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Understanding the magnetic field penetration in superconductors via COMSOL

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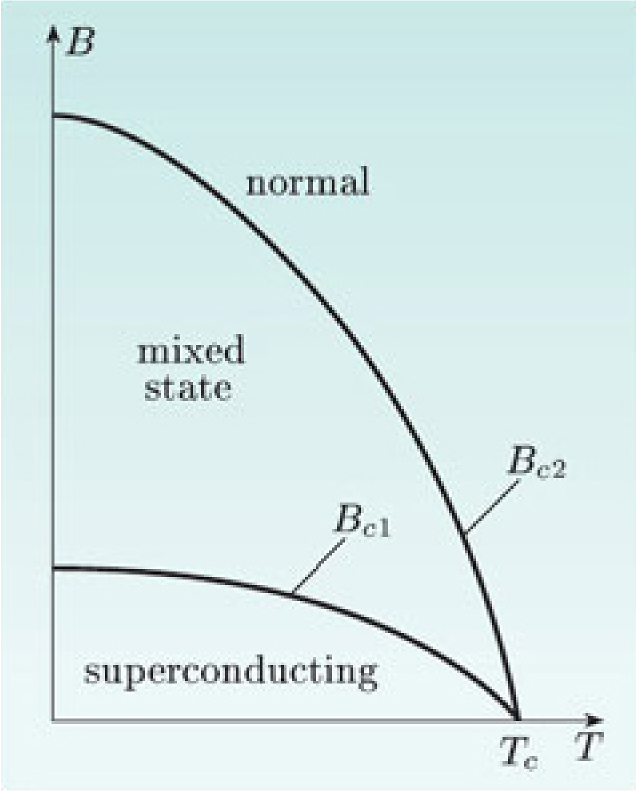
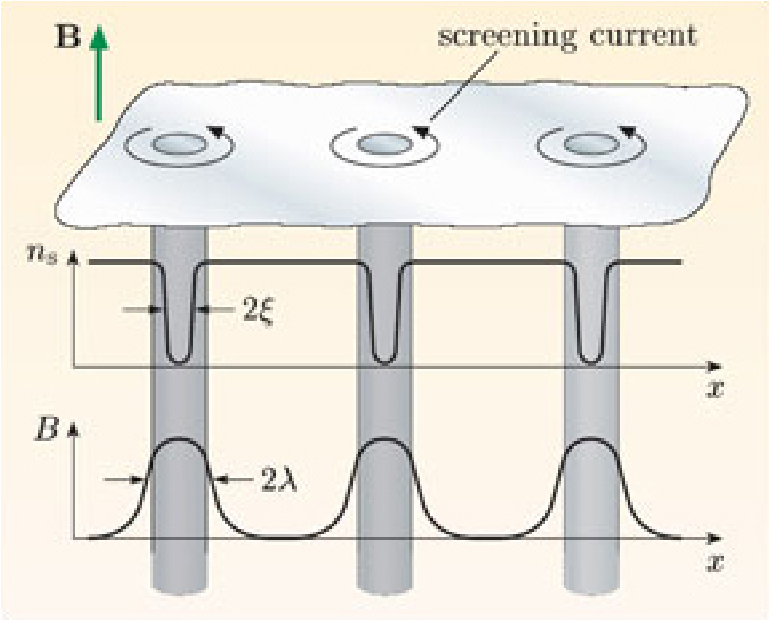
Outlines

- Time dependent Ginzburg-Landau theory
- Squared superconductor with a slit
- Macroturbulence instability

Type II superconductor





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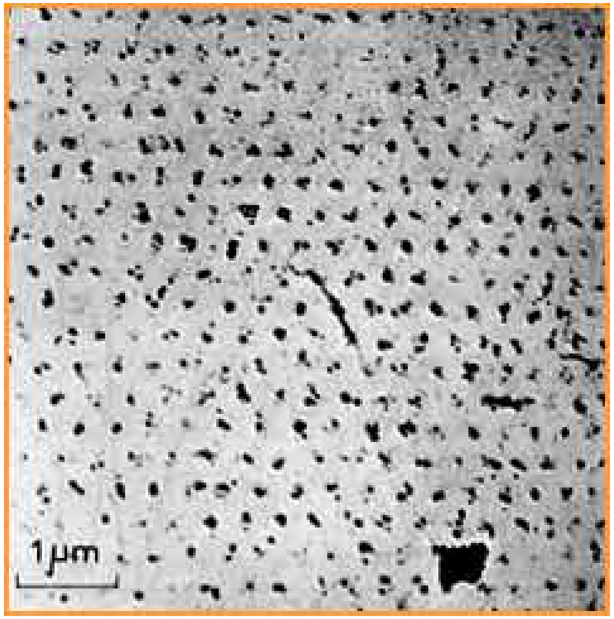
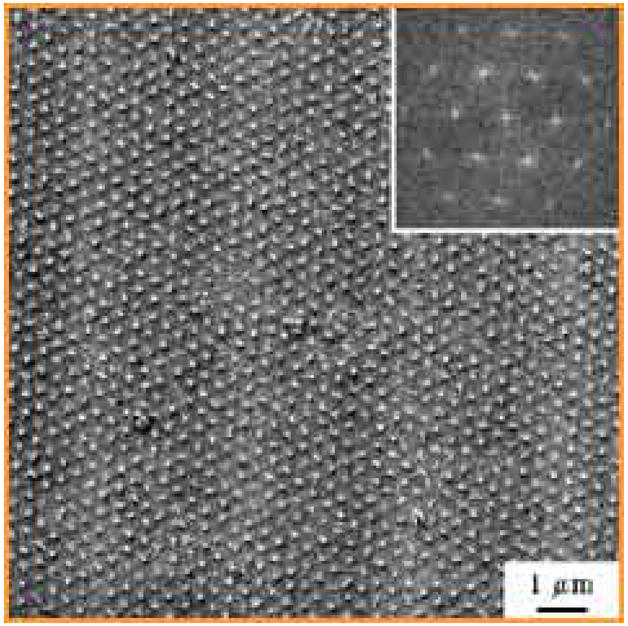
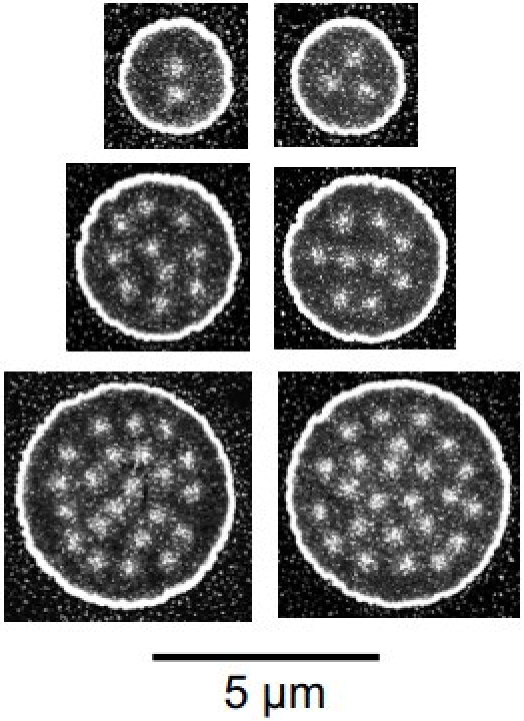
Type II superconductor



