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Modeling of Ultrasonic Near-Filed Acoustic Levitation: Resolving Viscous and Acoustic Effects

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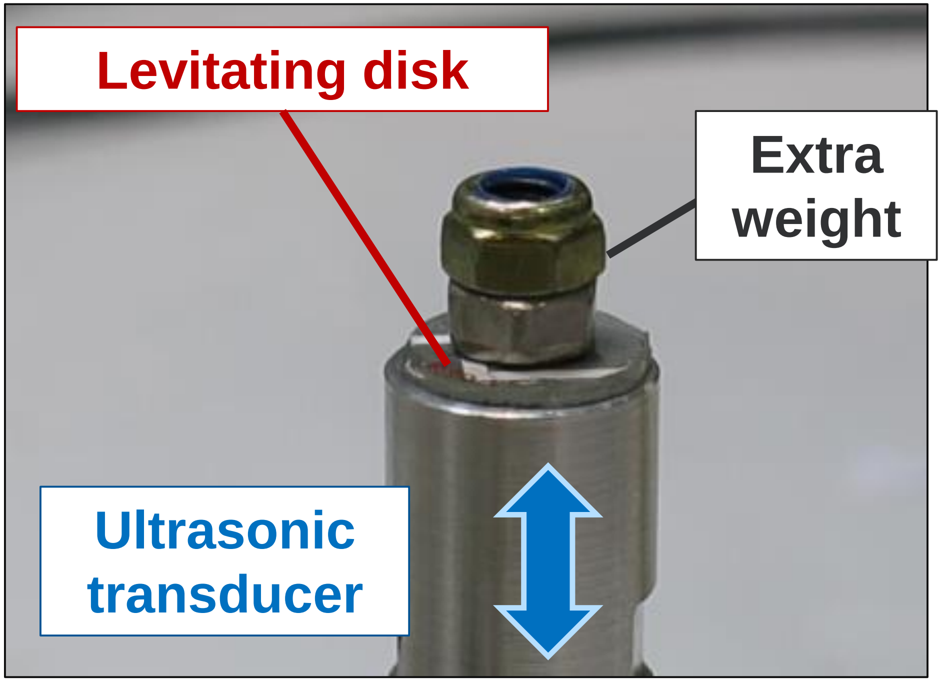
Ultrasonic Near-Field Acoustic Levitation Overview

Near-Field Levitation ***Levitation Heigth 5-500 µm***	
Viscous Mechanism *H < 2 BLT*	**Acoustic Mechanism** *H > 2 BLT*

*BTL – viscous boundary layer thickness, (for air @ 20 kHz approx. 20 µm)



- Allows levitation of moderately large objects
- Is used for contactless transportation systems (e.g. semiconductor industry)

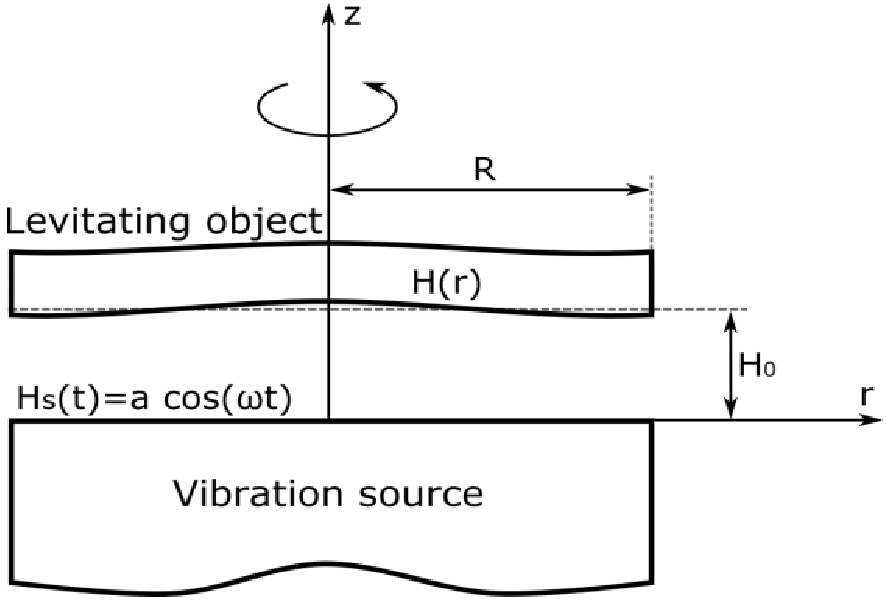
Details of analytical and experimental work can be found in
I. Melikhov, S. Chivilikhin, A. Amosov, R. Jeanson, Visco-acoustic model for near-field ultrasonic levitation, *submitted to Phys. Rev. E*

