* [σ] is then introduced, representing tractions, that is forces per unit area, acting on a generic infinitesimal cube in a solid continuum. The element σ_{ij} is the *j*-th component of the traction acting on the cube face normal to the *i*-th axis. These two tensors are linked, in the linear costitutive equation hypothesis, by the *stiffness tensor* [***C***]:

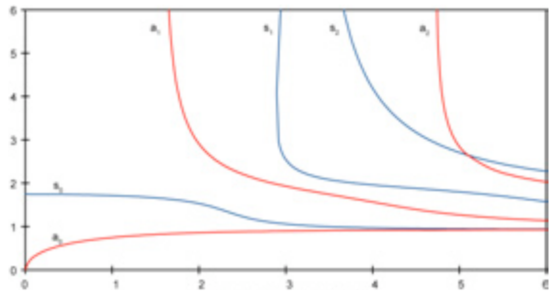

Figure 4. Normalized phase velocities of the first three symmetric and antisymmetric Lamb modes, as a function of normalized frequency.

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} \qquad (2)$$

By denoting with ***f*** the volume force density and with ρ the volume density of the material, the motion equation can be written as:

$$C_{ijkl}\frac{\partial^2 u_k}{\partial x_i \partial x_l} + f_j = \rho\,\ddot{u}_j \qquad (3)$$

Eqs. (1-3) holds for an acoustic wave traveling in an unloaded medium, but should be modified if an arbitrarily large pre-stress is present. This is due to the following non-linear effects associated to the pre-existing deformation on which the acoustic wave would be overlay [6]:

Geometric nonlinearity: the strain [ε] in eq. (1) is only a first-order approximation that does not hold for finite deformations. Furthermore a distinction should be made whether the stresses in [σ] are defined in terms of the original or deformed surfaces. The *large deformation* theory [5] addresses these issues.

Constitutive nonlinearity: the constitutive equation (2), too, is a first order approximation holding for small deformations.

In this work, as in [7], only the geometric nonlinearity will be taken into account.

To describe the behavior of a generic pre-stressed body where ultrasonic waves are excited, it is useful to define the following states:

Natural state: rest state with no applied or residual stress, small Greek letters are used for vectors and indices, such as in the position vector $\boldsymbol{\xi}$.

Initial state: equilibrium state where only the pre-stress is present, capital Latin letters are used (e.g. ***X***).

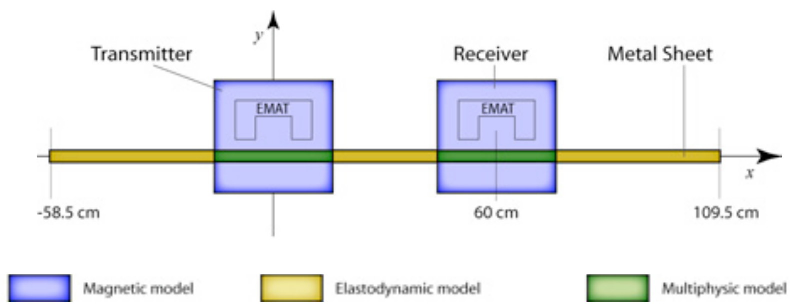

Figure 5. A scheme of the pitch-catch setup modeled with Comsol Multiphysics.

Final state: dynamic state where a small deformation is overlay on that due to the pre-stress, small latin letters are used (e.g. $\boldsymbol{x}$).

By using a Lagrangian approach, deformations are described by the *Green deformation tensor*

$$E_{\alpha\beta}^{(i)} = \frac{1}{2}\left(\frac{\partial X_K}{\partial \xi_\alpha}\frac{\partial X_K}{\partial \xi_\beta} - \delta_{\alpha\beta}\right) = \\ = \frac{1}{2}\left(\frac{\partial u_\alpha^{(i)}}{\partial \xi_\beta} + \frac{\partial u_\beta^{(i)}}{\partial \xi_\alpha} + \frac{\partial u_\gamma^{(i)}}{\partial \xi_\alpha}\frac{\partial u_\gamma^{(i)}}{\partial \xi_\beta}\right), \tag{4}$$

where the upper index *i* means that the initial state is being referred to, as in $\boldsymbol{u}^{(i)}=\boldsymbol{X}-\boldsymbol{x}$.

