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Dynamics of Slender Structures

(Beam Theory with COMSOL PDE Solver)

by

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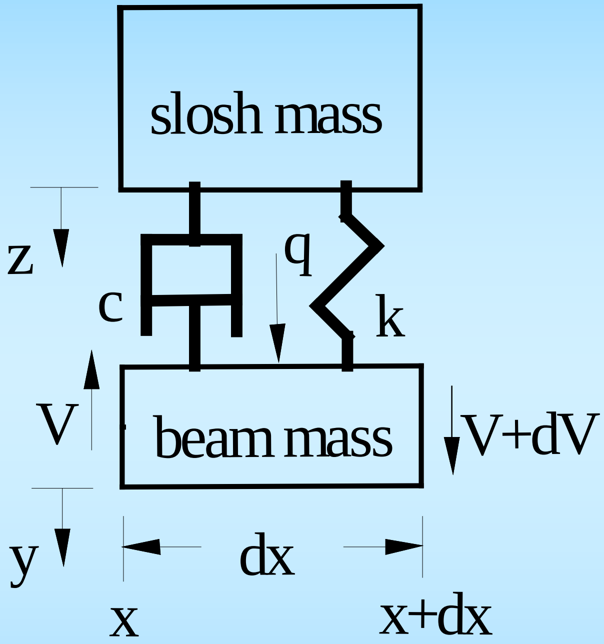
Problem Definition



$$dV + q\,dx = (\rho A\,dx)\frac{d^2 y}{dt^2}$$

$$\frac{\partial^2 M}{\partial x^2} \equiv \frac{\partial V}{\partial x} = -q + \rho A \frac{\partial^2 y}{\partial t^2}$$

$$M = -EI \frac{\partial^2 y}{\partial x^2}$$

$$\frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 y}{\partial x^2}\right) = q - \rho A \frac{\partial^2 y}{\partial t^2}$$

COMSOL Implementation

Coefficient Form for PDE Models

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} - c_a \frac{\partial^2 u}{\partial x^2} + au = f_a$$

Harmonic Solutions and Eigenvalue Solver

$$\partial^2 u / \partial t^2 = \lambda^2 u \qquad \partial u / \partial t = -\lambda u$$

$$-c_a \frac{\partial^2 u}{\partial x^2} + au = d_a \lambda u - e_a \lambda^2 u$$

$$\frac{\partial^2 M}{\partial x^2} = \rho A \lambda^2 y \qquad EI \frac{\partial^2 y}{\partial x^2} + M = 0$$

Scaled Equations

$\xi = x/L,\ \eta = y/L,\ m = ML/EI,\ \tau = t/t_c,$

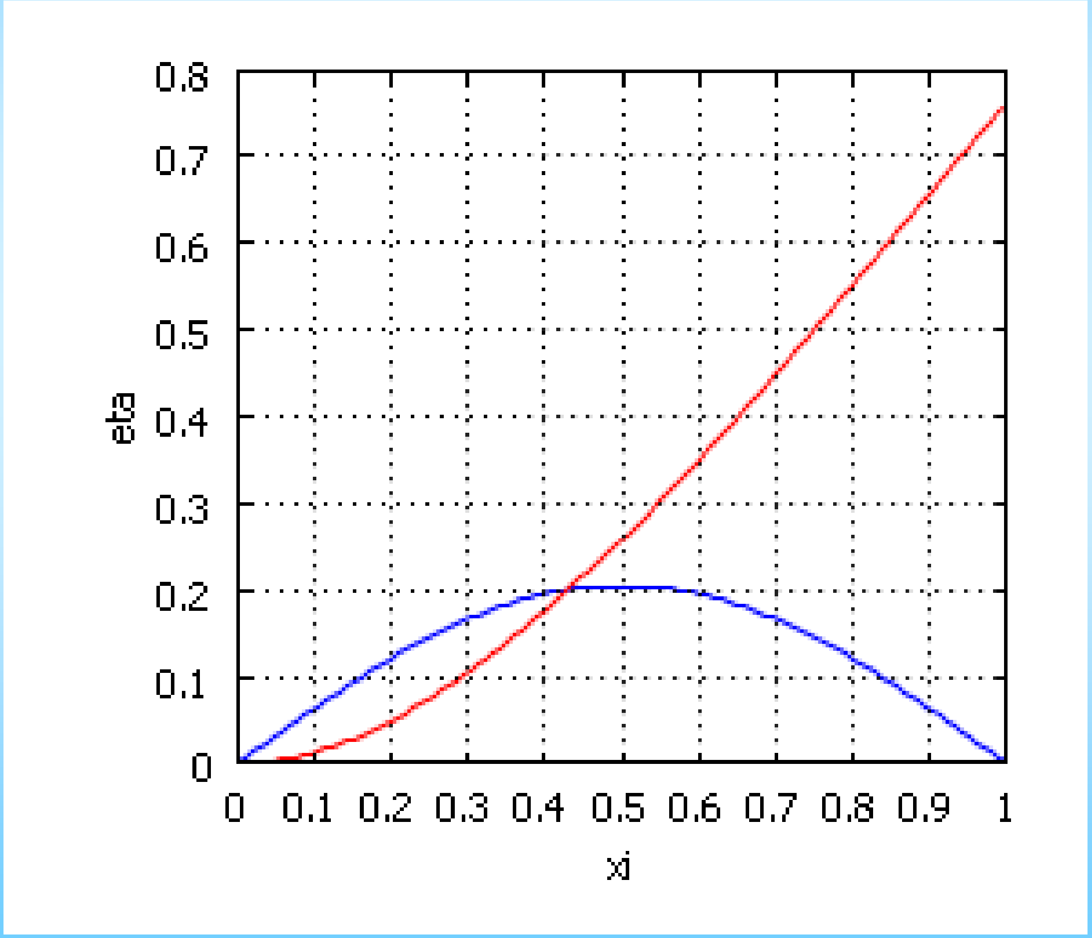
$$t_c = \sqrt{\frac{\rho A L^4}{EI}} = L^2\sqrt{\frac{\rho A}{EI}}$$