First image of Vortex lattice, 1967
Bitter Decoration
Pb-4at%In rod, 1.1K, 195G
U. Essmann and H. Trauble
Max-Planck Institute, Stuttgart
Physics Letters 24A, 526 (1967)



Abrikosov lattice in MgB2, 2003
Bitter Decoration
MgB2 crystal, 200G
L. Ya. Vinnikov et al.
Institute of Solid State Physics, Chernogolovka
Phys. Rev. B 67, 092512 (2003)



I.V. Grigorieva et al. arXiv:cond-mat/0506642
vortex configurations
in disks with *d ≈2, 3 and 5μm*
H =40 Oe..

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Time Dependent Ginzburg-Landau theory

$$\frac{\partial \Psi}{\partial t} = -\left(\frac{i}{\kappa}\vec{\nabla} + \vec{A}\right)^2 \Psi + \Psi - |\Psi|^2 \Psi \qquad (1)$$

$$\sigma \frac{\partial \vec{A}}{\partial t} = \frac{1}{2i\kappa}\left(\Psi^* \vec{\nabla}\Psi - \Psi\vec{\nabla}\Psi^*\right) - |\Psi|^2 \vec{A} - \vec{\nabla} \times \vec{\nabla} \times \vec{A} \qquad (2)$$

Boundary conditions

$$\vec{\nabla}\Psi \cdot \hat{n} = 0 \qquad (3)$$

$$\vec{\nabla} \times \vec{A} = \vec{B}_a \qquad (4)$$

$$\vec{A} \cdot \hat{n} = 0 \qquad (5)$$

Order parameter

$$\Psi = \psi_1 + i\psi_2$$

Applied magnetic field

$$A_x = -\frac{1}{2} B_a y$$

$$Ay = \frac{1}{2} B_a x$$

Equations 1 – 5 in matricial form in order to use COMSOL library

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot \Gamma = F$$

Using the definitions $\Psi = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ and $\nabla \equiv \begin{pmatrix} \partial_x \\ \partial_y \end{pmatrix}$

The equation (1) $\frac{\partial \Psi}{\partial t} = -\left(\frac{i}{\kappa} \vec{\nabla} + \vec{A} \right)^2 \Psi + \Psi - |\Psi|^2 \Psi$ become

$$\partial_t \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = -\left(\frac{i}{\kappa} \nabla + A \right)^2 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} - \left(u_1^2 + u_2^2 \right) \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\nabla \cdot \begin{pmatrix} -u_{1x} / \kappa^2 \\ -u_{1y} / \kappa^2 \end{pmatrix} + \left(u_3^2 + u_4^2 \right) u_1 + \frac{1}{\kappa} \left[\left(u_{3x} + u_{4y} \right) u_1 + u_3 u_{1x} + u_4 u_{1y} \right]$$

Thus the equation (1) is divided in two:

$$\partial_t u_1 + \nabla \cdot \begin{pmatrix} -u_{1x}/\kappa^2 \\ -u_{1y}/k^2 \end{pmatrix} = -(u_{3x} + u_{4y})u_1/\kappa - (u_{1x}u_3 + u_{1y}u_4)/\kappa - (u_3{}^2 + u_4{}^2)u_1 + u_1 - (u_1{}^2 + u_2{}^2)u_1$$

$$\partial_t u_2 + \nabla \cdot \begin{pmatrix} -u_{2x}/\kappa^2 \\ -u_{2y}/k^2 \end{pmatrix} = -(u_{3x} + u_{4y})u_2/\kappa - (u_{2x}u_3 + u_{2y}u_4)/\kappa - (u_3{}^2 + u_4{}^2)u_2 + u_2 - (u_1{}^2 + u_2{}^2)u_2$$

