Presented at the COMSOL Conference 2008 Boston

DEPARTMENT OF BIOMEDICAL ENGINEERING

Fluid-Structure Interaction Model of Active Eustachian Tube Function in Healthy Adult Patients

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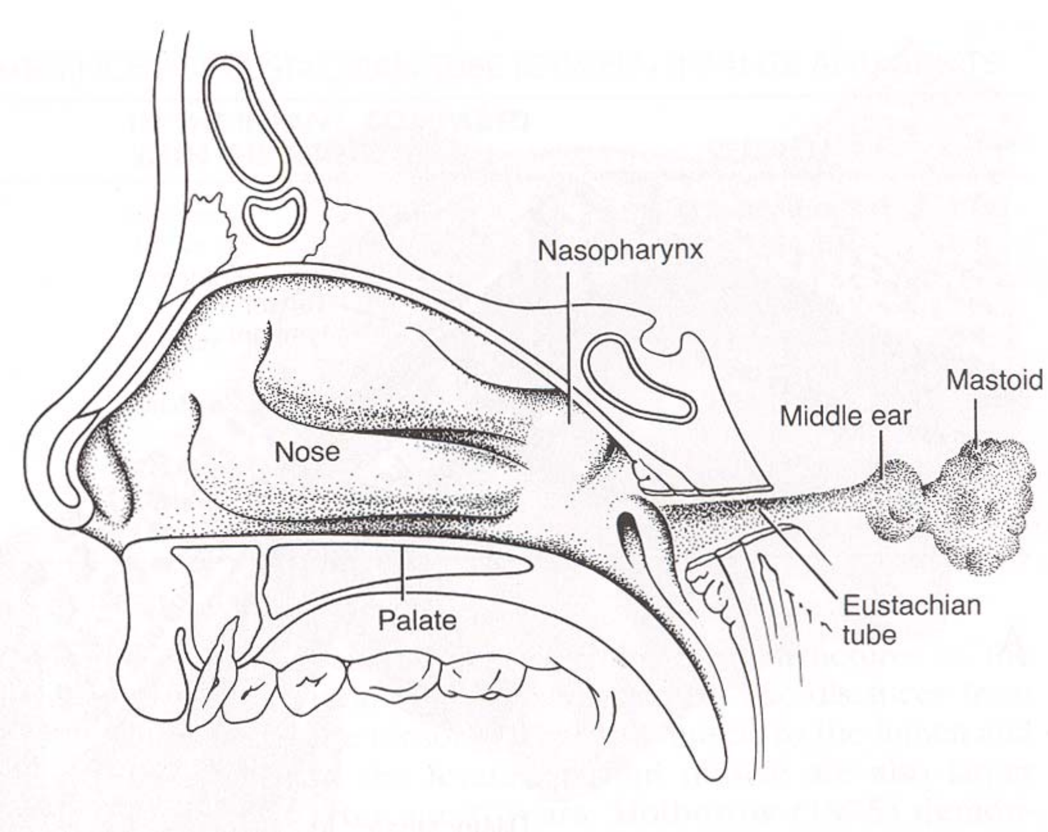
COMSOL Conference 2008

Background and Motivation



Reprinted from Bluestone, Charles D, and Klien, Jerome O., Otitis Media in Infants and Children Third edition, W.B. Saunders Company, 2001

Eustachian Tube (ET) connects the Middle Ear to the Nasopharynx.

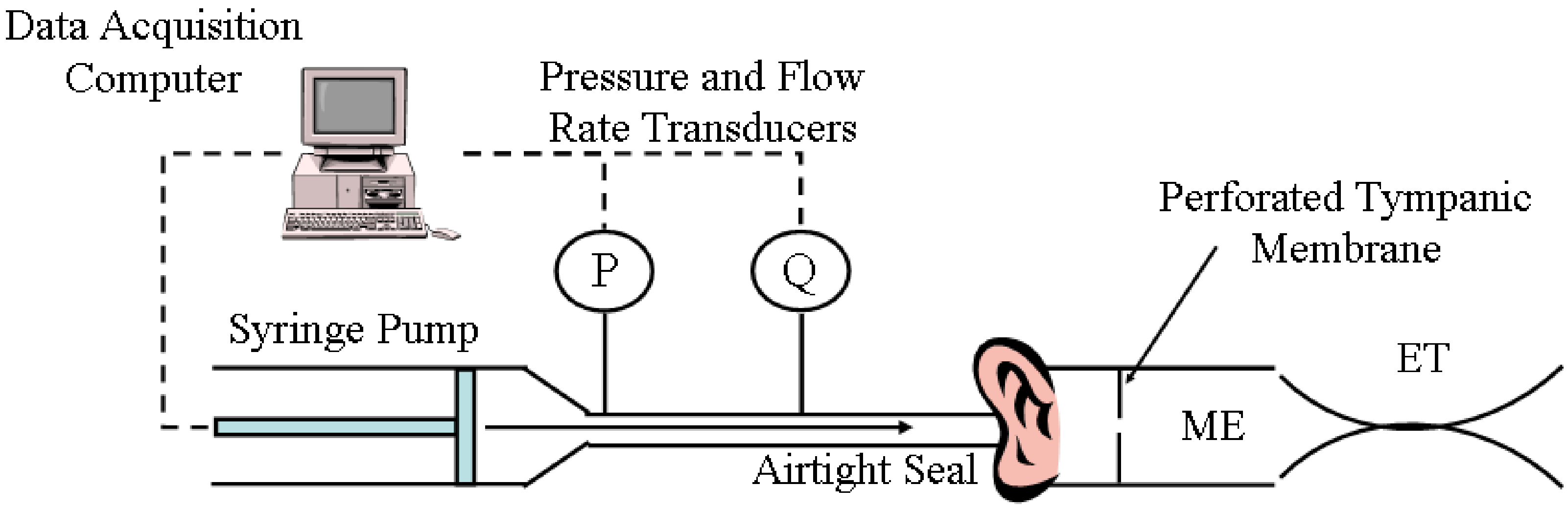
ET has three primary functions:

- Drainage of ME fluid
- Equilibrate ME pressure
- Protect ME from Pathogens

ET dysfunction has been directly linked to Otitis Media.

Otitis Media has an annual health-related cost of $4 billion dollars in the US. [1]

