Free convection in a square cavity partially filled with porous media with spatial wall temperature

Ammar Ibraheem Alsabery

email: ammar_e_2011@yahoo.com

Join work with: Habibis Saleh & Ishak Hashim

School of Mathematical Sciences
Universiti Kebangsaan Malaysia, 43600 Bangi Selangor, Malaysia

COMSOL CONFERENCE 2015
Kuala Lumpur, November, 11-12, 2015.

Convection

- The transfer of heat from one place to another by the movement of fluids.

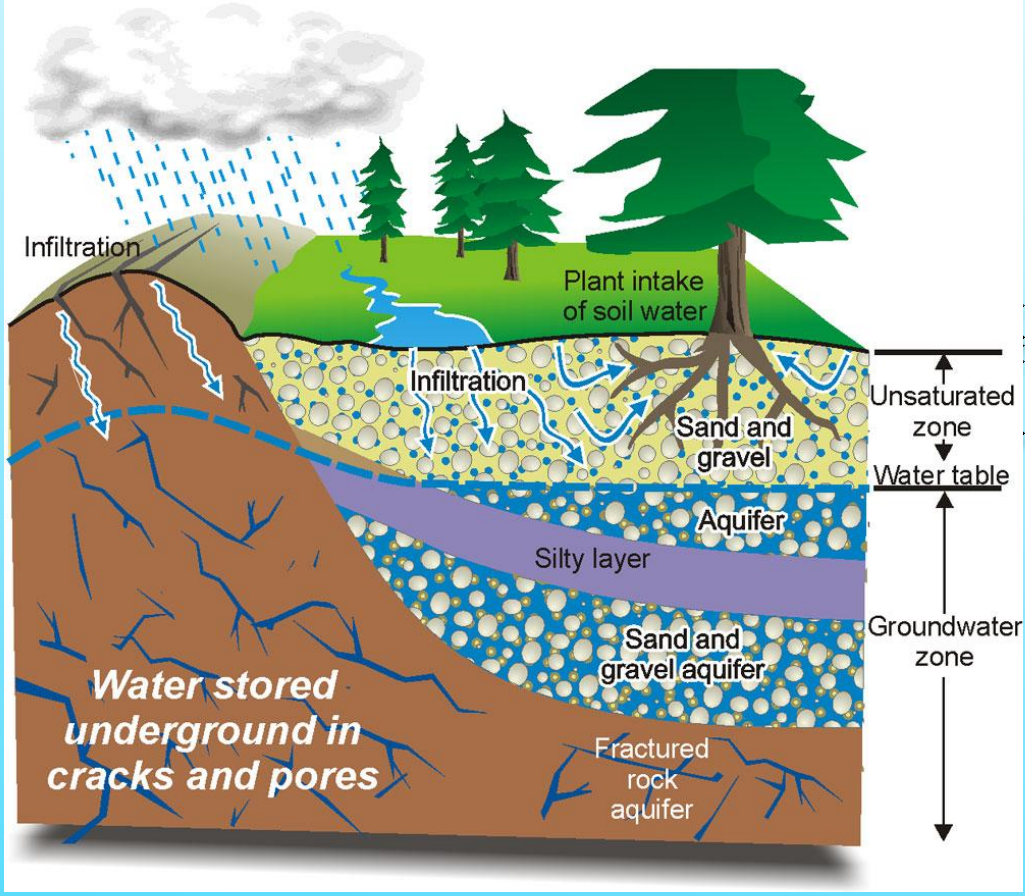
- Convection can be "forced" by movement of a fluid or by natural buoyancy forces alone, when the fluid is heated (natural convection).

- A porous medium: a material consisting of a solid matrix with an interconnected void.

- The study of natural convection in cavity partially-filled with porous media has importance in many fields of science, engineering, chemical engineering and industrial applications.