$$\frac{\partial^2 m}{\partial \xi^2} + \frac{\partial^2 \eta}{\partial \tau^2} = 0$$

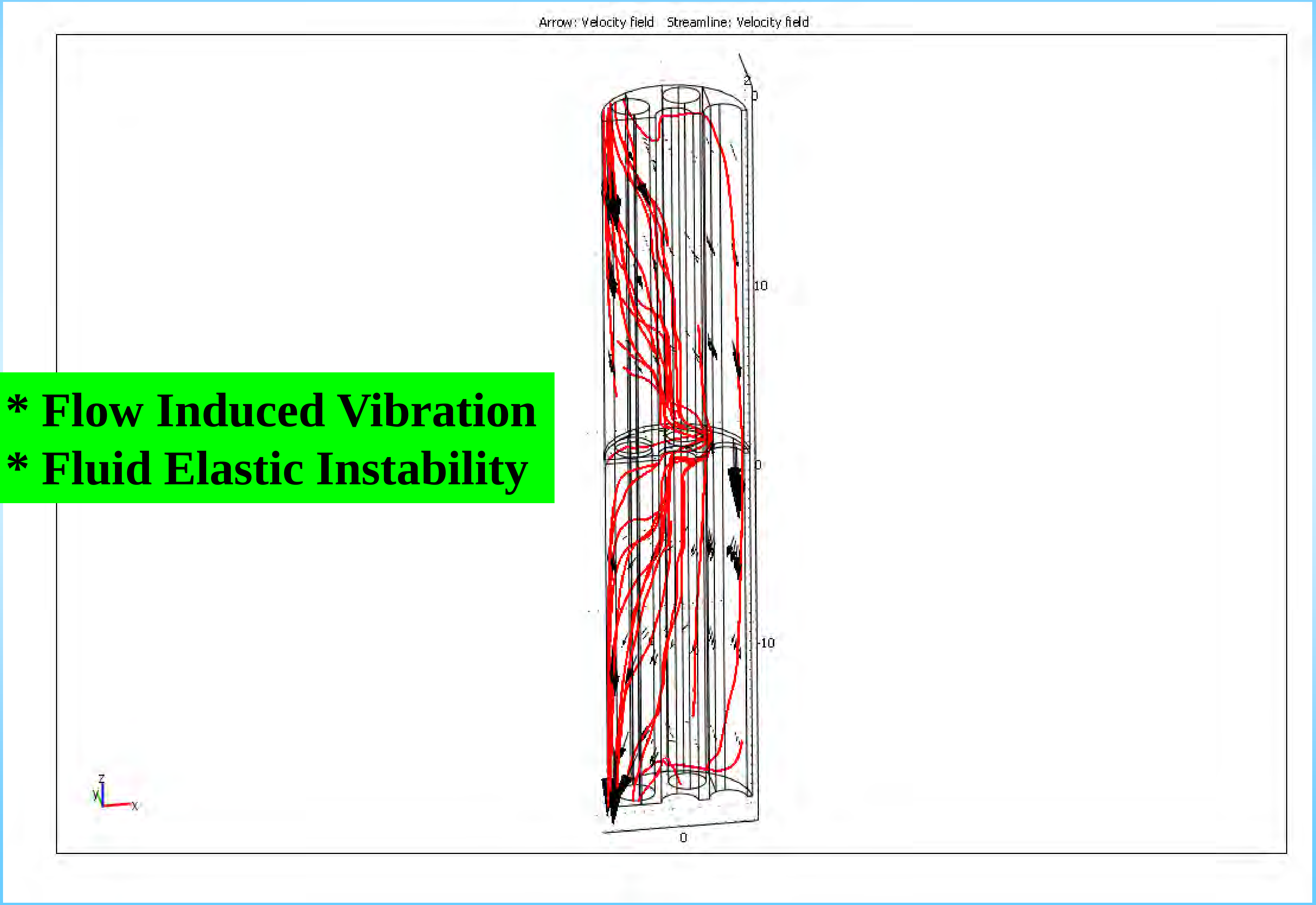
$$\frac{\partial^2 \eta}{\partial \xi^2} + m = 0$$