Similarly the equation (2) is divided into:

$$\sigma\partial_t u_3 = (u_1 u_{2x} - u_2 u_{1x})/\kappa - (u_1{}^2 + u_2{}^2)u_3 - \nabla \cdot (0, u_{4x} - u_{3y} - B_a)^T$$

$$\sigma\partial_t u_4 = (u_1 u_{2y} - u_2 u_{1y})/\kappa - (u_1{}^2 + u_2{}^2)u_4 - \nabla \cdot (-u_{4x} + u_{3y} + B_a, 0)^T$$

And the boundary conditions (3 - 5) can be resumed by

$$\nabla \cdot \begin{pmatrix} u_3 \\ u_4 \end{pmatrix} = u_{3x} + u_{4y} + u_5$$

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COMSOL → $$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot \Gamma = F$$

$$\hat{d}_a = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \sigma & 0 & 0 \\ 0 & 0 & 0 & \sigma & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \Gamma = \begin{pmatrix} \left[-u_{1x}/\kappa^2, -u_{1y}/\kappa^2\right]^T \\ \left[-u_{2x}/\kappa^2, -u_{2y}/\kappa^2\right]^T \\ \left(0, u_{4x} - u_{3y} + B_a\right)^T \\ \left(-u_{4x} + u_{3y} + B_a, 0\right)^T \\ \left(u_3, u_4\right)^T \end{pmatrix}$$

$$F = \begin{pmatrix} F_1 & F_2 & F_3 & F_4 & F_5 \end{pmatrix}^T$$

$$F_1 = -\left(u_{3x} + u_{4y}\right)u_1 / \kappa - \left(u_{1x}u_3 + u_{1y}u_4\right)/\kappa - \left(u_3{}^2 + u_4{}^2\right)u_1 + u_1 - \left(u_1{}^2 + u_2{}^2\right)u_1$$

$$F_2 = -\left(u_{3x} + u_{4y}\right)u_2 / \kappa - \left(u_{2x}u_3 + u_{2y}u_4\right)/\kappa - \left(u_3{}^2 + u_4{}^2\right)u_2 + u_1 - \left(u_1{}^2 + u_2{}^2\right)u_2$$

$$F_3 = \left(u_1 u_{2x} - u_2 u_{1x}\right)/\kappa - \left(u_1{}^2 + u_2{}^2\right)u_3$$

$$F_4 = \left(u_1 u_{2y} - u_2 u_{1y}\right)/\kappa - \left(u_1{}^2 + u_2{}^2\right)u_4$$

$$F_5 = u_{3x} + u_{4y} + u_5$$

Density of energy

$$H_{tot} = H_{\sup} + H_{mag} + H_{\text{int}}$$

with

$$H_{\text{int}} = \frac{i}{\kappa}\vec{A}\cdot\left[\left(\nabla\Psi\right)\Psi^* - \Psi\left(\nabla\Psi^*\right)\right] + \left|\vec{A}\right|^2\left|\Psi\right|^2$$

$$H_{mag} = \left(\vec{B}_a - \vec{\nabla}\times\vec{A}\right)^2$$

$$H_{suc} = -\left|\Psi\right|^2 + \frac{1}{2}\left|\Psi\right|^4 + \frac{1}{\kappa^2}\left|\nabla\Psi\right|^2$$

FEM - COMSOL

$$H_{suc} = \left(u_{1x}{}^2 + u_{2y}{}^2\right)/\kappa^2 - \left(u_1{}^2 + u_2{}^2\right) + \frac{1}{2}\left(u_1{}^2 + u_2{}^2\right)^2$$

$$H_{mag} = \left(B_a - u_{4x} - u_{3y}\right)^2$$

$$H_{\text{int}} = \frac{1}{\kappa}\left[\left(u_{2x}u_1 - u_{1x}u_2\right)u_3 + \left(u_{2y}u_1 - u_{1y}u_2\right)u_4\right] + \left(u_3{}^2 + u_4{}^2\right)\left(u_1{}^2 + u_2{}^2\right)$$

The energy of the system

$$\mathrm{E}_{tot} = \int_\Omega H_{tot}(x,y,t)\,dxdy$$

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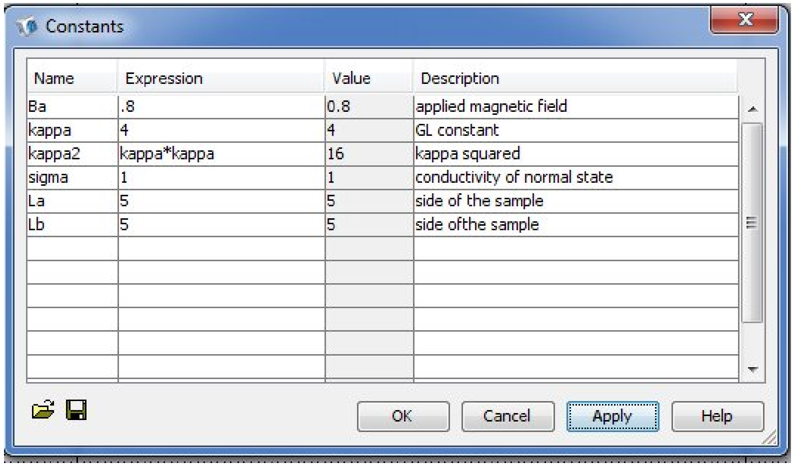
Finite Element Method (COMSOL)