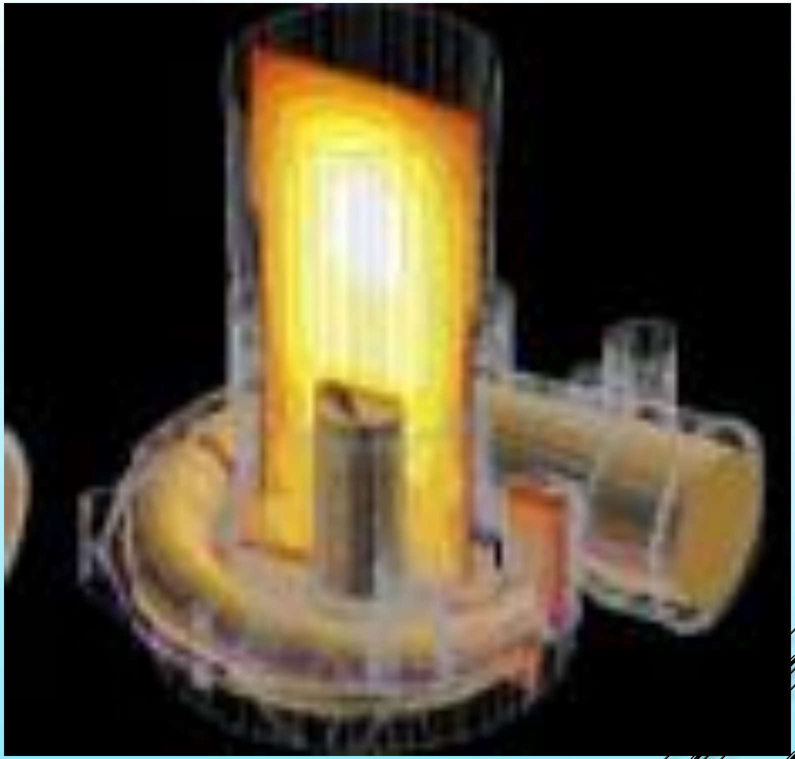
Ground Water

Heated ground water due to hot intrusion may rise in a narrow fractured zone. As the heated water rises, it eventually encounters a cooler rock formation that sandwiches the permeable vertically slender space. This causes heat transfer between the hot water and the colder surrounding rocks.

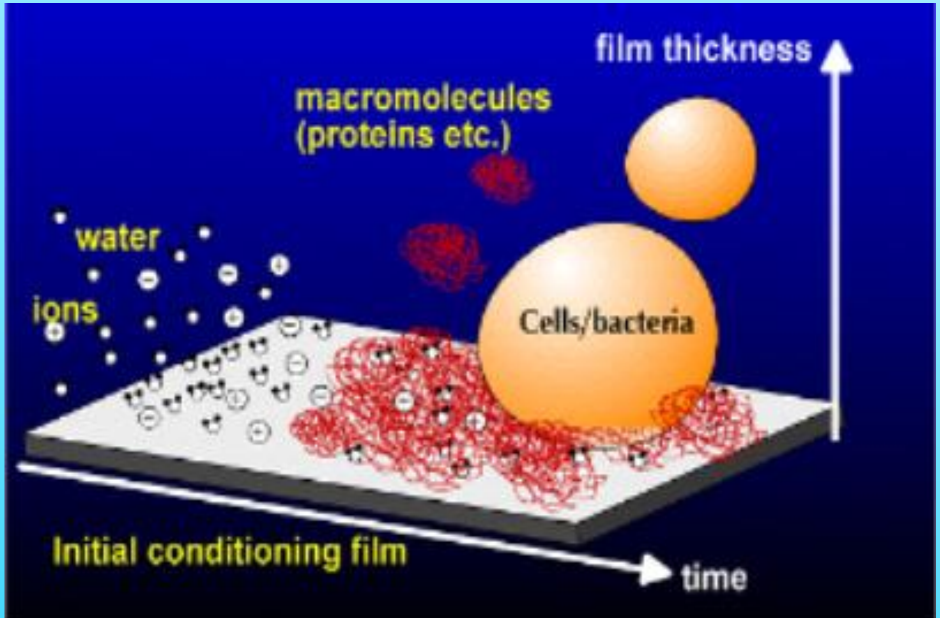


Partially-Filled Porous Medium

- Solidification of castings



- The biofilm growth



Heat Transfer in Cavity

1. Beavers and Joseph (1967) investigated the simple situation of the boundary conditions between a porous media and a homogeneous fluid.

2. Singh and Thorpe (1995) conducted a comparative study of different models for the investigation of natural convection in a confined fluid and overlying porous layer.

3. Saeid and Mohamad (2005) studied numerically the natural convection in a porous cavity with spatial sidewall temperature variation using finite element method.

Study the effect of spatial wall temperature on free convection in a square cavity partially filled with porous media has not been undertaken yet.