Modeling Goals



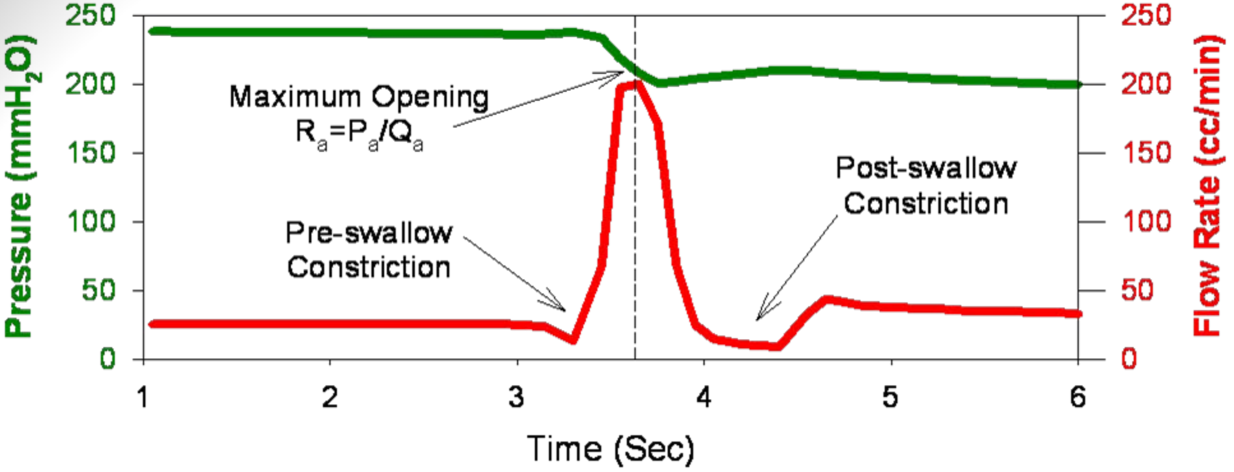
- In the clinics, we can perform a diagnostic procedure called a Forced Response Test (FRT)
  - Patient is hooked up to experimental apparatus shown above and asked to swallow

- First Goal: Create a transient COMSOL model that can replicate FRT results to serve as model validation

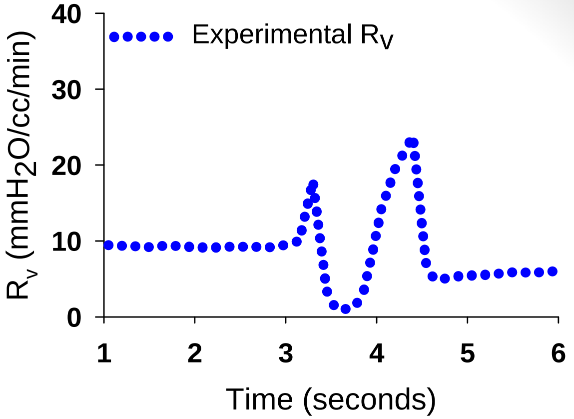
- Use validated model to explore other parameters involved in ET function

Experimental Data

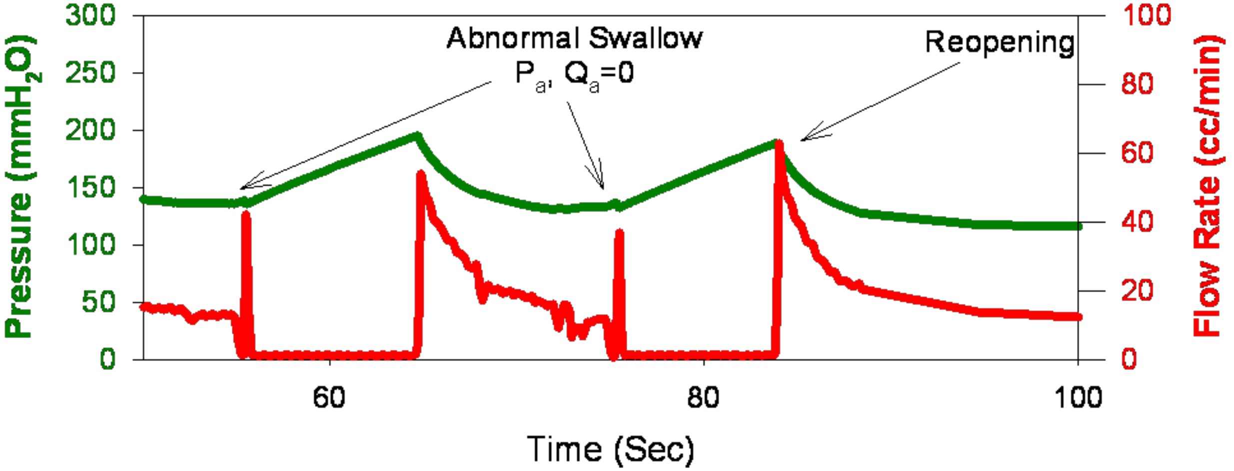
Normal ET Function:

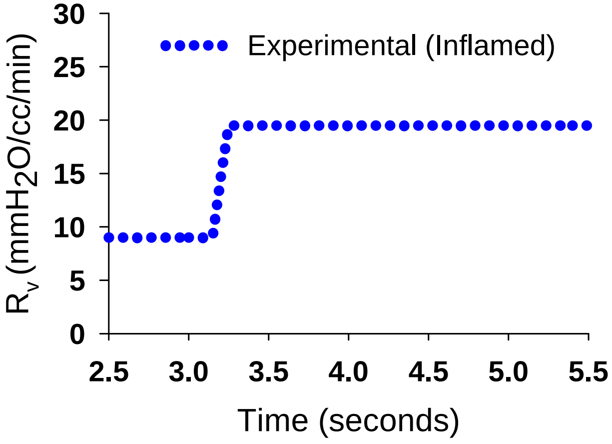


Resistance = P/Q



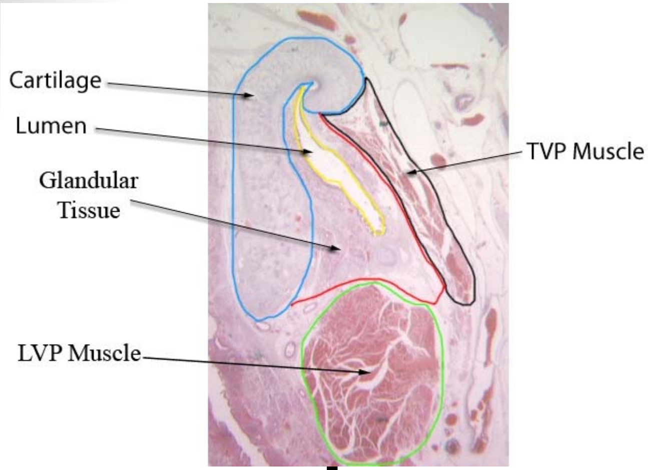
Patients with ET dysfunction:



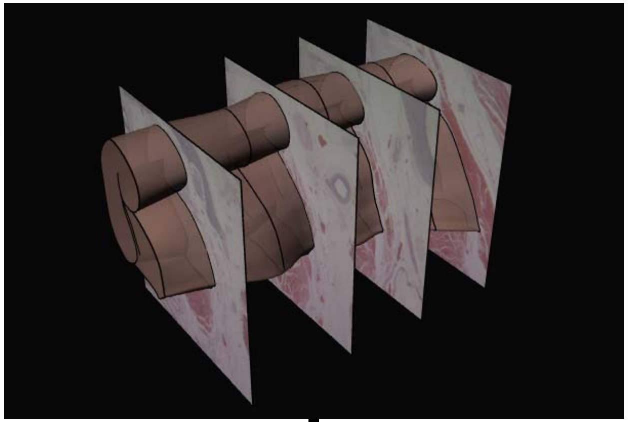


Structure of the Eustachian Tube

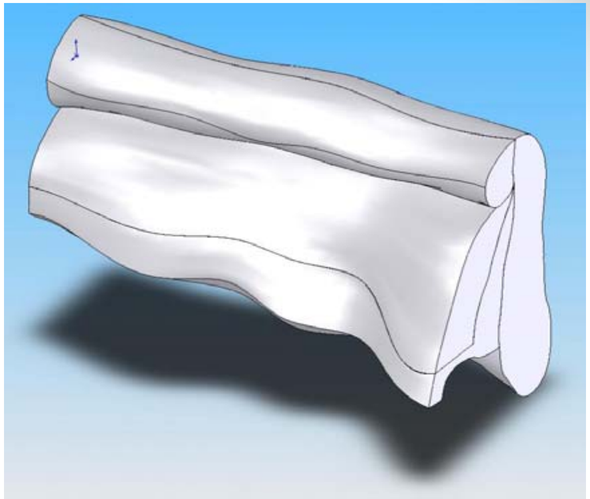
Define Cross Sections

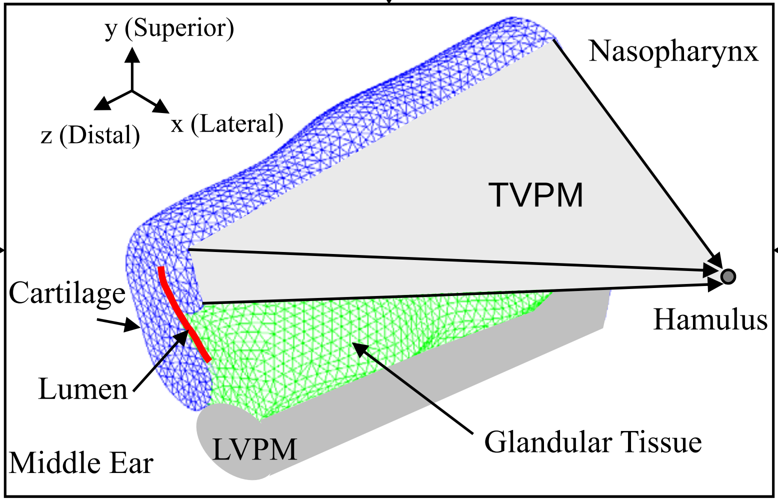


Loft Cross Sections

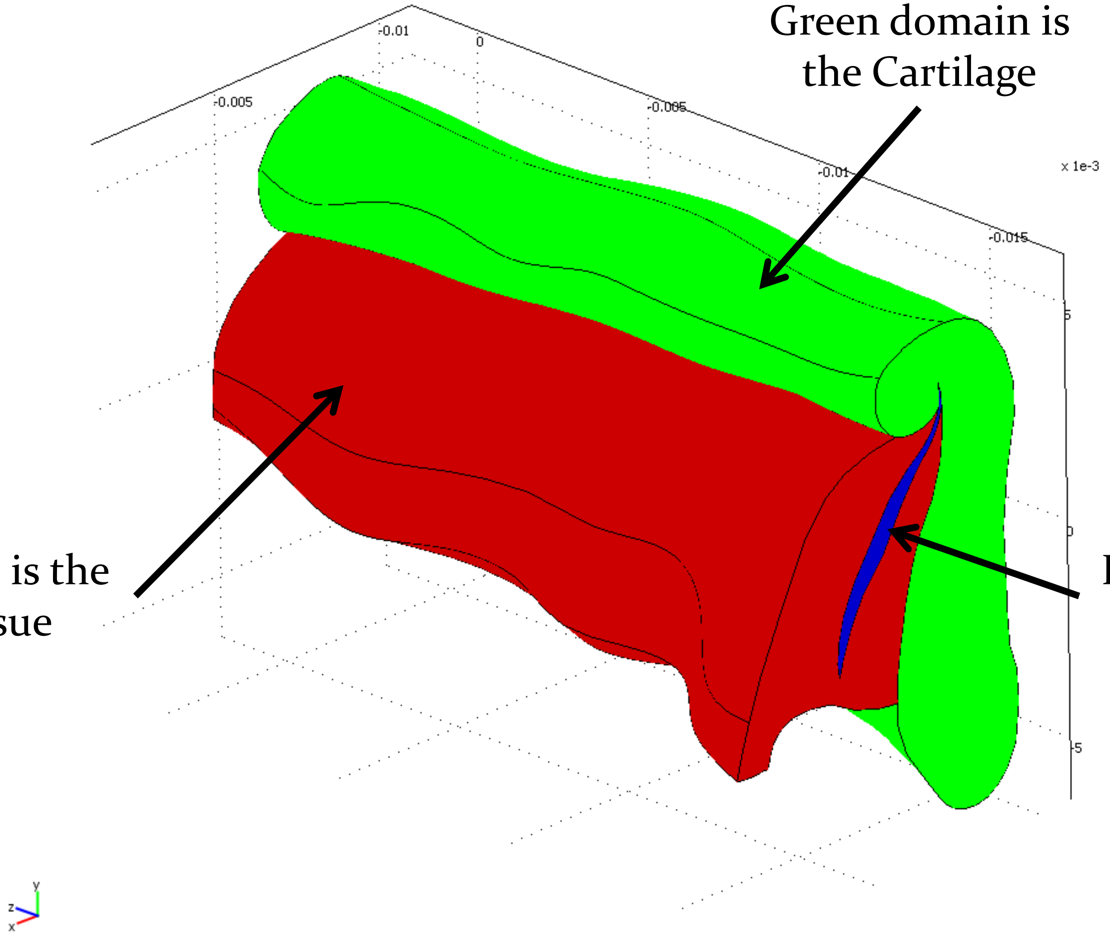


Full 3D CAD Model





Final Analyzed 3D Geometry



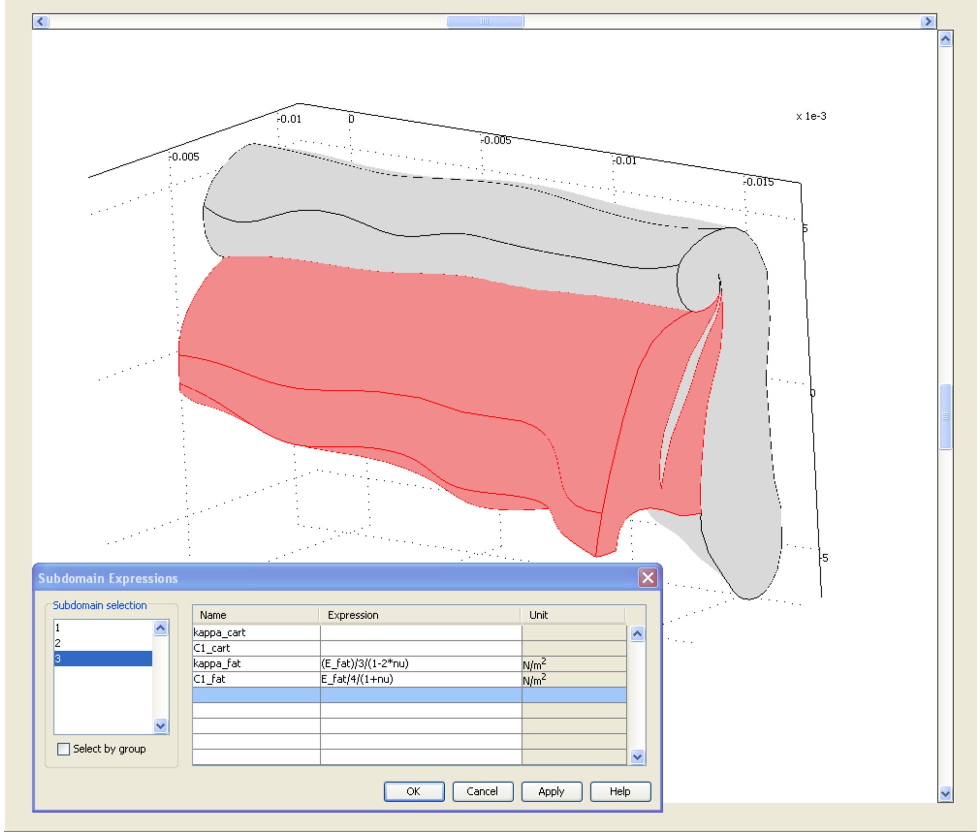
Green domain is the Cartilage

Red domain is the Fatty Tissue

Blue domain is the Lumen Opening

Solid Mechanics Properties

Both Cartilage and Glandular Tissue are represented by Mooney-Rivlin Hyperelastic material models.