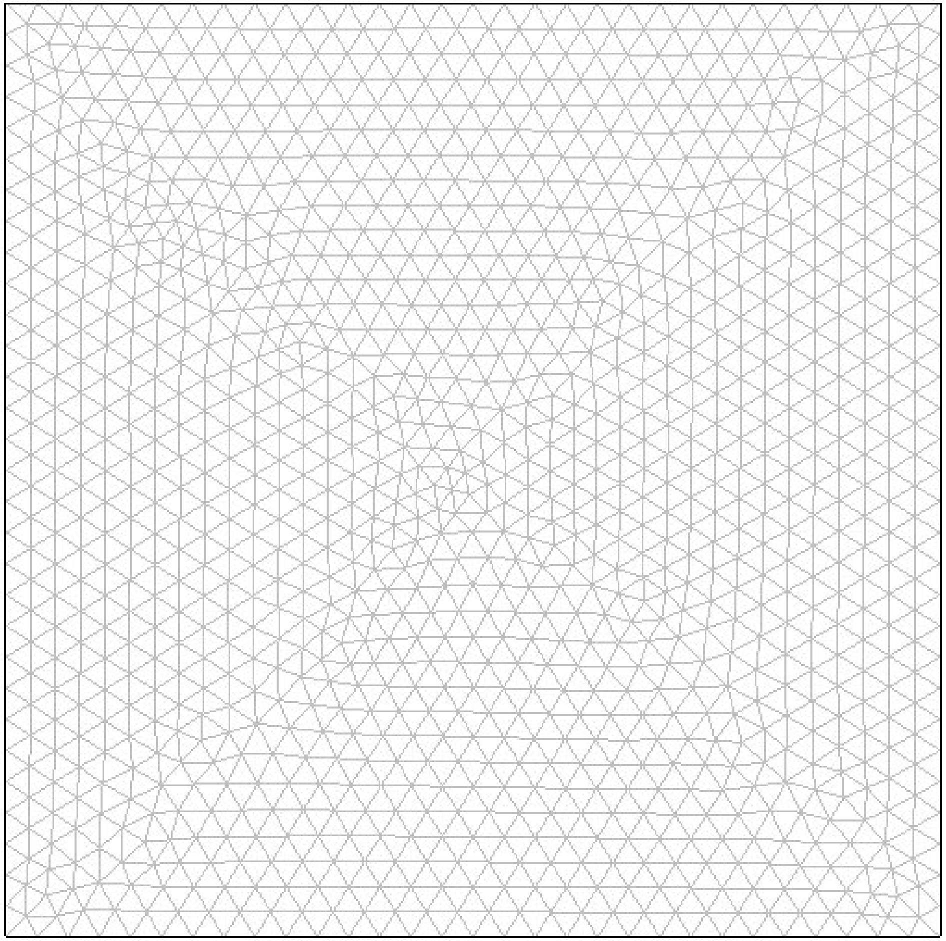
$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot \Gamma = F$$



Constants

Name	Expression	Value	Description
Ba	.8	0.8	applied magnetic field
kappa	4	4	GL constant
kappa2	kappa*kappa	16	kappa squared
sigma	1	1	conductivity of normal state
La	5	5	side of the sample
Lb	5	5	side ofthe sample

OK Cancel Apply Help



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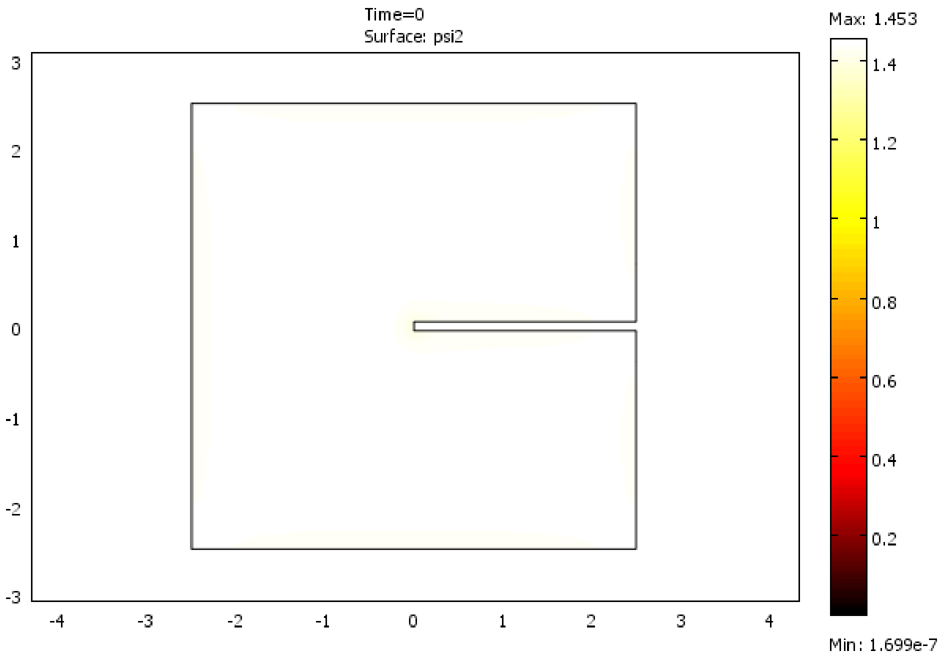
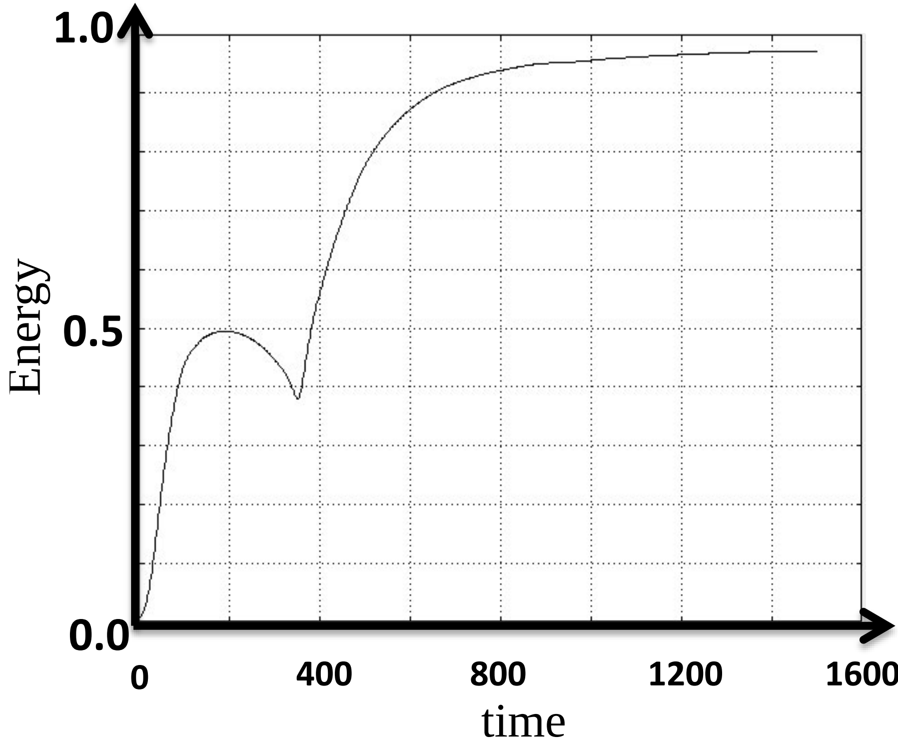
Squared superconductor with a slit