Mathematical Formulation

The conservation equations for mass, momentum and energy equations for the porous layer:

1. $$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0,$$

2. $$u_p = -\frac{K_p}{\mu}\frac{\partial p_p}{\partial x},$$

3. $$v_p = -\frac{K_p}{\mu}\frac{\partial p_p}{\partial y} + \frac{K_p \beta_p g}{\nu}\left(T_p - T_c\right),$$

4. $$u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} = \frac{\partial^2 T_p}{\partial x^2} + \frac{\partial^2 T_p}{\partial y^2}.$$

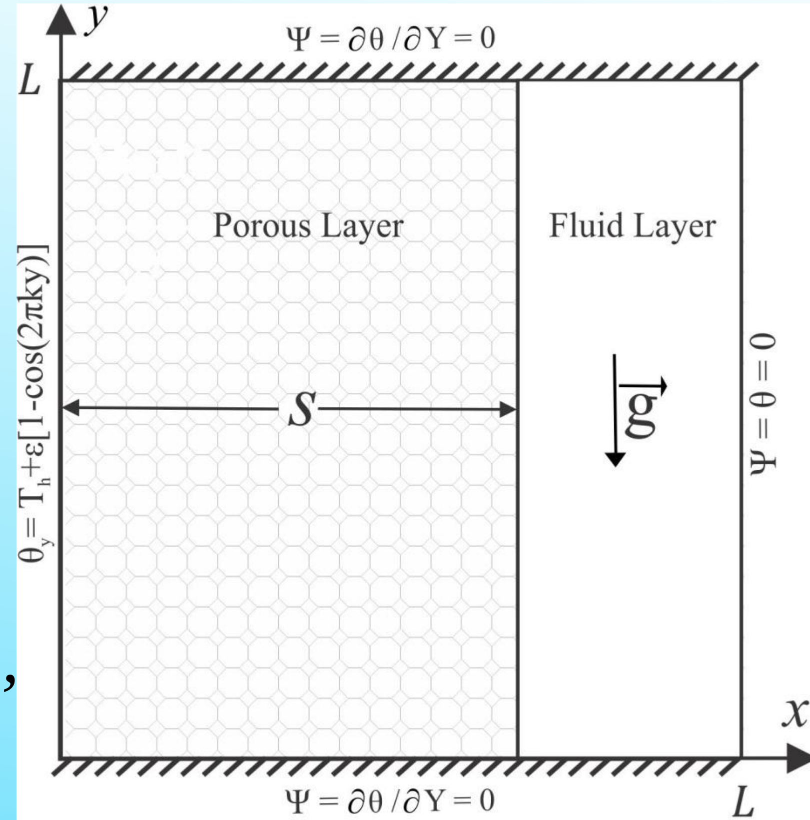


Fig. 1.

The conservation equations for mass, momentum and energy equations for the homogenous fluid layer are:

5
$$\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} = 0,$$

6
$$u_f \frac{\partial u_f}{\partial x} + v_f \frac{\partial u_f}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p_f}{\partial x} + \nu\left(\frac{\partial^2 u_f}{\partial x^2} + \frac{\partial^2 u_f}{\partial y^2}\right),$$

7
$$u_f \frac{\partial v_f}{\partial x} + v_f \frac{\partial v_f}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p_f}{\partial x}\nu\left(\frac{\partial^2 v_f}{\partial x^2} + \frac{\partial^2 v_f}{\partial y^2}\right) + \rho g \beta\left(T_f - T_c\right),$$

8
$$u_f \frac{\partial T_f}{\partial x} + v_f \frac{\partial T_f}{\partial y} = \left(\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2}\right).$$

The non-dimensional variables are:

9

$$\Psi = \frac{\psi}{\alpha_m \varphi L}, \theta_p = \frac{T_p - T_c}{T_h - T_c}, \theta_f = \frac{T_p - T_c}{T_h - T_c},$$

$$U = \frac{Lu}{\alpha}, V = \frac{L_v}{\alpha}, X = \frac{x}{L}, Y = \frac{y}{L}.$$

10

$$T_h(y) = \bar{T}_h + \varepsilon\left(\bar{T}_h - T_c\right)\left[1 - cos\left(\frac{2\pi ky}{L}\right)\right].$$

The governing equations for the porous layer can be written as:

11

$$\frac{\partial^2 \Psi_p}{\partial X^2} + \frac{\partial^2 \Psi_p}{\partial Y^2} = -RaDa\frac{\partial \theta_p}{\partial X},$$

12

$$\frac{\partial \Psi_p}{\partial X}\frac{\partial \theta_p}{\partial Y} - \frac{\partial \Psi_p}{\partial Y}\frac{\partial \theta_p}{\partial X} = \frac{\lambda_p}{\lambda_f}\left(\frac{\partial^2 \theta_p}{\partial X^2} + \frac{\partial^2 \theta_p}{\partial Y^2}\right).$$