$\lambda = -9.8696i = -\pi^2 i$

Cantilever: $\lambda = -3.515i$

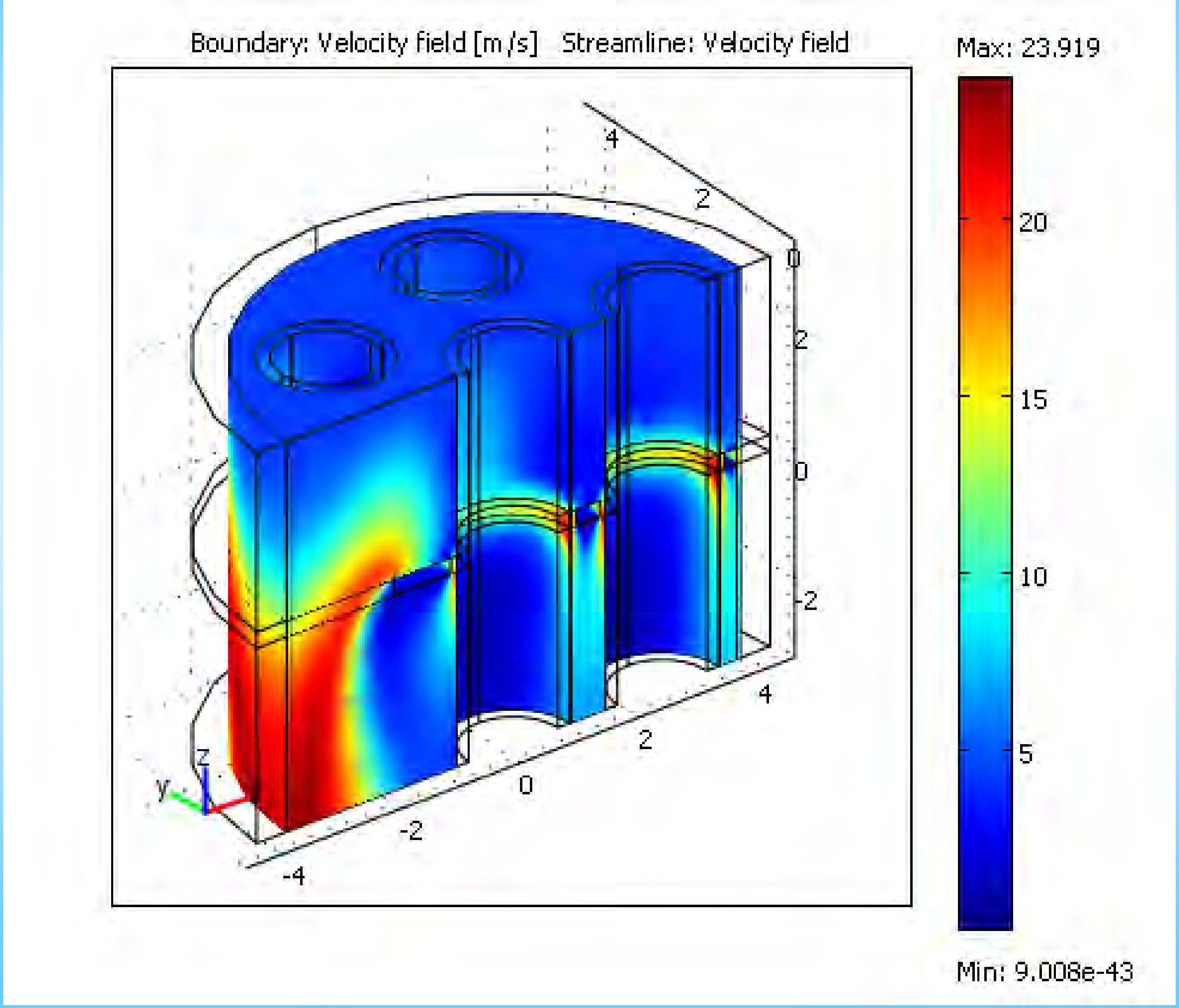


Typical Heat Exchanger with Tubes