Physics of Near-Field Levitation

- The air flow between sound source and object is described by
  - Transient Navier-Stokes equations for compressible fluid
  - Energy equation
  - Equation of gas state
  - Viscosity-temperature dependence (Sutherland's law)

- Leading-order model assumptions
  - $H_0/R \ll 1$
  - $H_0/\lambda_{acoustic} \ll 1$
  - $\epsilon = \frac{a}{H_0} \ll 1$

➡ Pressure, density, viscosity are *z*-independent



Equations to Solve (Non-dimensional)
First-Order (Frequency Domain)

(u, v) - velocity vector;
p - pressure;
γ - adiabatic index;
K, Σ - non-dimensional constants;
i - imaginary unit

- Continuity

$$i\frac{p_1}{\gamma} + \frac{1}{r}\frac{\partial}{\partial r}(r\bar{u}_1) = \frac{i}{2}$$

$$\partial_r p_1\Big|_{r=1} = iKp_1$$

- Longitudinal velocity

$$i\gamma K^2 u_1 = -\frac{\partial p_1}{\partial r} + \Sigma\frac{\partial^2 u_1}{\partial z^2}$$

$$u_1\Big|_{z=0} = u_1\Big|_{z=1} = 0$$

$$\bar{u}_1 = \int_0^1 u_1\, dz$$

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- Transversal velocity

$$v_1 = -\int_0^z \left(\frac{i}{\gamma}p_1 + \frac{1}{r}\frac{\partial}{\partial r}(ru_1)\right) dz' + \frac{i}{2}$$

Equations to Solve (Non-dimensional) Second-Order (Time-Averaged Values)

$[a_1 b_1]_0 = a_1 b_1^* + a_1^* b_1$
where * denotes complex conjugation;
M - constant came from Sutherland's law

- Continuity

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\left\{\bar{u}_2 + \left[\left(\frac{p_1}{\gamma} - 1/2\right)\bar{u}_1\right]_0\right\}\right) = 0$$