Name	Expression	Unit
kappa_cart		
C1_cart		
kappa_fat	(E_fat)/3/(1-2*nu)	N/m^2
C1_fat	E_fat/4/(1+nu)	N/m^2

E_{cart}=300kPa [2,3] E_{Gland}=50kPa [2,3]

$$\kappa = \frac{E}{3(1-2\upsilon)} \qquad C_1 = \frac{E}{4(1+\upsilon)}$$

$$W_{hyp} = C_{01}\left(\bar{I}_2 - 3\right) + \frac{1}{2}\kappa\left(J-1\right)^2$$

$$P = \frac{\partial W_{hyp}}{\partial \nabla \mathbf{u}} \text{ where } \mathbf{u} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

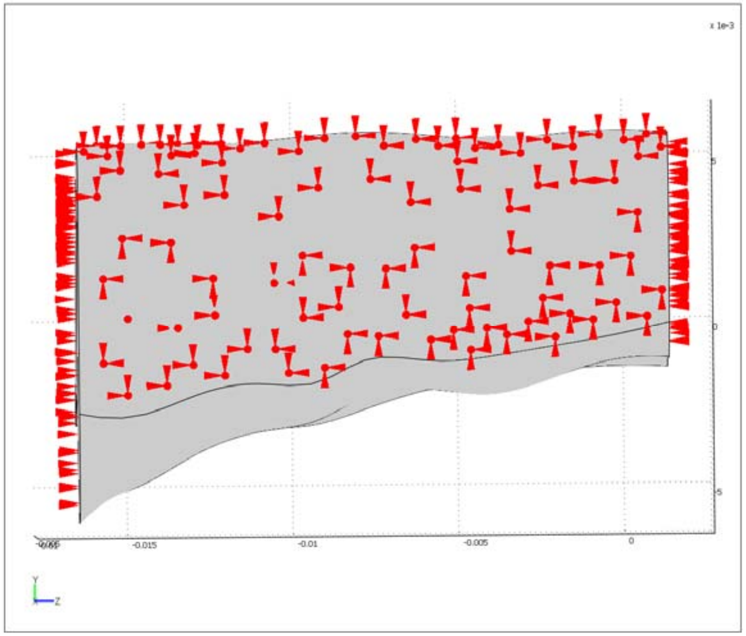
Solid Boundary Conditions

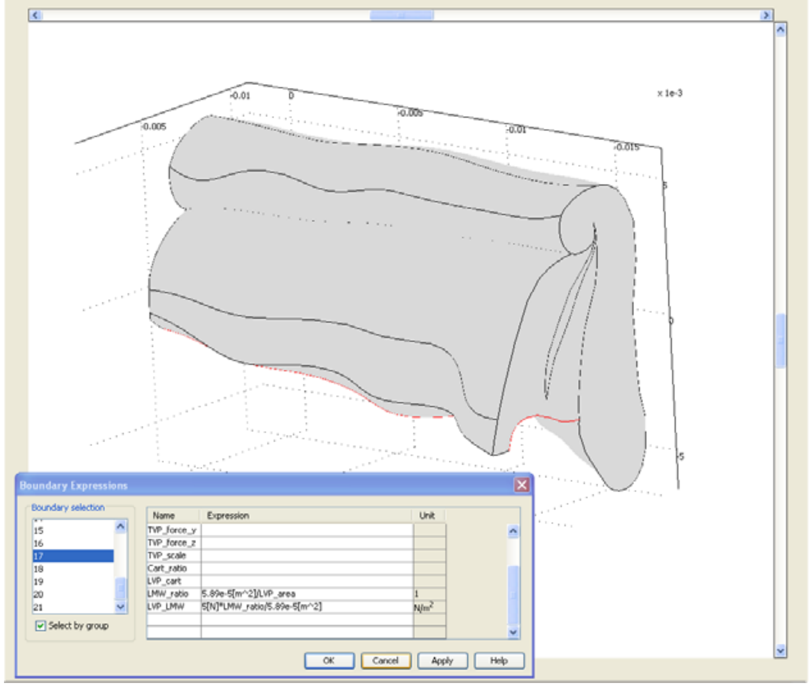
Boundary Conditions:

- Attachment of ET to the cranial base of the skull
- Attachment of soft tissue to the bony portions of the ET at the proximal and distal ends

Applied Loads:

- Tensor Veli Palatini (TVP) muscle force
- Levator Veli Palatini (LVP) muscle force
- Fluid forces