The governing equations for the fluid layer can be written as:

13
$$U_f \frac{\partial U_f}{\partial X} + V_f \frac{\partial U_f}{\partial Y} = -\frac{\partial P_f}{\partial X} + \left(\frac{\partial^2 U_f}{\partial X^2} + \frac{\partial^2 U_f}{\partial Y^2} \right),$$

14
$$U_f \frac{\partial V_f}{\partial X} + V_f \frac{\partial V_f}{\partial Y} = -\frac{\partial P_f}{\partial Y} + \left(\frac{\partial^2 V_f}{\partial X^2} + \frac{\partial^2 V_f}{\partial Y^2} \right) + \frac{Ra}{Pr}\theta,$$

15
$$U_f \frac{\partial \theta_f}{\partial X} + V_f \frac{\partial \theta_f}{\partial Y} = \frac{1}{Pr} \left(\frac{\partial^2 \theta_f}{\partial X^2} + \frac{\partial^2 \theta_f}{\partial Y^2} \right),$$

The dimensionless boundary conditions of Eqs. (11)-(15) are:

16

$$\Psi(0,Y)=0, \quad \theta_p(0,Y)=0.5+\varepsilon[1-cos(2\pi kY)],$$
$$\Psi(1,Y)=0, \quad \theta_f(1,Y)=-0.5,$$
$$\Psi(X,0)=\Psi(X,1)=0, \quad \frac{\partial\theta(X,0)}{\partial Y}=\frac{\partial\theta(X,1)}{\partial Y}=0,$$

At the interface by using the matching conditions:

17

$$U^{+}=-V^{-},$$
$$\frac{\partial V}{\partial X}=\bar{\alpha}(U^{+}-V^{-})/\sqrt{Da},$$
$$\theta|_{X=s^{+}}=\theta|_{X=s^{-}},$$
$$\lambda_p\frac{\partial\theta}{\partial X}s^{+}=\lambda_f\frac{\partial\theta}{\partial X}s^{-}.$$

The local Nusselt number along the hot and the cold walls, which are defined, respectively, by:

18 $$Nu_h = \frac{hL}{\lambda} = -\left(\frac{\partial \theta}{\partial X}\right)_{X=0} ,$$

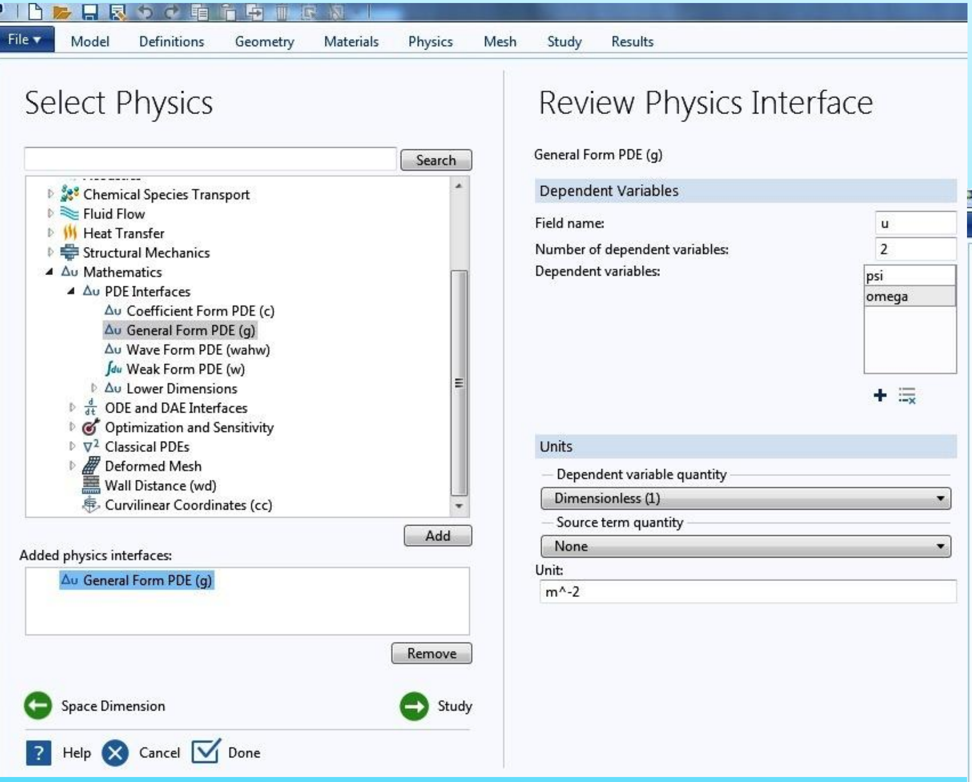
19 $$Nu_c = \frac{hL}{\lambda} = -\left(\frac{\partial \theta}{\partial X}\right)_{X=1} .$$