$$p_2\Big|_{r=1} = 0$$

- Longitudinal velocity

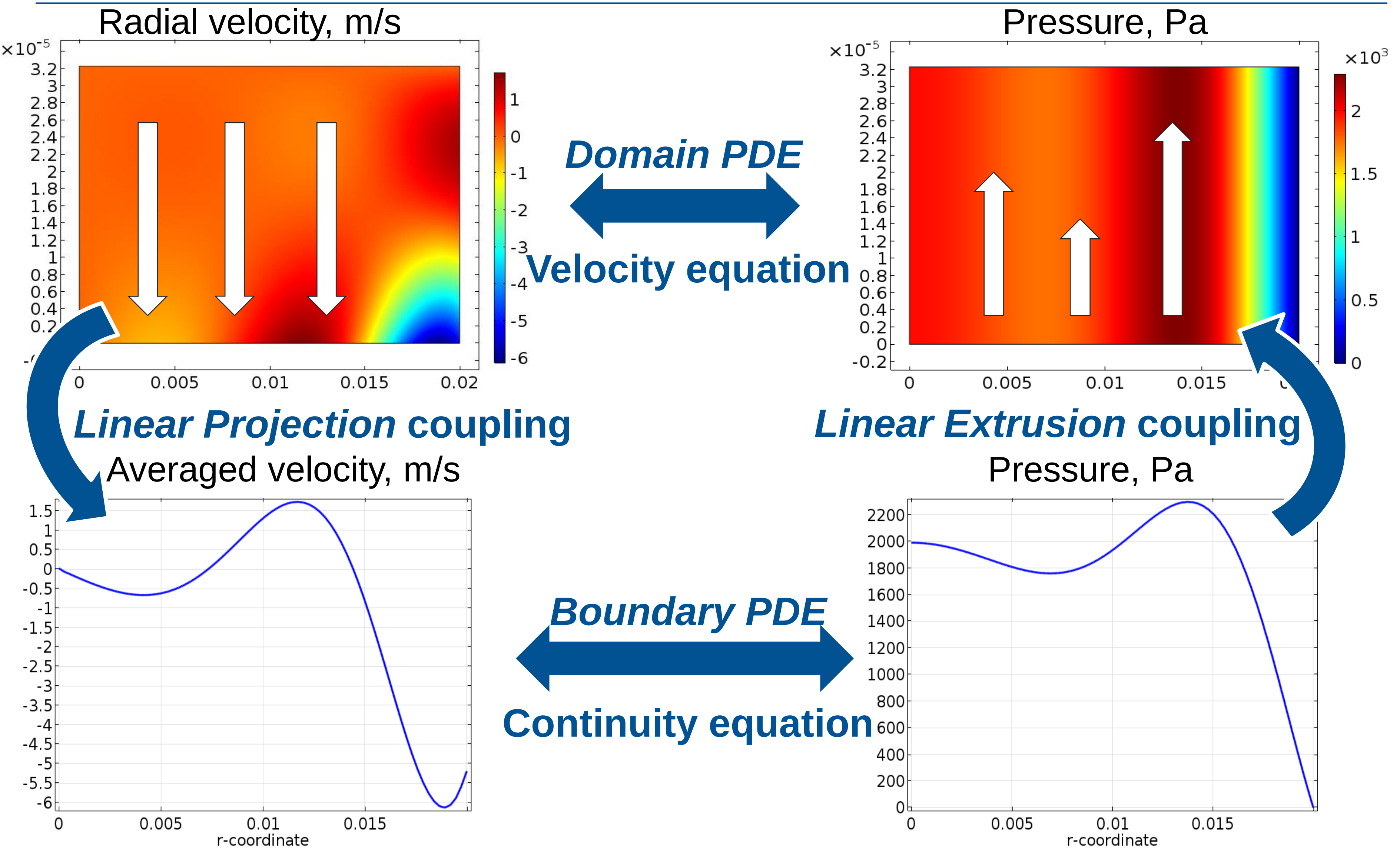
$$\gamma K^2\left(\frac{i}{\gamma}(p_1 u_{-1}^* - p_{-1}^* u_1) + \left[u_1\frac{\partial u_1}{\partial r} + v_1\frac{\partial u_1}{\partial z}\right]_0\right)$$
$$= -\frac{\partial p_2}{\partial r} + \Sigma\frac{\partial^2}{\partial z^2}(u_2 + M[p_1 u_1]_0)$$

$$u_2\Big|_{z=0} = -\frac{1}{2}\frac{\partial}{\partial z}[u_1 + u_1^*]_0, \qquad u_2\Big|_{z=1} = 0$$