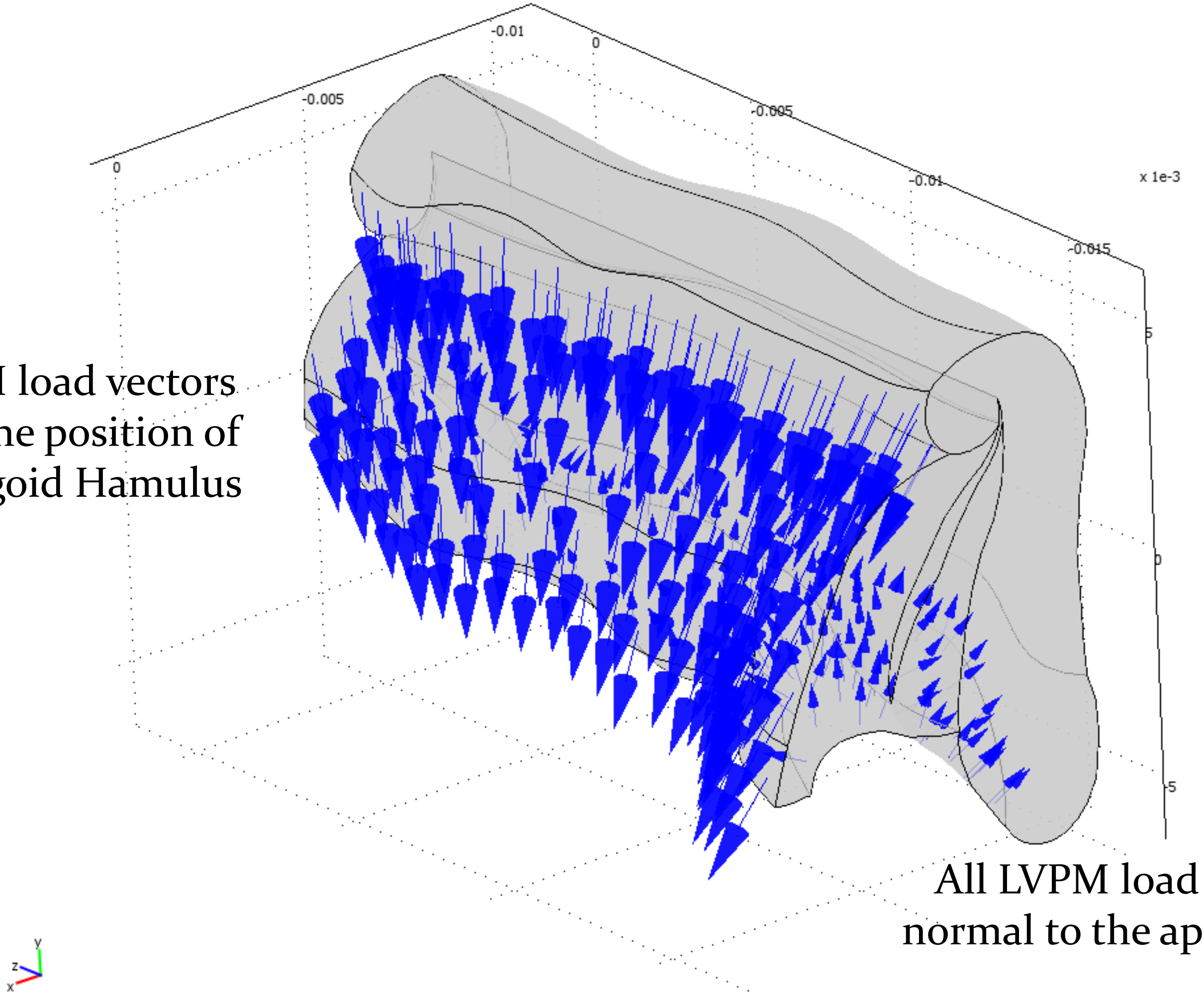




☑ Select by group

Name	Expression	Unit
TVP_force_y		
TVP_force_z		
TVP_scale		
Cart_ratio		
LVP_cart		
LMW_ratio	5.89e-5[m^2]/LVP_area	1
LVP_LMW	5[N]*LMW_ratio/5.89e-5[m^2]	N/m²

Solid Boundary Conditions (cont)



All TVPM load vectors point to the position of the Pterygoid Hamulus

All LVPM load vectors are normal to the applied surface

Solid-Fluid Coupling

Arbitrary Lagrange-Eulerian (ALE) equations

$$\frac{\partial^2 x_i}{\partial X_j^2} = 0 \quad \text{where} \quad x_i = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \& \quad X_j = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

In the Solid Domain, fluid force is applied on the interface boundaries

$$T_x = \sigma n_x = (-p\mathbf{I} + \eta(\nabla\mathbf{u} + (\nabla\mathbf{u})^T))n_x$$

$$T_y = \sigma n_y = (-p\mathbf{I} + \eta(\nabla\mathbf{u} + (\nabla\mathbf{u})^T))n_y$$

$$T_z = \sigma n_z = (-p\mathbf{I} + \eta(\nabla\mathbf{u} + (\nabla\mathbf{u})^T))n_z$$

In Fluid Domain, velocities from solid deformation is applied to interface boundaries

$$u_2 = \frac{\partial u}{\partial t}$$

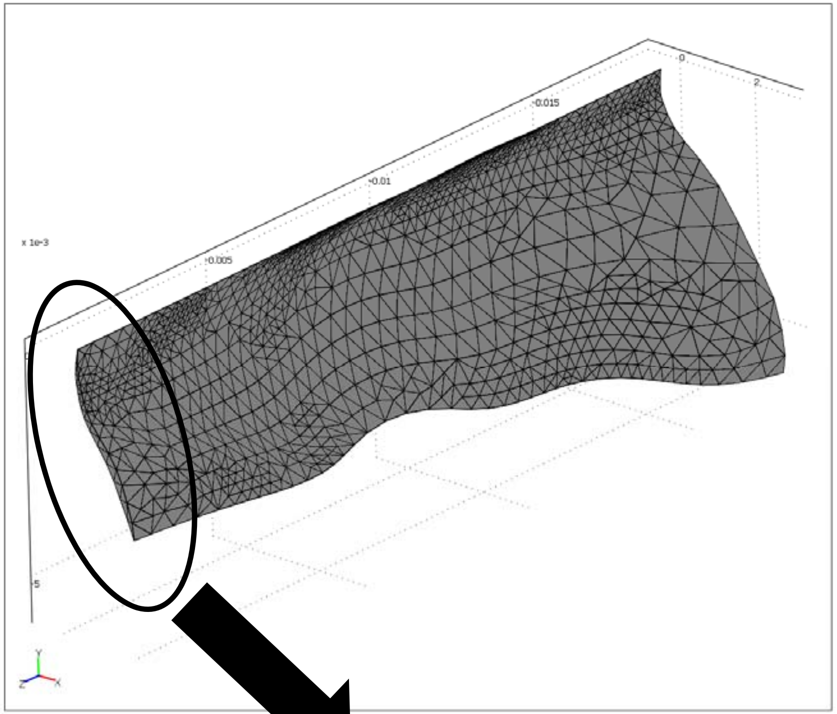
$$v_2 = \frac{\partial v}{\partial t} \quad \text{where}$$

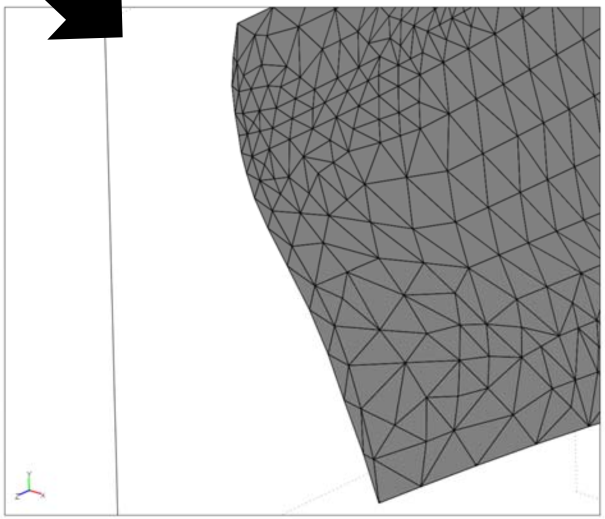
$$w_2 = \frac{\partial w}{\partial t}$$

u, v, and w are the solid displacements and u_2, v_2, and w_2 are the fluid velocities

Fluid Domain

Due to the complexity of the geometry, a coarse, tetrahedral mesh is used with high order elements to accommodate the large displacements without any element collapse.





