On the use of a diffusion equation model for sound energy flow prediction in acoustically coupled spaces

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Abstract: This paper studies sound energy flows through an aperture across two coupled spaces using a diffusion equation model. The so-called double sloped sound energy decay is believed to be caused by the sound energy exchange through the aperture coupling the two rooms. However, little light has been shed on this aspect. The diffusion equation model is chosen to model the sound energy flow due to its high efficiency and accuracy on room-acoustic prediction. The room diffusion equation is first solved by COMSOL, then the time dependent energy flow is visualized via COMSOL arrow plotting, which reveals the sound energy feedback through the aperture. In addition, this study also discloses a “flipping-over” characteristic of the energy flow decay which is shown to be highly dependent on the size and the location of the aperture.

Keywords: Coupled spaces, diffusion equation model.

1. Introduction

Acoustically coupled spaces are several spaces acoustically connected through apertures (windows, doors, etc.). This is an interesting subject since sound energy can travel back and forth between coupled spaces, and in certain cases, the sound energy first flows from one room to the other rooms and then flows back, which results in non-exponential sound energy decay. In a single-volume room, the sound energy decay usually follows an exponential decay profile. The so-called double-sloped energy decay can happen in two acoustically coupled spaces where the room with the source (primary room) is less reverberant than the coupled room (secondary room). This phenomenon has attracted many room-acousticians and has been studied for a long time[1,2]. Generally, in a double-sloped decay curve, the sound energy decays at a fast rate for the first few milliseconds and then decays at a noticeablely slower rate afterwards. The benefit of this type of decay profile is that, the clarity is satisfied by the first decay and the reverberence is effected by the second decay.

Room-acoustic design incorporating coupled spaces (double-sloped decay) has already been realized in several places[2], including Festival Hall in Tampa, FL; the Great Hall in Hamilton, Ontario; Lucerne Concert Hall in Lucerne, Switzerland; the Myerson-McDermott Hall in Dallas, TX; and Verizon Hall in Philadelphia, PA. However, there are still numerous aspects of coupled spaces that are not well understood, and need further investigations.

The objective of this paper is to study particularly the sound energy flow using a recently emerging geometrical acoustic model, i.e., diffusion equation model. The diffusion equation model is chosen because it is highly efficient[3-7] and inherently suitable for calculating the sound energy flow.

2. Governing equations

Given the slow rates of energy change with time, the gradient of the sound energy density $w(\mathbf{r},t)$ at position $\boldsymbol{r}$ and time $\boldsymbol{t}$ in the room under investigation causes the sound energy flow vector $\boldsymbol{J}$ [3,4],

$$\mathbf{J} = -D\ \mathbf{grad}\ w(\mathbf{r},t)\,, \qquad (1)$$

where $D = \lambda c/3$ is defined as diffusion coefficient with λ being the mean free path of the room. The expression in eq. (1) follows the physical analogy with the diffusion of particles in a scattering medium[3], it assumes sound particles traveling along straight lines at sound speed c in the room with diffusely reflecting walls. In any room with sound sources, the change of the sound energy density per unit time is expressed as a diffusion equation

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D\nabla^2 w(\mathbf{r},t) + c\,m\,w(\mathbf{r},t) = q(\mathbf{r},t), \qquad \in V \tag{2}$$

where the subroom denoted by domain V has a source term $q(\mathbf{r},t)$, which is a delta Dirac function when studying the impulse response of a room. The term $c\,m\,w(\mathbf{r},t)$ accounts for air dissipation in the room(s) [7]. Equation (3) is the interior equation in domain V, being subject to the boundary condition on the interior surface S,

$$D\frac{\partial w(\mathbf{r},t)}{\partial n} + \frac{c\alpha}{2(2-\alpha)}\; w(\mathbf{r},t) = 0\,. \tag{3}$$

It has have demonstrated that the diffusion equation with this boundary condition can model surfaces with a wide range of absorption coefficients without losing significant accuracy.