$$E_{tot} = \int_{\Omega} H_{tot}(x, y, t)\,dxdy$$



L=5, k=4, Ba=0.55Hc2



J Supercond Nov Magn (2014) 27:1143–1152
DOI 10.1007/s10948-013-2390-2

ORIGINAL PAPER

Magnetic Flux Penetration in a Mesoscopic Superconductor with a Slit

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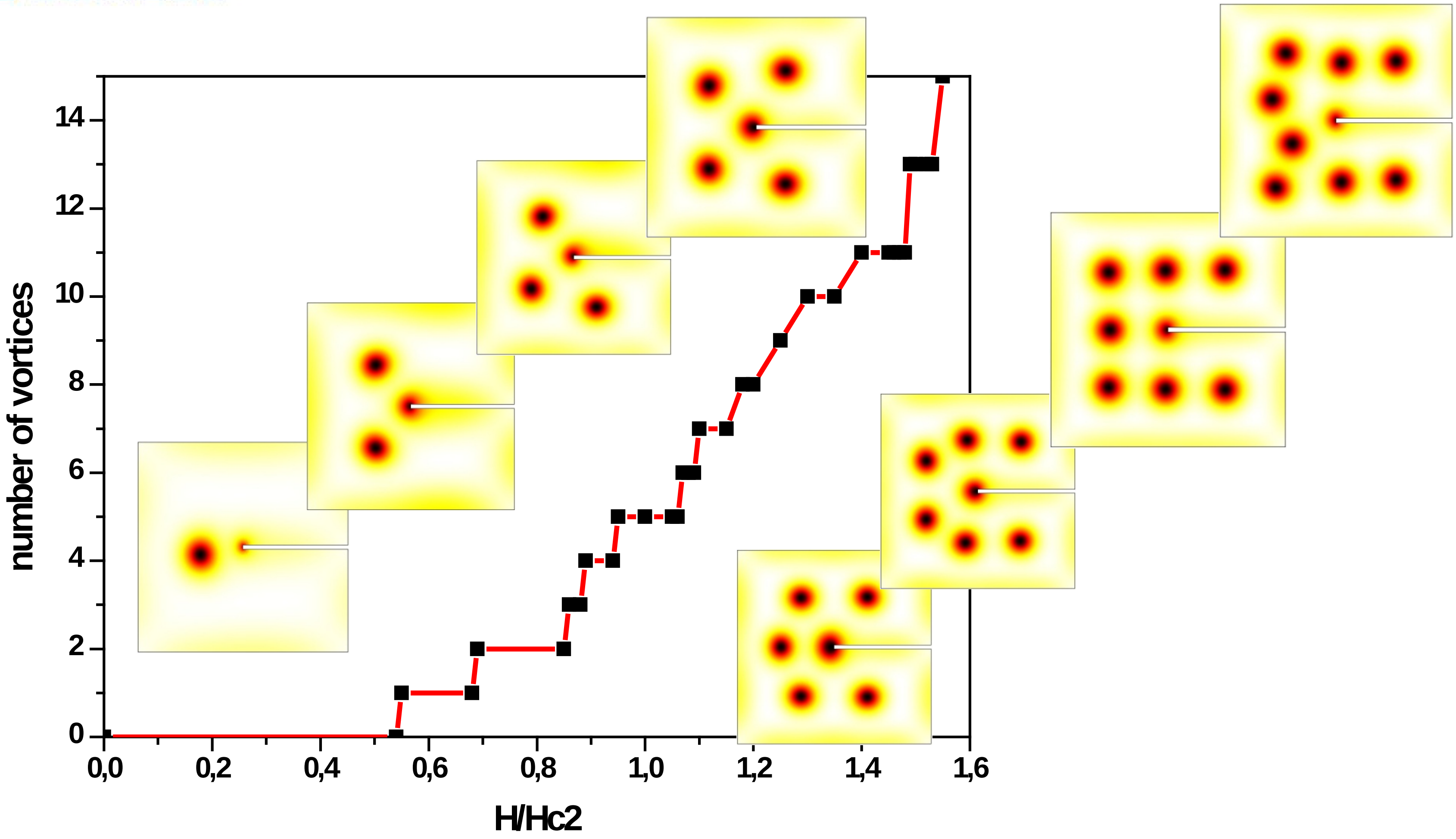
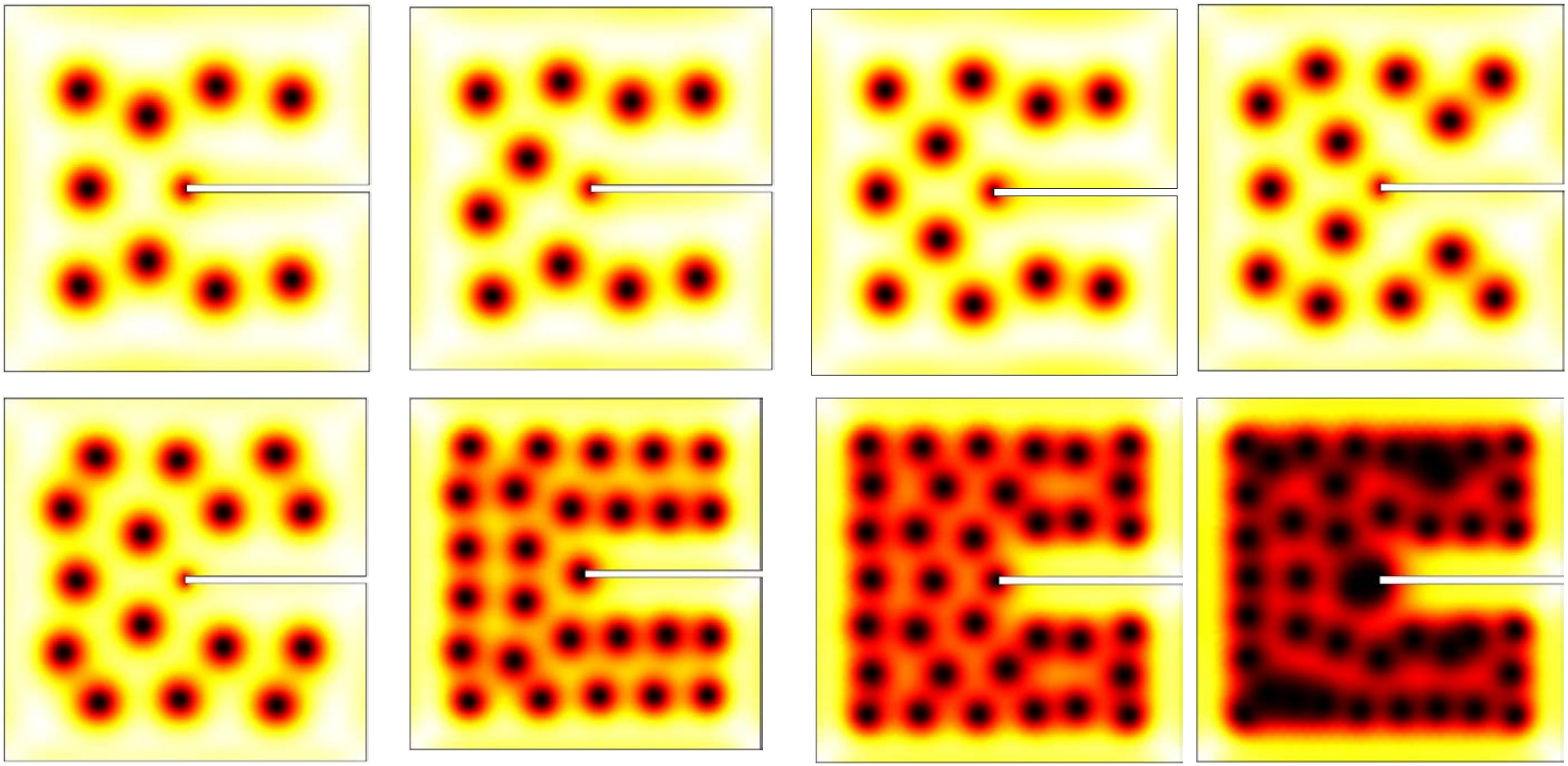
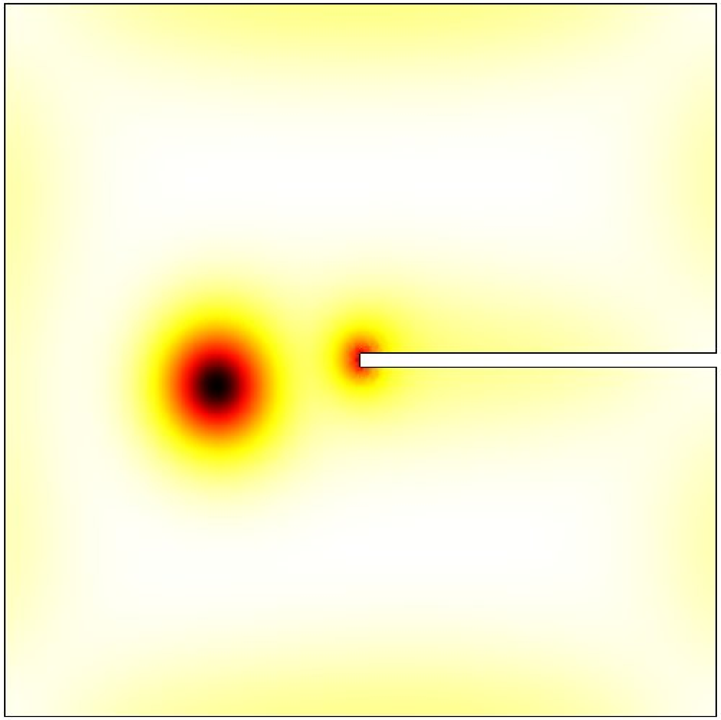
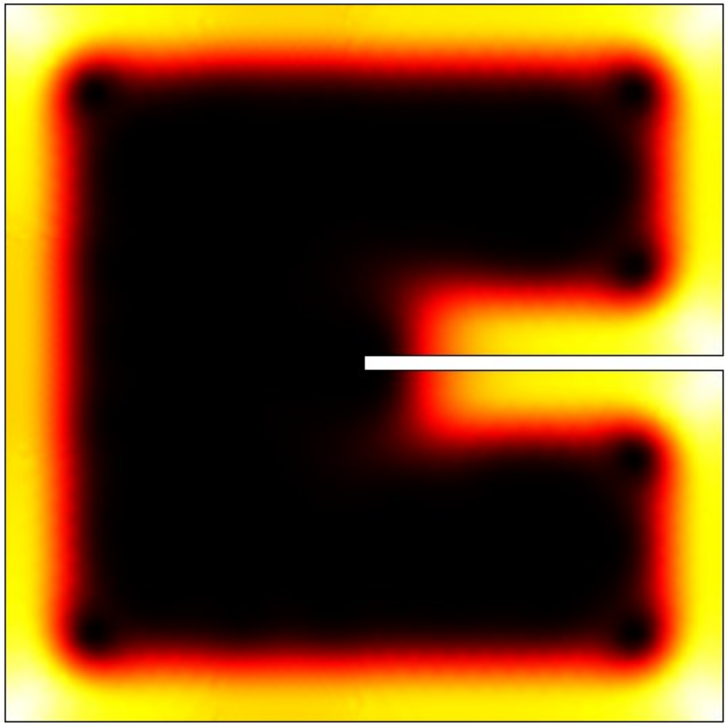