Incompressible Continuity and Navier-Stokes equations

$$\rho \frac{\partial v_i}{\partial x_i} = 0$$

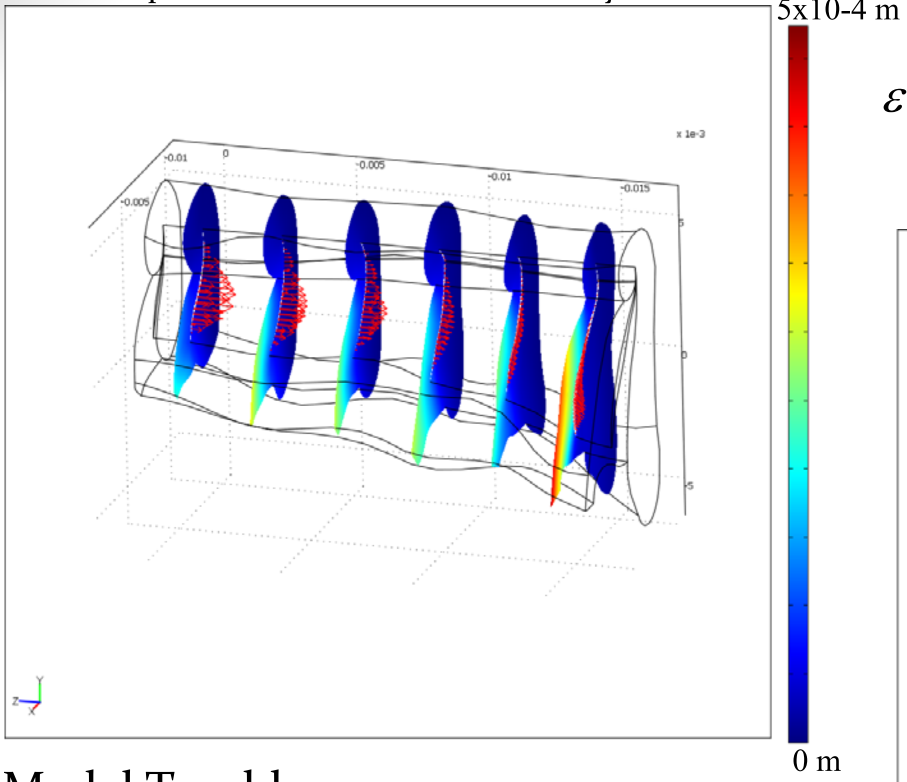
$$\rho \frac{\partial v_i}{\partial t} + \rho v_j \frac{\partial v_i}{\partial x_i} = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 v_i}{\partial x_i \partial x_j}$$

where $v_i = \begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix}$

Fluid equations are solved in the moving reference frame!!

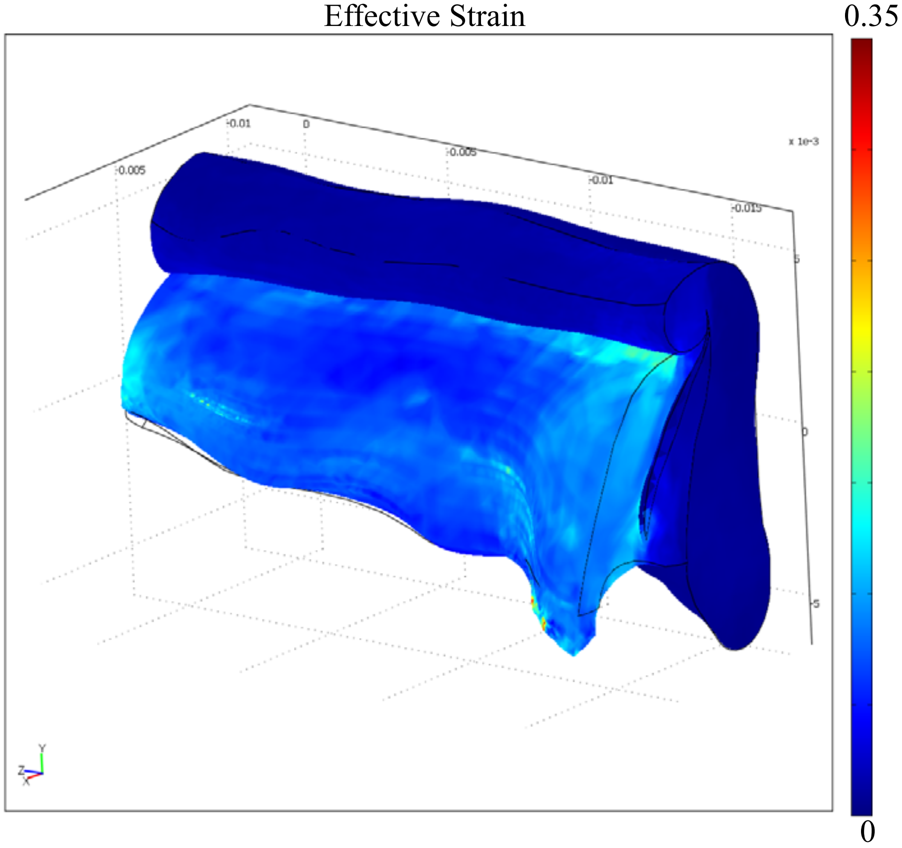
Model Results

Solid Displacement field and Fluid Velocity vectors



$$\varepsilon_{eff} = \sqrt{\frac{2}{3}(\varepsilon_X^2 + \varepsilon_Y^2 + \varepsilon_Z^2) + 2(\varepsilon_{XY}^2 + \varepsilon_{XZ}^2 + \varepsilon_{YZ}^2)}$$

