Time Dependent Dirac Equation FEM Solutions for Relativistic Quantum Mechanics

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Abstract

INTRODUCTION: The Dirac equation is employed in particle physics and historically provided the first combined application of quantum mechanics and relativity theory by introducing a four component wave function {Ψi}. This wave function described the behavior of fermion type particles (e.g. electrons) and further predicted the existence of anti particles (e.g positrons) even before they were observed experimentally. COMSOL is used for obtaining a simpler 2D wave function { Ψ1(x,y,t),Ψ4(x,y,t) } as a solution to the time dependent Dirac equation [1]. The probability of a particle at a spacial point is treated with the "wave function formulation" which involves solving field pde's for Ψi .
USE OF COMSOL MULTIPHYSICS®: The Coefficient-Form PDE "time dependent" study is employed . Known solution examples are solved to test the COMSOL Dirac transient solving capabilities, followed by examples with no exact solution. When the wave vector k lies in the xy plane, the 4 component Ψi uncouples into two components for i=1&4.
RESULTS: Bar Model: The building block of Dirac theory is the plane wave solution, as illustrated with a 2-D finite bar (Fig. 1a inset), which is end driven with {Ψ1}oExp(-iωt) and truncated with Ψ1=0 B.C. . The wave propagation is shown in Fig.1a illustrating a comparison of the exact vs FEM transient solution. The dotted steady state limit [1], is shown for comparison.
Radial Cylindrical Model: A 2-D (Fig. 1b inset) semi-infinite circular annulus region inner surface is end driven with {Ψ1}oExp(-iωt) and the outer surface is terminated with an absorbing B.C. . Good agreement between FEM and Exact Steady State Limit solutions is achieved for{Ψ1,Ψ4} components at time snapshot t′=t/(2π/ω) = 9.
Cylindrical Scattering Model: A plane wave {Ψ1}oExp(kx-iωt) impinges upon a triangular pattern of three cylinders, using a ∂Ψ1/∂n =0 B.C. on each cylinder surface. The |Ψ1| is shown in Fig. 1c , where the presence of multiple scatters and resulting interference patterns is illustrated.
Two Slit Model: An incident PW enters an infinite domain via two slits. In Fig.1d, it is observed that waves emerging from the slits interact, forming bands of constructive and destructive interference. The probability density [1], ρ*(x,y) = |Ψ1|^2 + |Ψ4|^2, represents the probability/volume of a particle being at (x,y) . The solution shows the destructive interference result that at cut x′=x/λ=1.9, the in-line ρ*(x,y) is ≈.272 times smaller than the ±λ/2 off-line ρ*.
CONCLUSION: The agreement between the Exact-FEM solution validations was good. Solutions to the PW incident upon a two slit barrier, produced patterns showing bands of null zones due to destructive interference.
References:

[1] Paul Strange, Relativistic Quantum Mechanics, Cambridge University Press Cambridge UK, 1998.

Figures used in the abstract

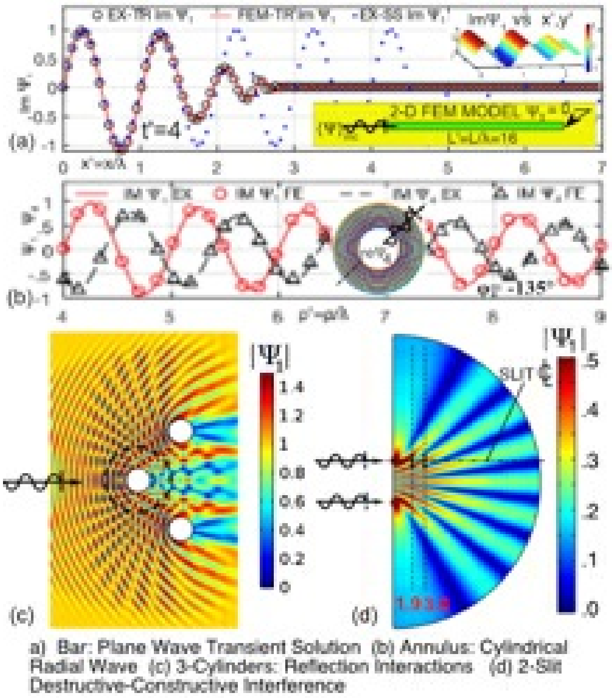

Figure 1: Validation FEM vs Exact solutions to the Dirac Equations.