In addition, the time-dependent decaying function at location $\mathbf{r}$ of energy flow levels is defined as

$$J_L(\mathbf{r},t) = 10 \;\; \log\left[\left(\frac{\partial w(\mathbf{r},t)}{\partial x}\right)^2 + \left(\frac{\partial w(\mathbf{r},t)}{\partial y}\right)^2 + \left(\frac{\partial w(\mathbf{r},t)}{\partial z}\right)^2 \right]^{\frac{1}{2}}, \tag{4}$$

with the diffusion coefficient D being neglected, since relative levels of the energy flow in each room are of major concern.

3. Sound energy flows

We herein study a few important and intriguing aspects of sound energy flows in two acoustically coupled spaces. Figure 1 illustrates the geometry of the coupled spaces in this study. The aperture is a square with a width 3m, and spans from y = 0.32 m to y = 3.32 m. The smaller room is the primary room with an acoustic source located at (-5, 2.4, 1.3) m.

By recognizing that the double-sloped decay or energy flow feedback normally only occurs when the primary room is less reverberant than the secondary room. The absorption coefficients on the walls in each room are carefully chosen as 0.2 and 0.1, for the primary room and the secondary room, respectively. This configuration leads to the fact that the reverberation times in two rooms are 0.66s, and 1.78s. Both numbers are predicted by Eyring equation.

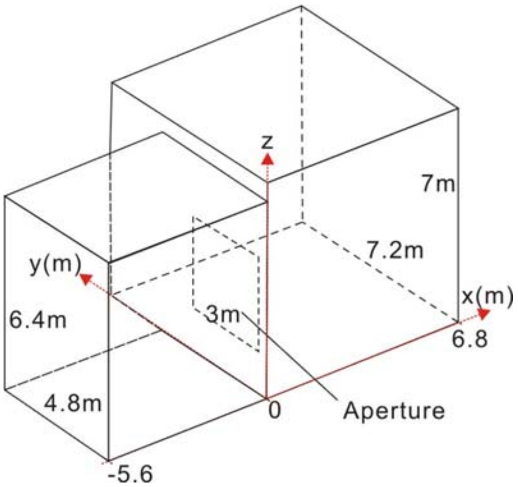

Figure 1. Dimensions and co-ordinate definition of the two coupled rooms.

3.1 Sound energy flow feedback

This section addresses the sound energy flow feedback predicted by the diffusion equation model. To calculate the time-dependent energy flow, an impulsive sound energy is generated by the acoustic source to excite the room. The diffusion equation model is solved by COMSOL, the PDE modes, with approximately 8,000 meshes which fulfills the mesh condition determined by the mean-free path length[4]. For more details regarding the numerical implementing of the diffusion equation model, the readers are referred to refs [4-6].

After solving the sound energy density *w*, the energy flow can be easily shown by COMSOL arrow plotting. Figure 2 shows the two-dimensional plots at the height of 3m for different times, and clearly reveal the sound energy feedback which approximately occur at t=0.1s in this case. In other words, the sound energy flows back from the secondary to the first room at around t=0.1s. This happens because at the first beginning, the sound energy in the primary room is always higher since the source is inside the primary room. However, the sound energy in the secondary room decays more slowly than the primary room does, so at a certain time, the energy in the secondary room becomes higher and flows back to the primary room. To verify this statement, additional simulations have also been executed and manifest that when the primary room is equally or more reverberant than the secondary room, the energy feedback does not exist in this case.

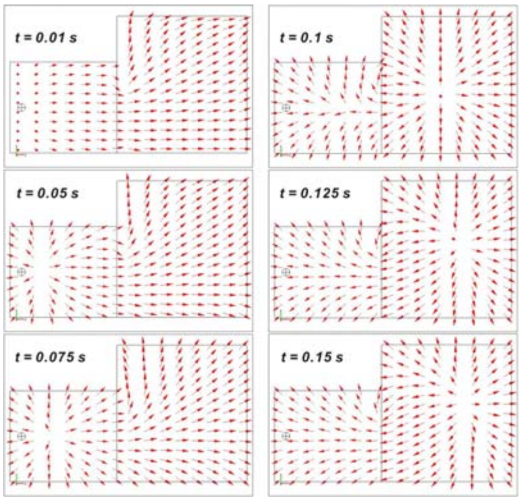

Figure 2. Time-dependent energy flows in two coupled spaces for different times.