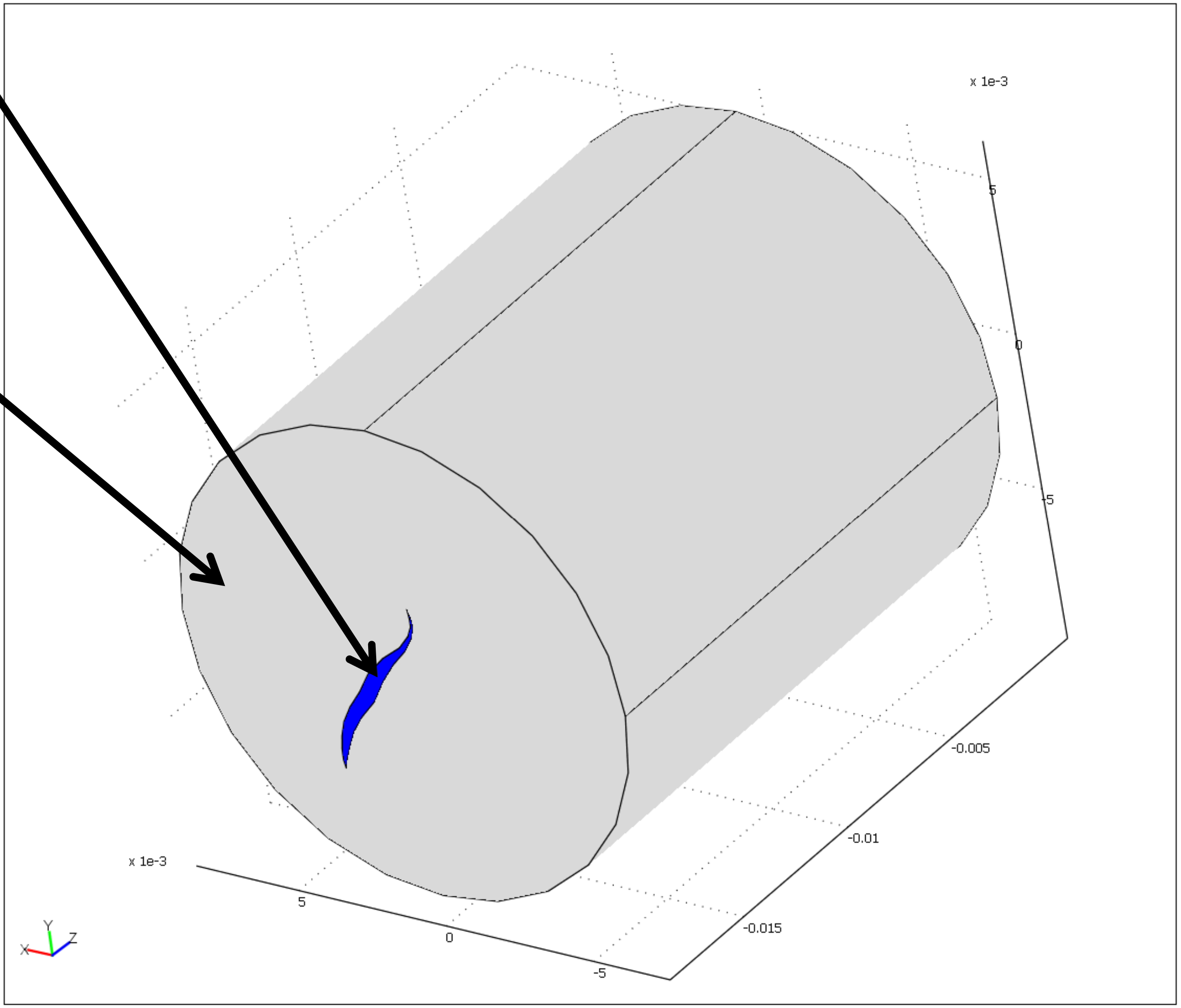
Effective Strain



Model Troubles:

- Static solutions only
- Model unstable
- Fictitiously viscous fluid domain
- Solid → Fluid coupling only

Test Model

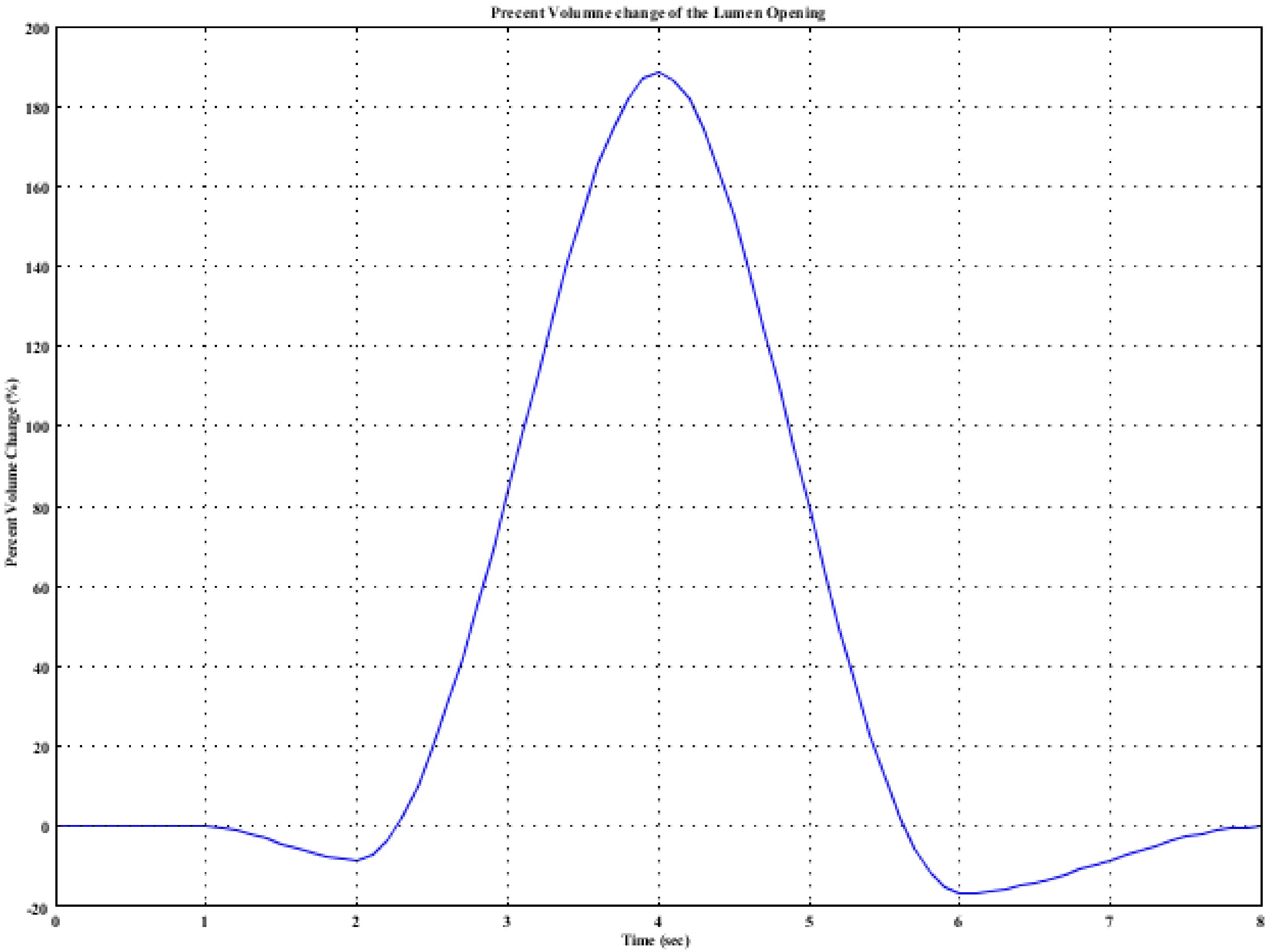


Anatomically correct lumen opening

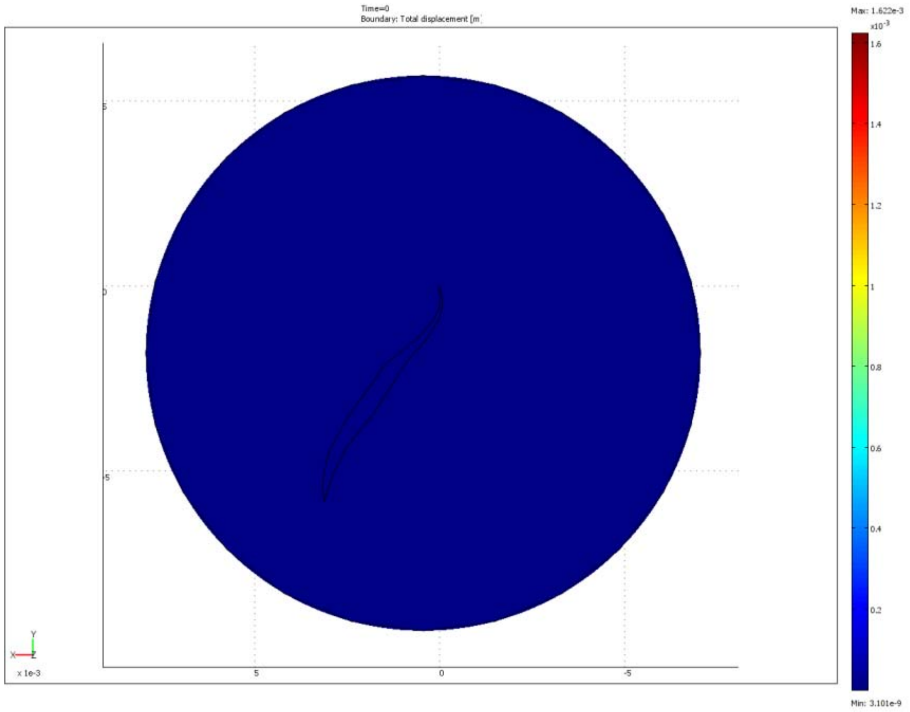
Lumen opening surrounded by cylinder of cartilage