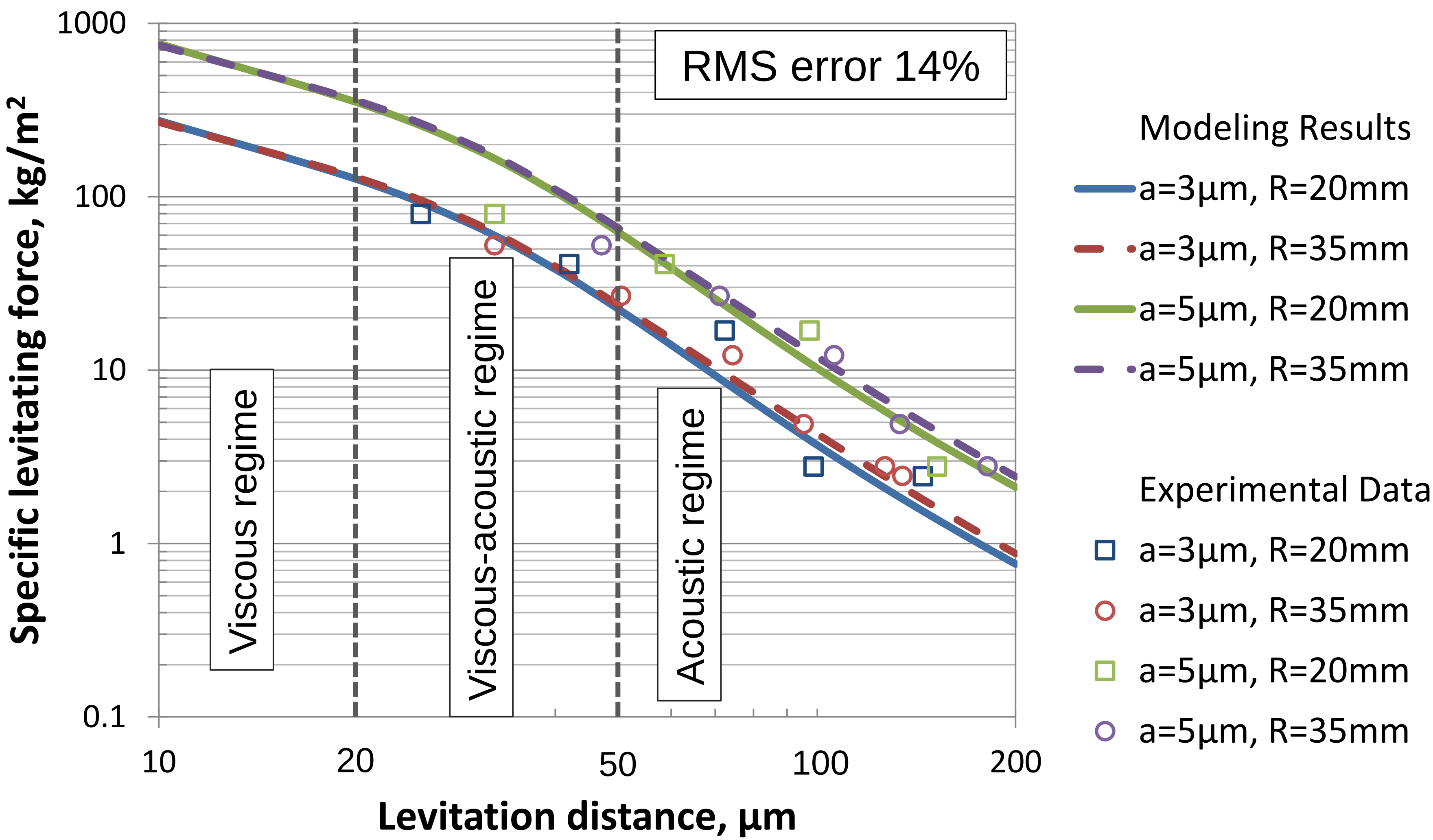
$$\bar{u}_2 = \int_0^1 u_2 dz - (\bar{u}_1 + \bar{u}_1^*)/2$$