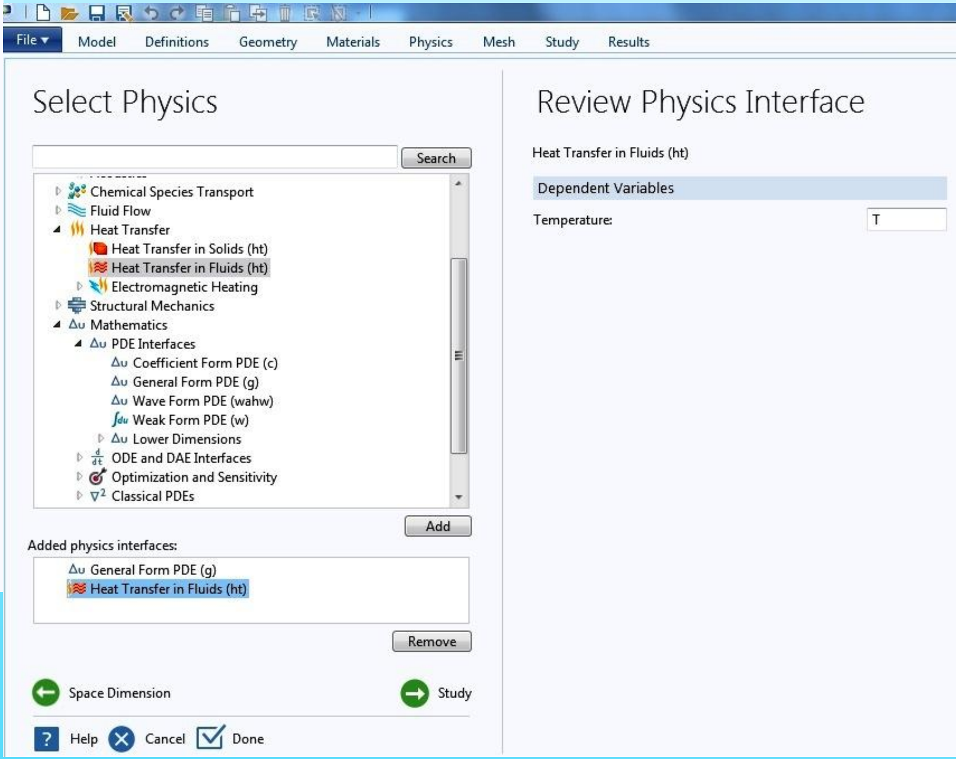
Finally, the average Nusselt number can be defined based on the average heat transfer coefficient and is given by:

20 $$\overline{Nu} = \frac{\bar{h}L}{\lambda} = \int_0^1 Nu\,dY .$$

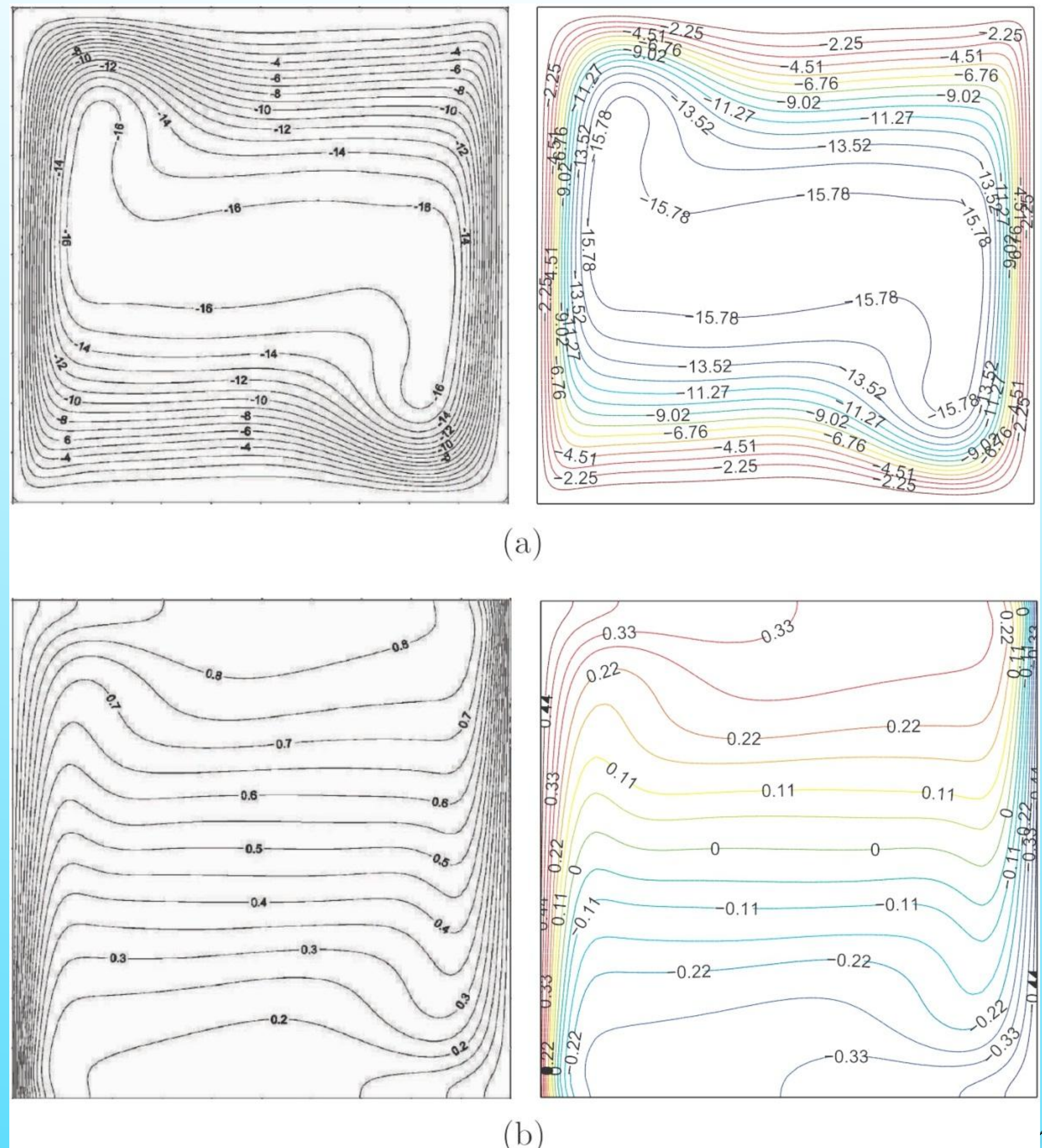
Numerical Method and Validation

Galerkin finite element method (GFEM), the governing equations subject to the boundary conditions are solved numerically using the CFD software package COMSOL Multiphysics.





Validation



Saeid and Mohamad (2005)

Fig. 2.