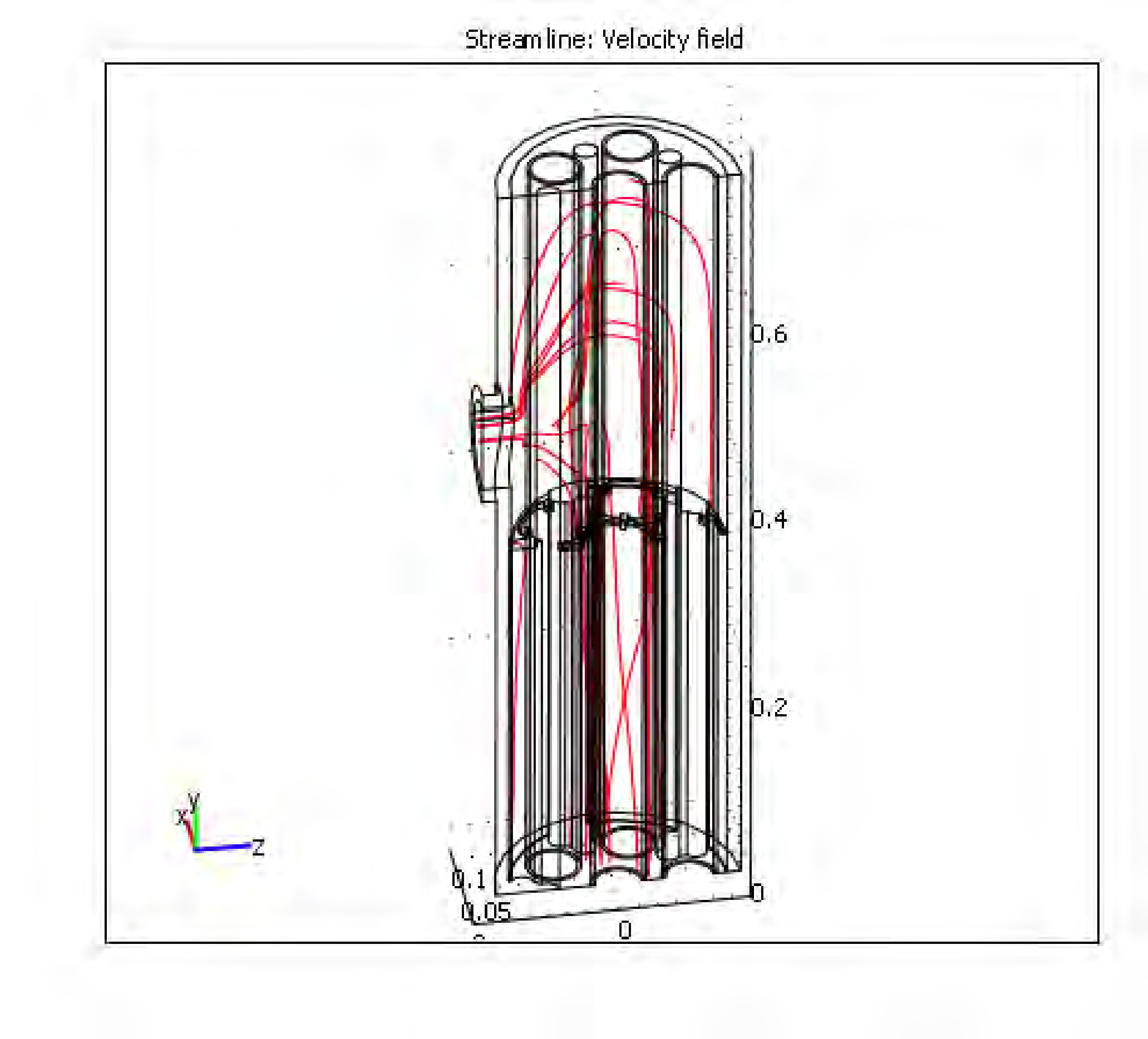


*** Flow Induced Vibration**
*** Fluid Elastic Instability**

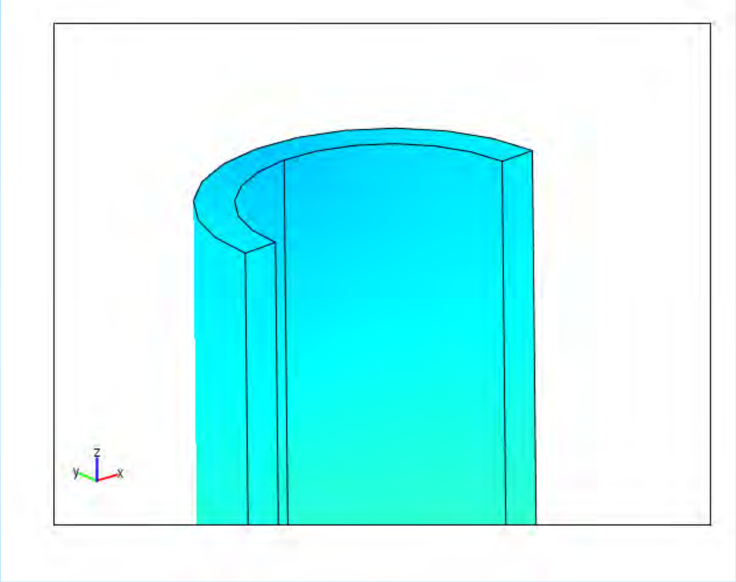