By subtracting it from the analogous final deformation tensor, the *incremental deformation tensor* can be computed:

$$E_{\alpha\beta} = E_{\alpha\beta}^{(f)} - E_{\alpha\beta}^{(i)} \simeq \\ \simeq \frac{1}{2}\left(\frac{\partial u_\alpha}{\partial \xi_\beta} + \frac{\partial u_\beta}{\partial \xi_\alpha} + \frac{\partial u_\gamma^{(i)}}{\partial \xi_\alpha}\frac{\partial u_\gamma}{\partial \xi_\beta} + \frac{\partial u_\gamma}{\partial \xi_\alpha}\frac{\partial u_\gamma^{(i)}}{\partial \xi_\beta}\right), \tag{5}$$

where $\boldsymbol{u}=\boldsymbol{u}^{(f)}-\boldsymbol{u}^{(i)}$ is the incremental displacement, due to the acoustic wave only.

The following two stress tensors are needed:

Cauchy stress [σ]: ratio of forces in deformed state to surfaces in deformed state.

II Piola-Kirchhoff stress [*S*]: ratio of forces in un-deformed state to surfaces in deformed state.

The motion equation can be written in terms of [$\sigma^{(f)}$], with *b* now being the volume force density:

$$\frac{\partial \sigma_{ij}^{(f)}}{\partial x_i} + \rho^{(f)} b_j = \rho^{(f)} \ddot{u}_j^{(f)} = \rho^{(f)} \ddot{u}_j \tag{6}$$

or in terms of [$S^{(f)}$]:

$$\frac{\partial}{\partial X_K}\left(\mathcal{S}_{KL}^{(f)}\frac{\partial x_j}{\partial X_L}\right) + \rho^{(i)} b_j = \rho^{(i)} \ddot{u}_j, \tag{7}$$

with $J = j$. By imposing the equilibrium condition on the initial state

$$\frac{\partial \sigma_{KL}^{(i)}}{\partial X_L} = \frac{\partial \mathcal{S}_{KL}^{(i)}}{\partial X_L} = 0, \tag{8}$$

by acknowledging the identity $S_{IJ}^{(i)} = \sigma_{IJ}^{(i)}$, and by ignoring incremental terms of order higher than the first one, eq. (7) can be written as:

$$\frac{\partial}{\partial X_I}\mathcal{S}_{IJ} + \sigma_{IL}^{(i)}\frac{\partial^2 u_J}{\partial X_I \partial X_L} + \rho^{(i)} b_J = \rho^{(i)} \ddot{u}_J \tag{9}$$

Finally, assuming an homogeneous pre-deformation ad not taking into account the constitutive nonlinearity, the equation of motion reads

$$C'_{IJKL}\frac{\partial^2 u_K}{\partial X_I \partial X_L} + \rho^{(i)} b_J = \rho^{(i)} \ddot{u}_J \tag{10}$$

$$C'_{IJKL} = C_{IJKL} + \sigma_{IL}^{(i)}\delta_{JK} \tag{11}$$

where [$\boldsymbol{C'}$] is the equivalent stiffness tensor, depending on the material stiffness and on its pre-stress in the initial state.

Thanks to eq. (10), the acoustoelastic effect can be modeled as an added anisotropy in the stiffness tensor, thus still allowing the use of a linear equation for the acoustic wave propagation.

3. Use of COMSOL Multiphysics

In order to model the generation, propagation and reception of ultrasonic waves in the setup of Figure 5**,** two kind of application modes are used: the AC/DC Quasi static magnetic mode for the EMAT transducers, and the Structural Mechanics Plane Strain mode for the acoustic wave. The geometry is bi-dimensional, and includes a generic section where induced currents are out-of-plane, while displacements and strains are in-plane.

Both EMATs are constituted by a C-shaped permanent magnet, while they differ regarding their coil. The transmitting one is a double meander coil whose spacing was chosen as to optimally excite the a_0 mode at 20 KHz, while the receiving one is a multi-layer solenoid coil. As lead is non ferromagnetic, the only generating and receiving mechanisms are via the Lorentz forces and the Foucault currents.