3.2 Sound energy flow decay

It is also interesting to study the sound energy flow amplitude decay by using eq. (4). Figure 3 presents the energy flow decays in the primary room. Four positions at R1 (-0.05, 2.5, 3) m, R2 (-0.05, 4.5, 3) m, R3 (-2, 4.5, 3) m, R4 (-2, 2.5, 3) m are chosen. Distinct “dips” of the energy flows are found for R1 and R4, both being along the line y = 2.5 m. The energy flow dips occurs at around 105 ms and 110 ms, which is the time the energy feedback from the secondary room is about to happen observed from Fig. 2. The dip of the energy flow level strongly indicates that, in a certain area, when the energy feedback happens or about to happen, the magnitude of the energy flow abruptly drops and rises along with flipping over the flow direction, thus forming a sharp dip in the energy flow decay function. For R2 and R3, being both along the line at y = 4.5 m, the energy-flows decay constantly, showing no abrupt transition. Additional simulations show that, the dip is more likely to take place between y = 0.1m and y = 4m close to the aperture. Notice that the aperture ranges from y=0.3 m to y=3 m, implying that the energy-flow dip area (refers to the area where the flow dips occur) is very likely to depend on the aperture size and location. Moreover, the flow direction-flipping is most intense along the line y=2m, which is approximately the center line of the aperture. The energy flow at R1 decays rapidly before the ‘dip’, and has the largest energy flow amplitude after the flipping-over, which has occurred because R1 is in the vicinity of the aperture.

Furthermore, it has been found that, (not shown here) in the proximity of the source, the “dip” also occurs. However, this is not caused by the energy feedback from the secondary room. Initially when the energy decays from an impulse, the energy is strongest around the source and the energy flows point outwards from the source. With time progressing, the region with the strongest energy somehow moves from the source to the other place (usually not far away from the source) in the room and stays steady, leading to that the energy flows point outwards from a different spot, and causing the energy flow directions flip over, which is the reason why “dip” occurs.

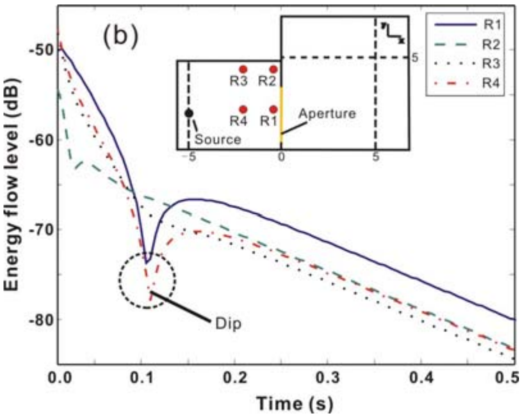

Figure 3. Time-dependent energy flows decay in the primary room.

4. Conclusions

In this paper, the diffusion equation model is employed to study both the sound energy flow directions and the energy flow decays for a coupled room system. The diffusion equation model is chosen for this study since it is able to calculate the time-dependent sound energy (which leads to the energy flow immediately) anywhere in the room on the order few minutes and has been proved to be relatively accurate.

The simulation results suggest that:
1. When the primary room is less reverberant than the secondary room, the energy flow feedback from the secondary room exists. The energy flow feedback is a result of the energy flow direction flipping-over, and the time when the flipping-over occurs is different for different locations in the coupled rooms.

2. The energy flow amplitude decay shows an interesting "dip" characteristic, which is also believed to be the product of the energy flow direction flipping-over. This "dip" characteristic does not appear to be anywhere in the room but is highly dependent on the size and the location of the aperture.

More details regarding our work on coupled spaces might be found in near future in refs [8-9].

5. References

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