Heat Exchanger Flow at Baffle

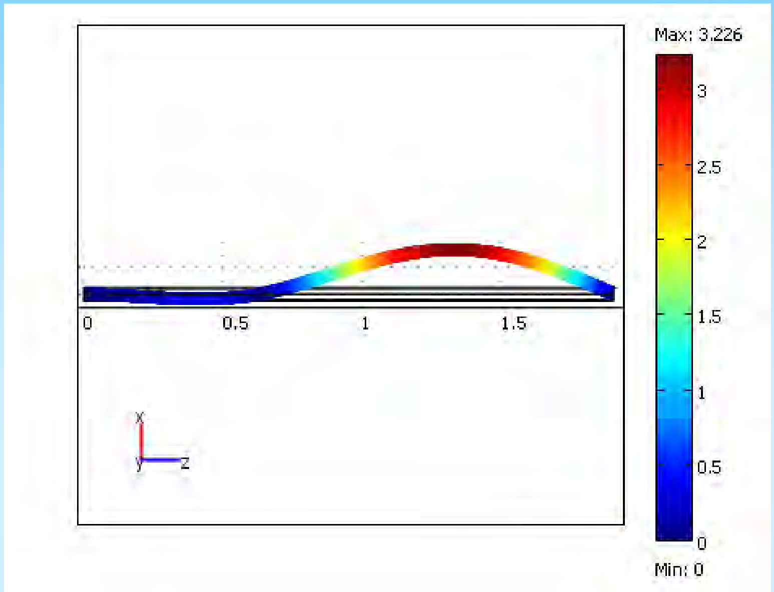


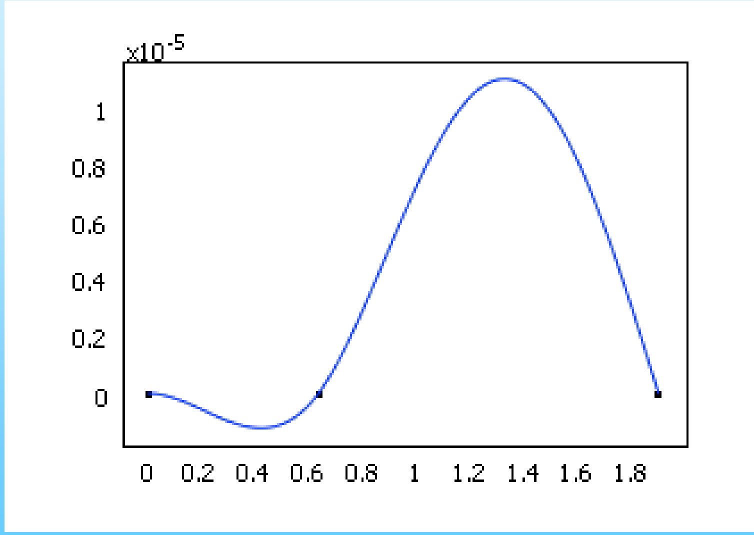
Heat Exchanger with Inlet Cross-Flow