The acoustoelastic correction is implemented in Comsol by simply imposing an anisotropic stiffness tensor for the lead sheet in the Plane Strain mode, so that it equals the tensor [***C'***] for a given pre-stress configuration.

Finally, the multiphysic coupling is unidirectional, from the AC/DC transmitting mode to the Plane Strain propagation mode, and from this one to the AC/DC receiving mode.

4. Results

The Comsol model described so far is used to simulate the propagation of an ultrasonic pulse excited by one damped cycle of a 20 KHz sinusoidal current flowing through the transmitting EMAT coil (Figure 6).

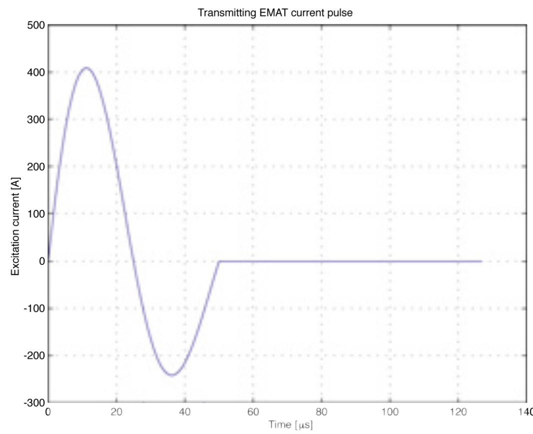

Figure 6: excitation pulse

In Figure 7 the time behavior of the receiving EMAT coil voltage for three applied stresses, uniaxially directed along the *x* axis, is plotted. As the increase in speed due to the acoustoelastic effect is quite small, in Figure 8 a close-up of the a_0 mode lower frequency components is provided. It can be anyway seen as the s_0 component is unaffected, while the a_0 is affected especially at low frequency, as reported in [7].

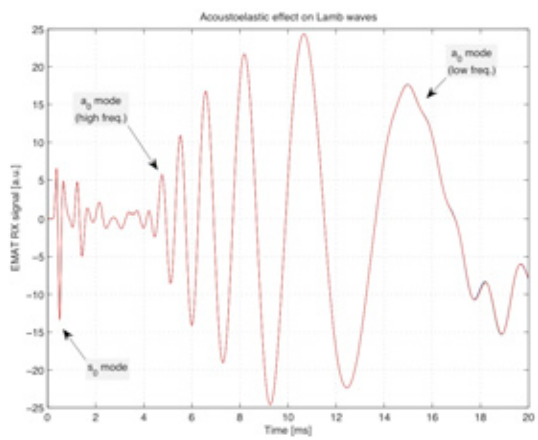

Figure 7: Lamb waves excited and received by two EMATs.

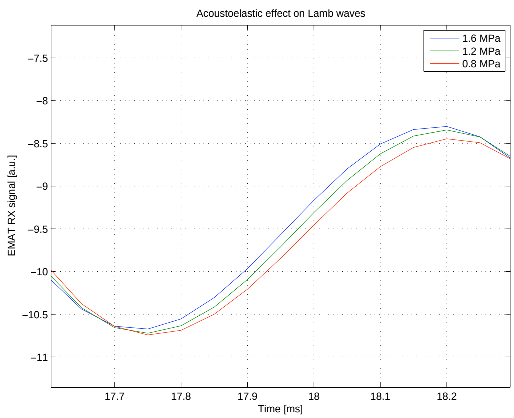

Figure 8: acoustoelastic effect.

7. Conclusions

In this work a first investigation on the modeling of the acoustoelastic effect in metal sheets with Comsol has been carried on. The theoretical basis of the ultrasonic waves propagation in pre-stressed solids have been recalled, and an equivalent stiffness tensor was derived and included in a Plane Strain mode. Excitation and reception of the acoustic waves via EMAT non-contact transducer has been successfully achieved, and it was possible to show a qualitative agreement between simulated data and the literature regarding the behavior of Lamb waves in pre-stressed media. Further developments will include the constitutive nonlinearity in the modeling of the acoustoelastic effect, by considering the use of higher order elastic constants.

8. References

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