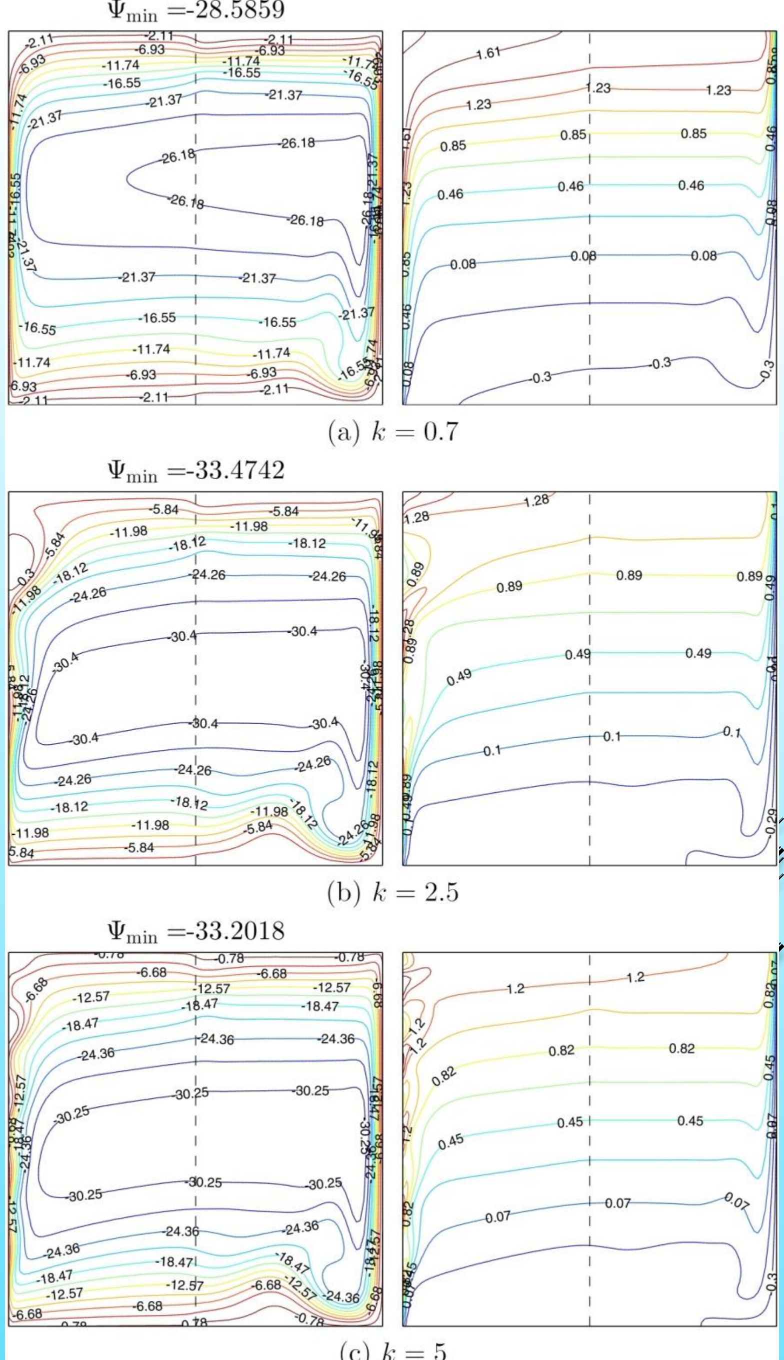
Present study

Results

$Ra = 10^7,\ Da = 10^{-4},\ \varepsilon = 0.5$ and $S = 0.5$.

It is found that when the wave number increases, the expansion of the streamline circulation cell tend to increases horizontally. The strength of the flow circulation increases with increasing k value up to 2.5.

The isotherm patterns are raised with high intensity and with irregular-shaped next to the left wall by the increase of the wave number, while near to the cold wall, the isotherm patterns occur with vertical lines.



(a) $k = 0.7$

(b) $k = 2.5$

(c) $k = 5$

Fig. 3.

Heat Transfer Rate

$Da = 10^{-4},\ k = 2.5$ and $\varepsilon = 0.5$

$Ra = 10^5,\ \varepsilon = 0.5$ and $S = 0.5$

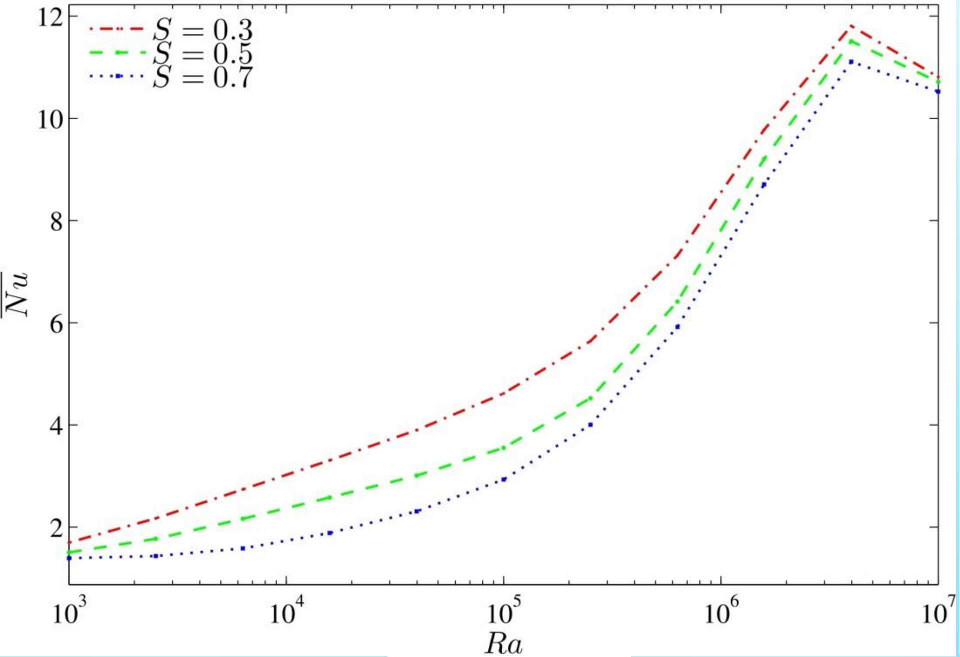


Fig. 4.