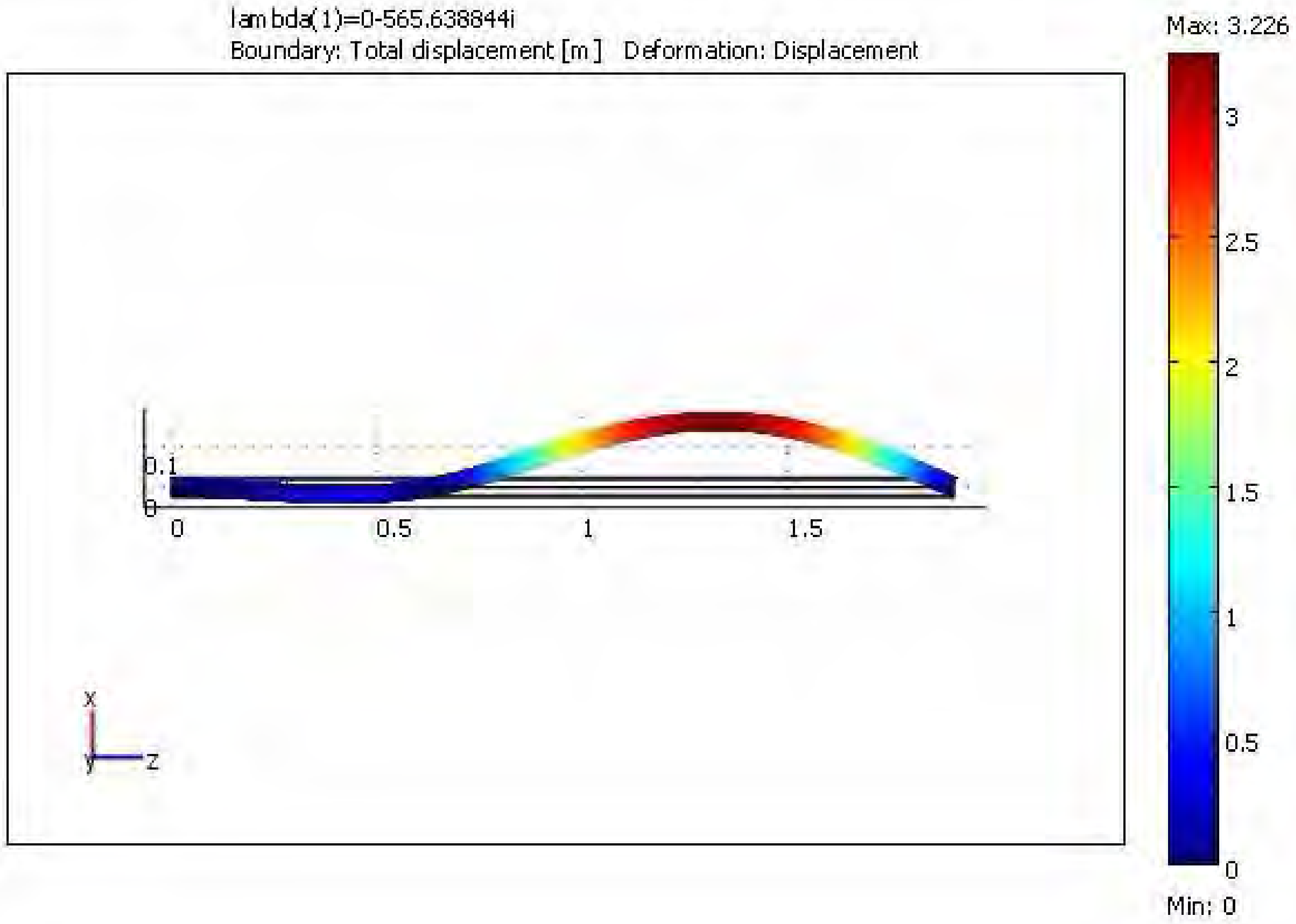


Model of Heat Exchanger Tube

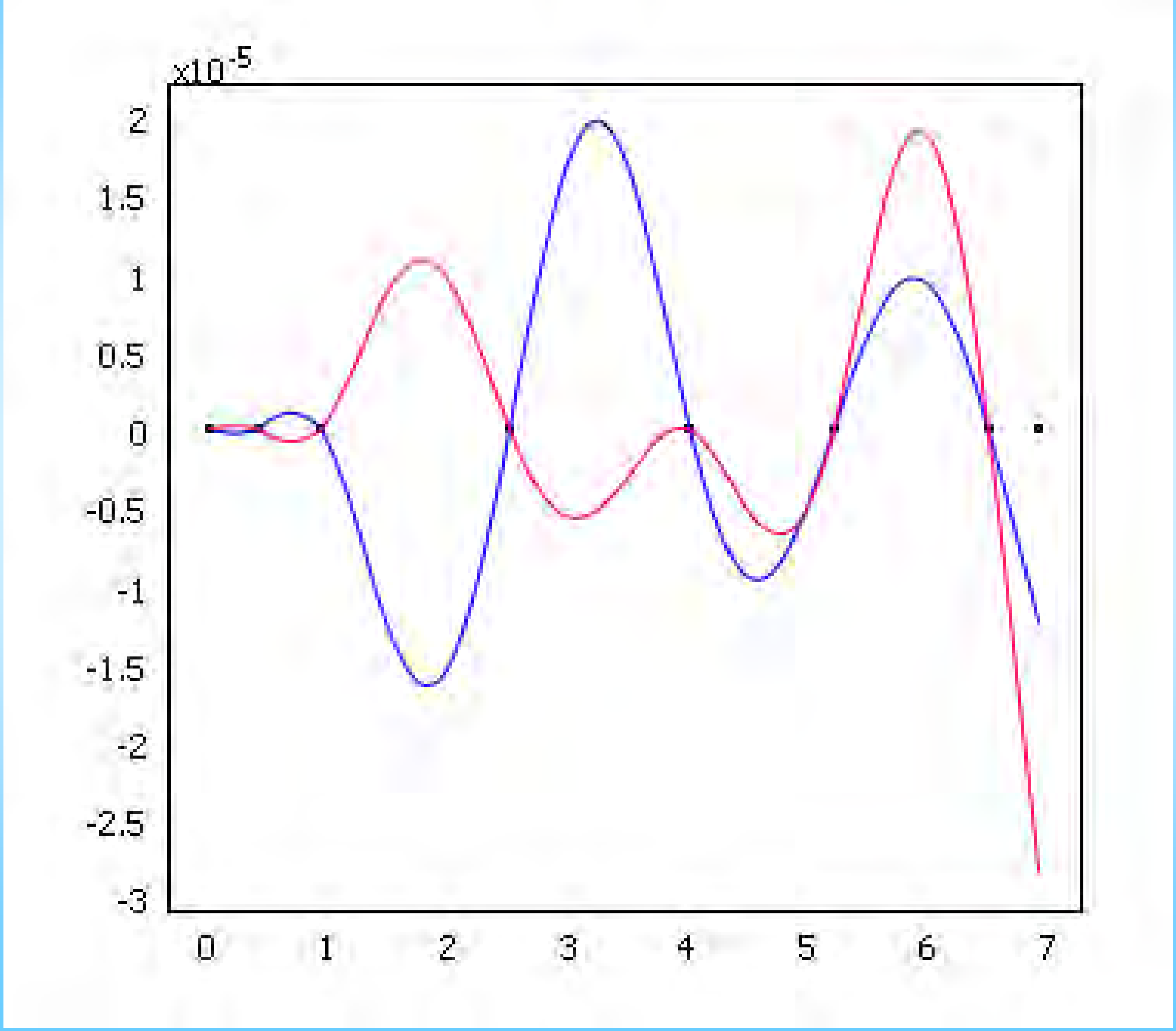







lambda(1)=0-565.638844i
Boundary: Total displacement [m] Deformation: Displacement
Max: 3.226
3
2.5
2
1.5
1
0.5
0
Min: 0
0.1
0
0
0.5
1
1.5
x
y
z