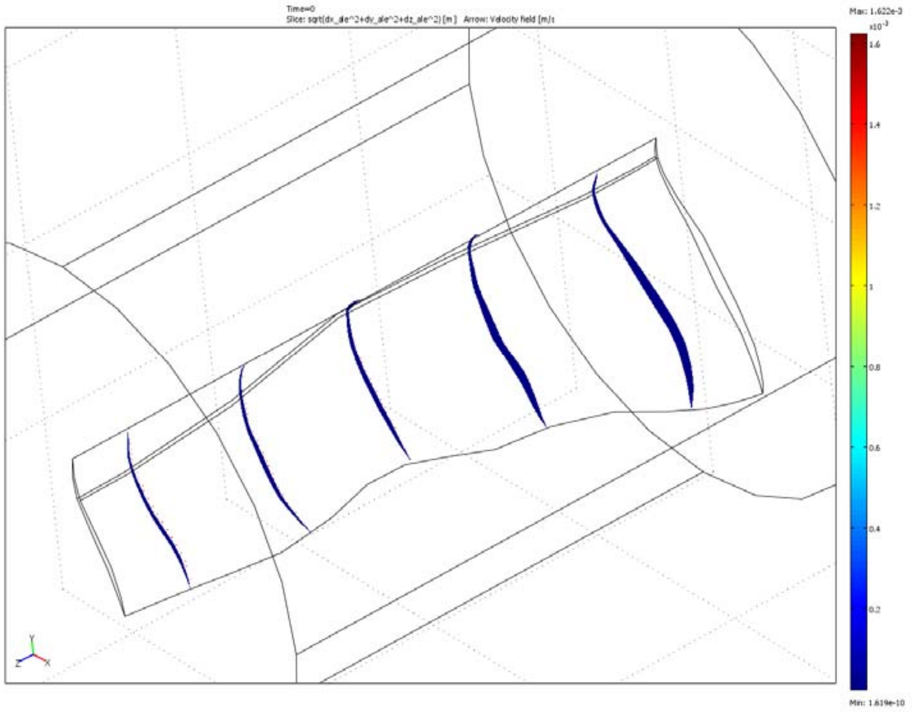
Cylinder is loaded with a uniform, outward radial force to expand lumen opening.

Test Model Results



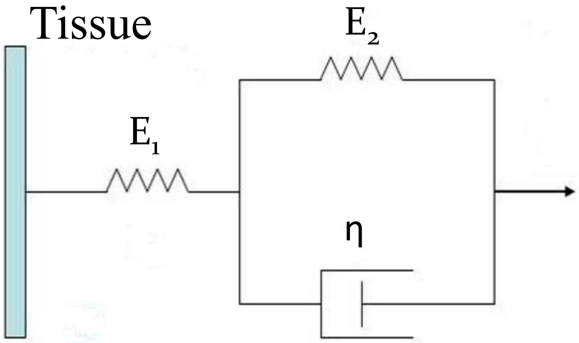
Test Model Results



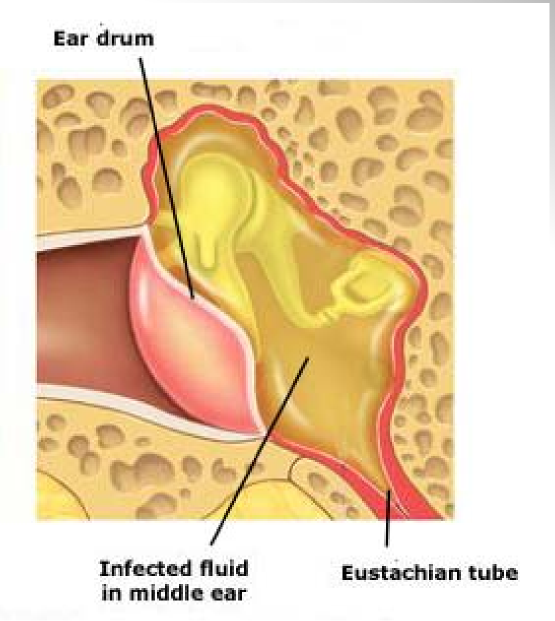


Future Work

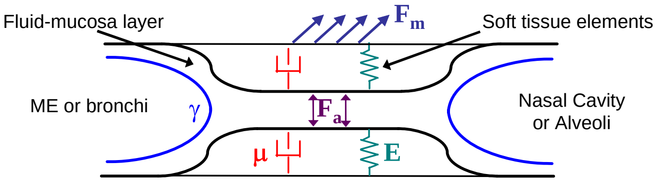
Incorporate a viscoelastic material model for the Fatty Tissue

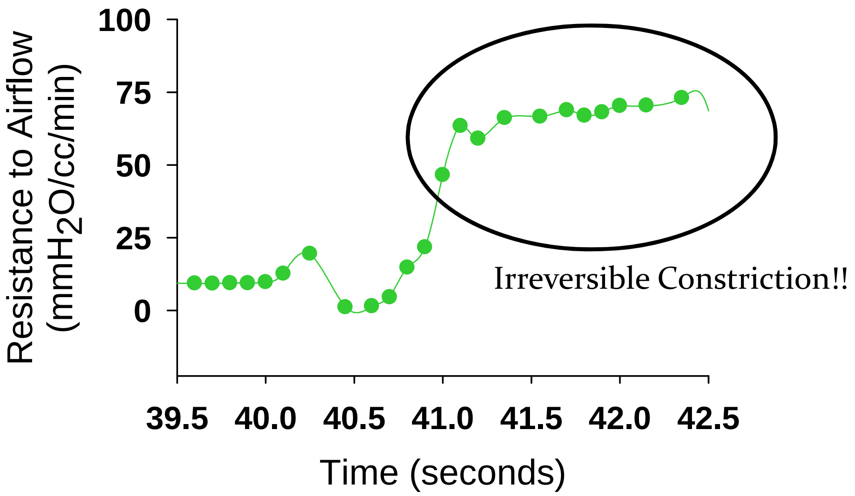


Simulate the drainage/flow of highly viscous mucus through the ET



Through the use of Steered Molecular Dynamics, create a multi-scale model which includes molecular adhesions





Acknowledgments

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References

1. Bondy, J., et al., *Direct expenditures related to Otitis media diagnoses: extrapolations from a pediatric medicaid cohort*. Pediatrics, 2000. **105**:p. 72
2. Hung, C.T., et al., *A paradigm for functional tissue engineering of articular cartilage via applied physiologic deformational loading. Ann Biomed Eng, 2004.* ***32: p. 35-49.***
3. Krouskop, T.A., et al., *Elastic moduli of breast and prostate tissues under compression. Ultrason Imaging, 1998.* ***20: p. 260-327.***