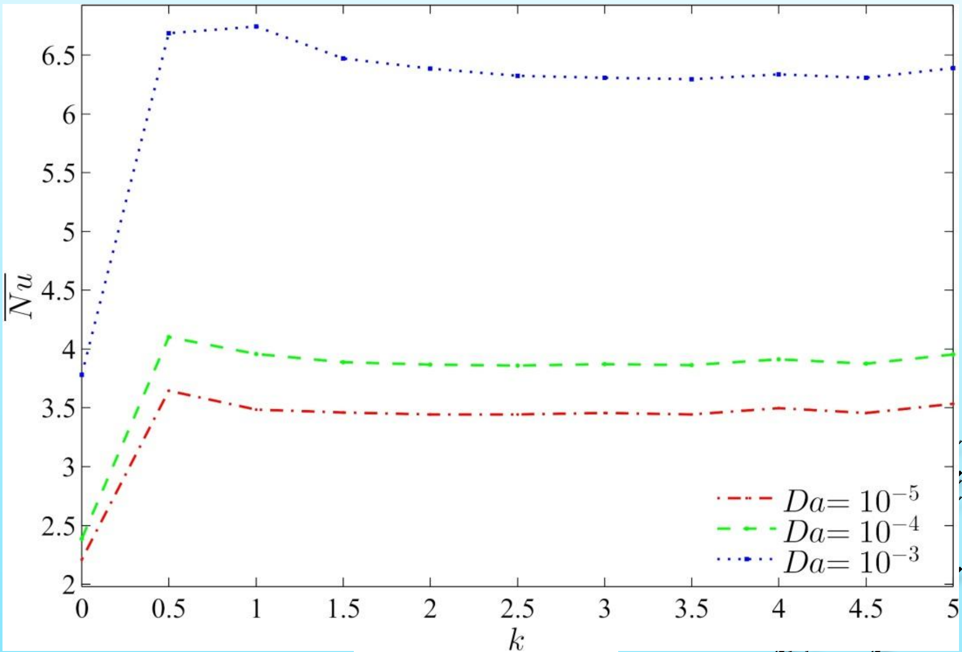
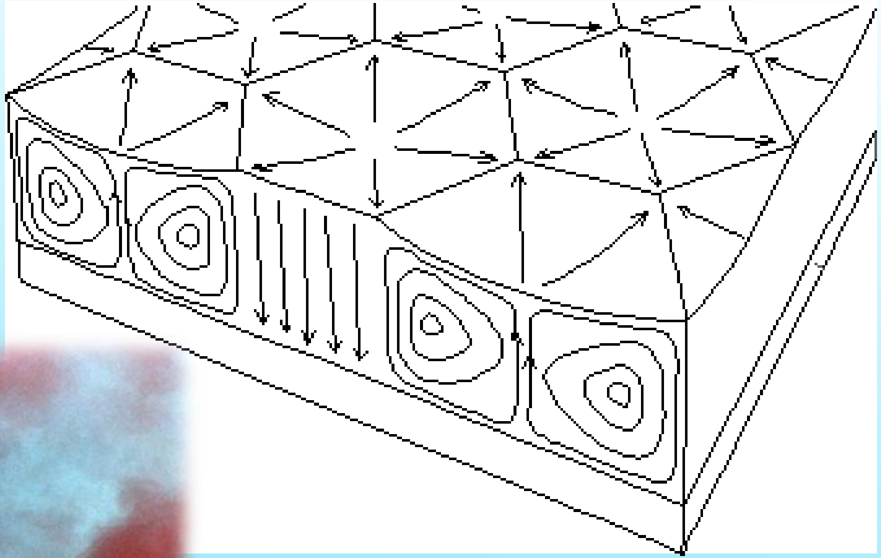


Fig. 5.

Increasing Rayleigh number leads to increase the average Nusselt number, due the fact that the fluid has higher thermal conductively than porous, the smaller porous thickness layer has stronger effect on the heat transfer rate which has higher average Nusselt number.

Future Works

- 3D
- Chaotic convection
- Turbulence flow
- Multiphase flow



THANK YOU