Coupled

COMSOL Implementation: 2D Axisymmetric Case
Equation Coupling

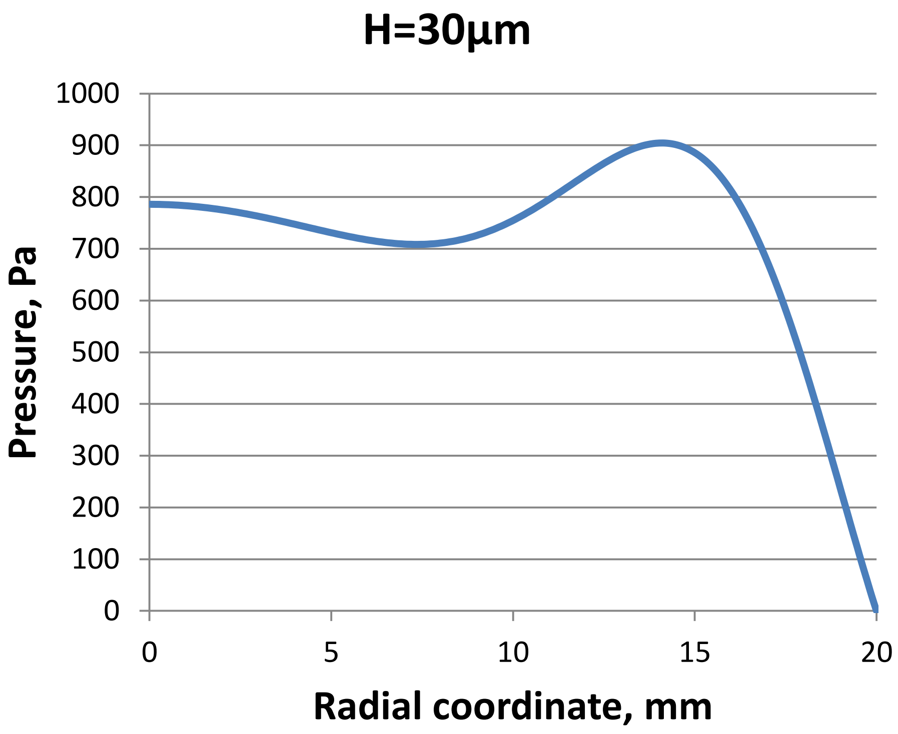


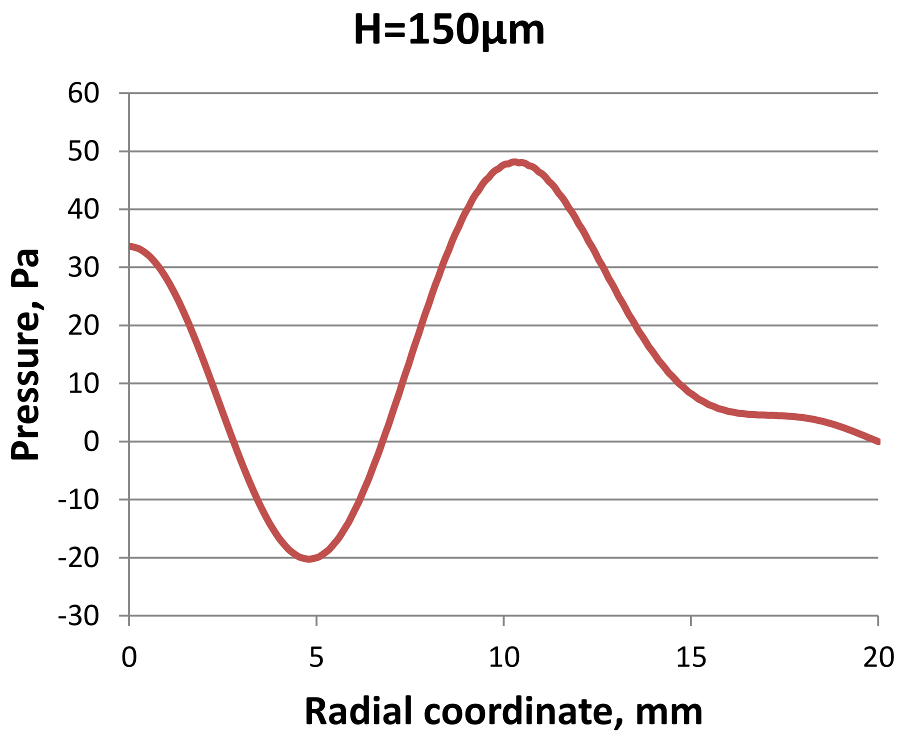
Modeling Results
Levitation Force vs. Levitation Distance



Modeling Results
Pressure Profile

- Pressure profile varies depending on the regime
  - Close-to-uniform pressure in the viscous regime
  - Non-uniform pressure in the acoustic regime





a=3μm, R=20mm, freq=20kHz

Conclusions

- We developed and experimentally validated a new efficient model of ultrasonic levitation covering wide range of air flow regimes
- The model consists of five linear stationary PDE – much easier to solve than initial transient non-linear problem
- COMSOL helps to formulate and solve non-standard equations in simple and elegant way

Math + ***Physics*** + COMSOL = ***Efficient, fast and accurate models***

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Non-Dimensional Variables

- Coordinates and time

$$r = \hat{r}/R, \qquad z = \hat{z}/H_0, \qquad t = \omega\hat{t}$$

- Velocities and pressure

$$v_z = \hat{v}_z/(\omega H_0), \qquad v_r = \hat{v}_r/(\omega R), \qquad p = \hat{p}/p_0$$

- Density, viscosity and temperature

$$\rho = \hat{\rho}/\rho_0, \qquad \mu = \hat{\mu}/\mu_0, \qquad T = \hat{T}/T_0$$

- Gap thickness

$$h^{(0)} = H(r)/H_0$$

- Non-dimensional quantities
  - Acoustic wave number $K = \left(\frac{\omega^2 R^2 \rho_0}{\gamma p_0}\right)^{1/2}$
  - Squeeze number $\Sigma = \frac{\mu\omega R^2}{p_0 H_0^2}$