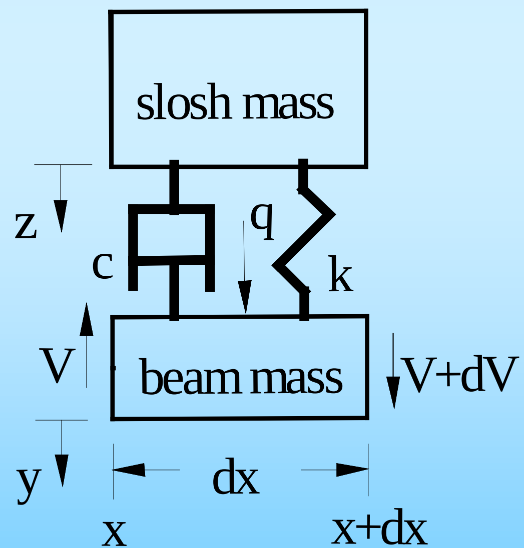
Continuous Beam Vibration Modes

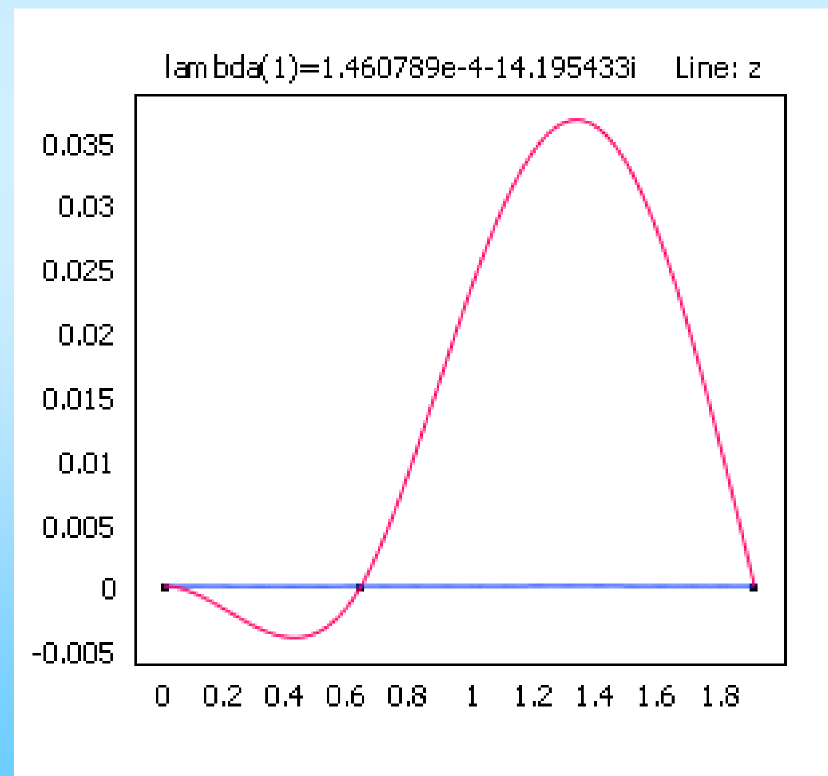


Effect of Sloshing

$$-[c_a]\frac{\partial^2 \vec{u}}{\partial x^2}+[a]\vec{u}=[d_a]\lambda\vec{u}-[e_a]\lambda^2\vec{u}$$

$$-\begin{bmatrix}0 & -1 & 0\\ -EI & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}y''\\ M''\\ z''\end{Bmatrix}+\begin{bmatrix}-k_s & 0 & k_s\\ 0 & 1 & 0\\ k_s & 0 & -k_s\end{bmatrix}\begin{Bmatrix}y\\ M\\ z\end{Bmatrix}=-\begin{bmatrix}-c_s & 0 & c_s\\ 0 & 0 & 0\\ c_s & 0 & -c_s\end{bmatrix}\begin{Bmatrix}\dot{y}\\ \dot{M}\\ \dot{z}\end{Bmatrix}-\begin{bmatrix}-\rho A & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -m_s\end{bmatrix}\begin{Bmatrix}\ddot{y}\\ \ddot{M}\\ \ddot{z}\end{Bmatrix}$$





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