number of vortices
0
2
4
6
8
10
12
14
0,0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
1,6
H/Hc2



Ba = 0.55 Hc2



Ba = 4 Hc2



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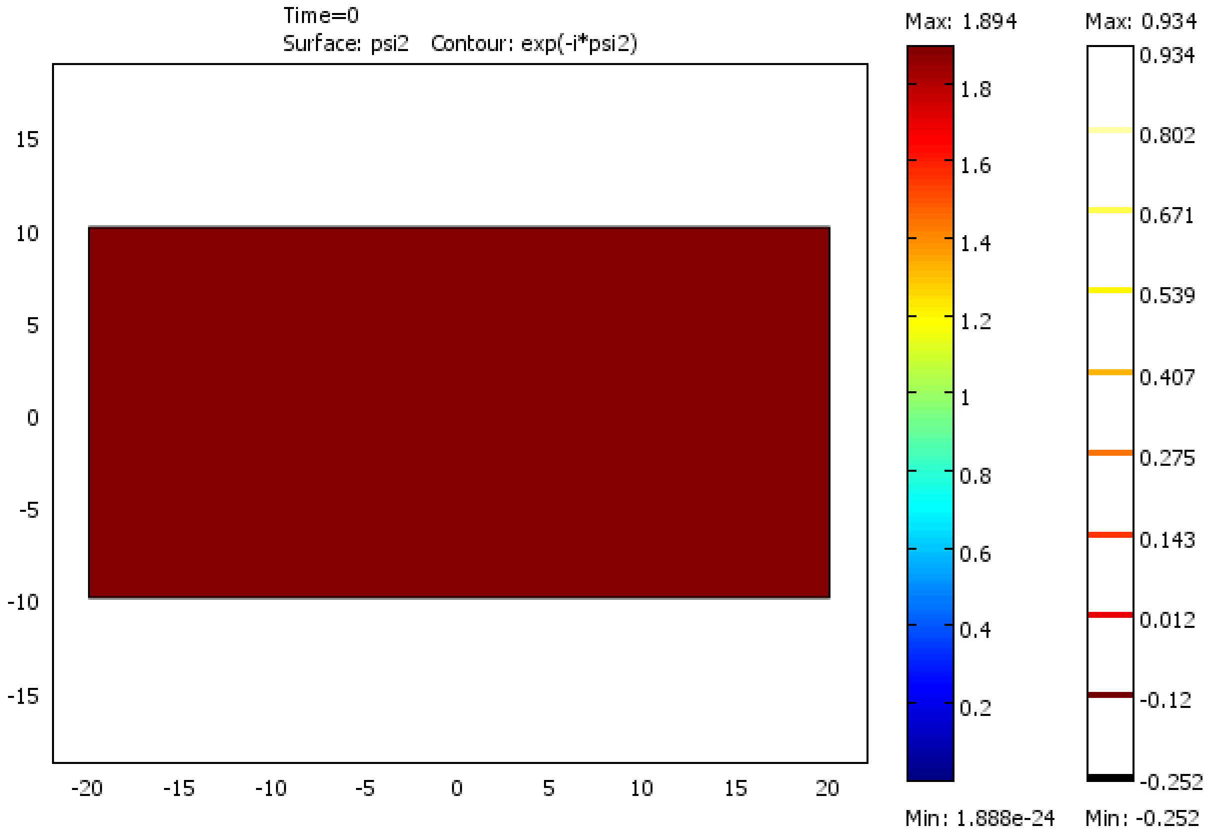
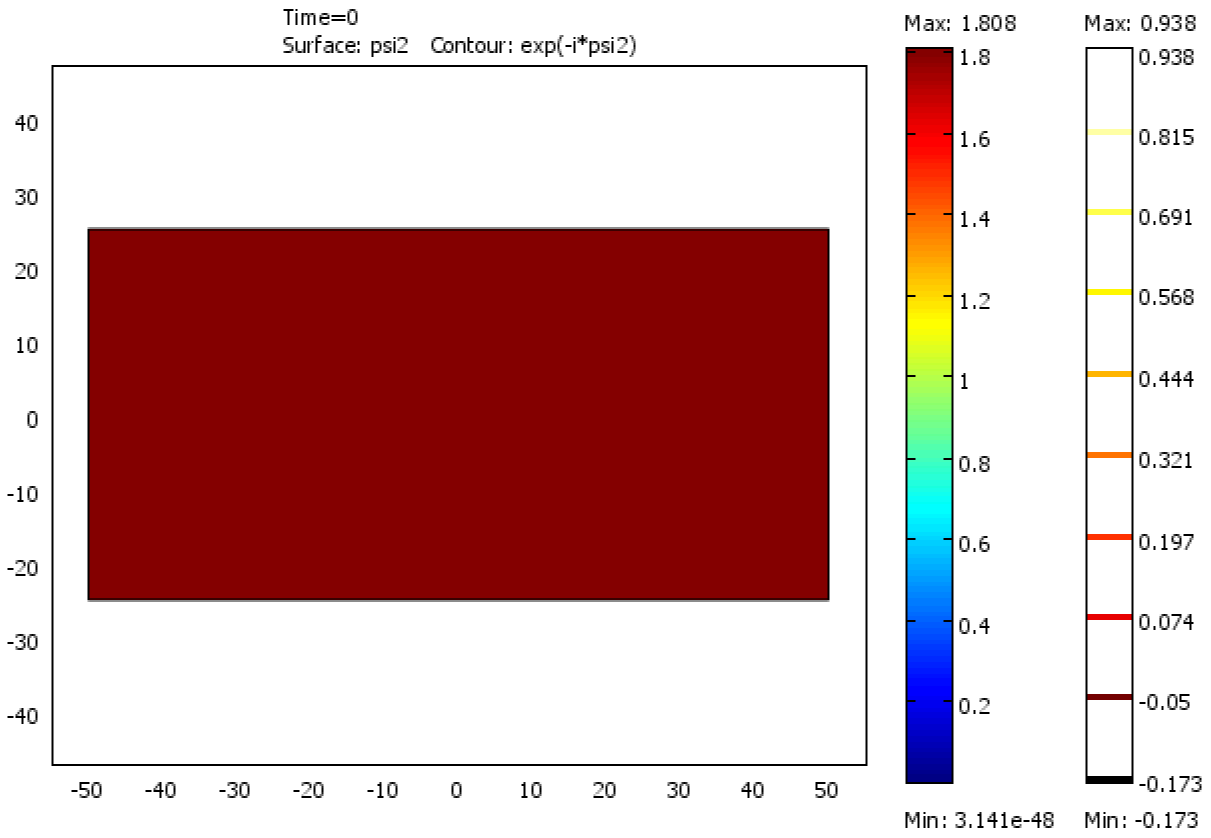
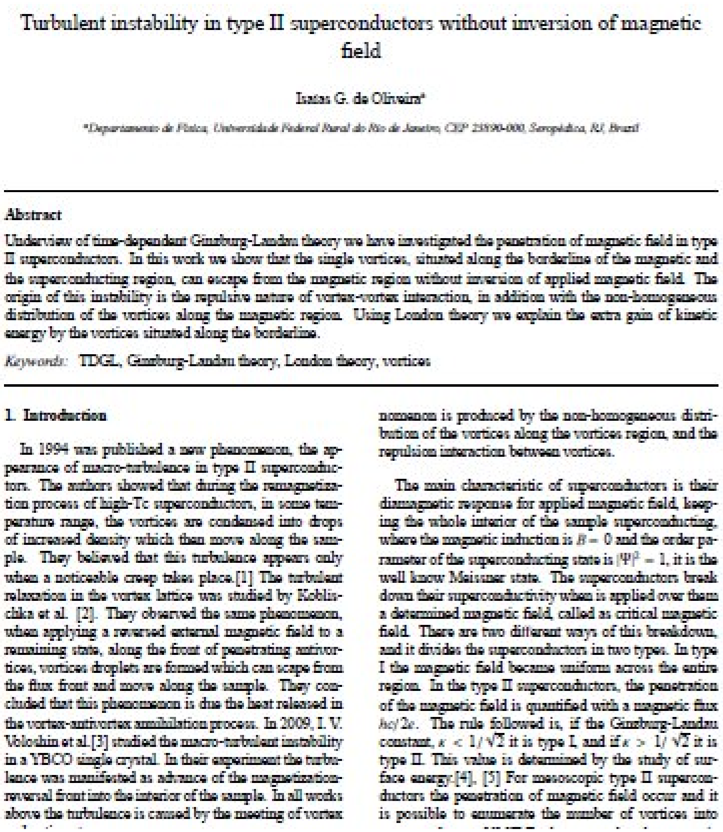
Magnetic field penetration in type II superconductor



Penetration of magnetic field in type I superconductor



Macroturbulent instability in superconductors



Turbulent instability in type II superconductors without inversion of magnetic field

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Abstract

Underview of time-dependent Ginzburg-Landau theory we have investigated the penetration of magnetic field in type II superconductors. In this work we show that the single vortices, situated along the borderline of the magnetic and the superconducting region, can escape from the magnetic region without inversion of applied magnetic field. The origin of this instability is the repulsive nature of vortex-vortex interaction, in addition with the non-homogeneous distribution of the vortices along the magnetic region. Using London theory we explain the extra gain of kinetic energy by the vortices situated along the borderline.

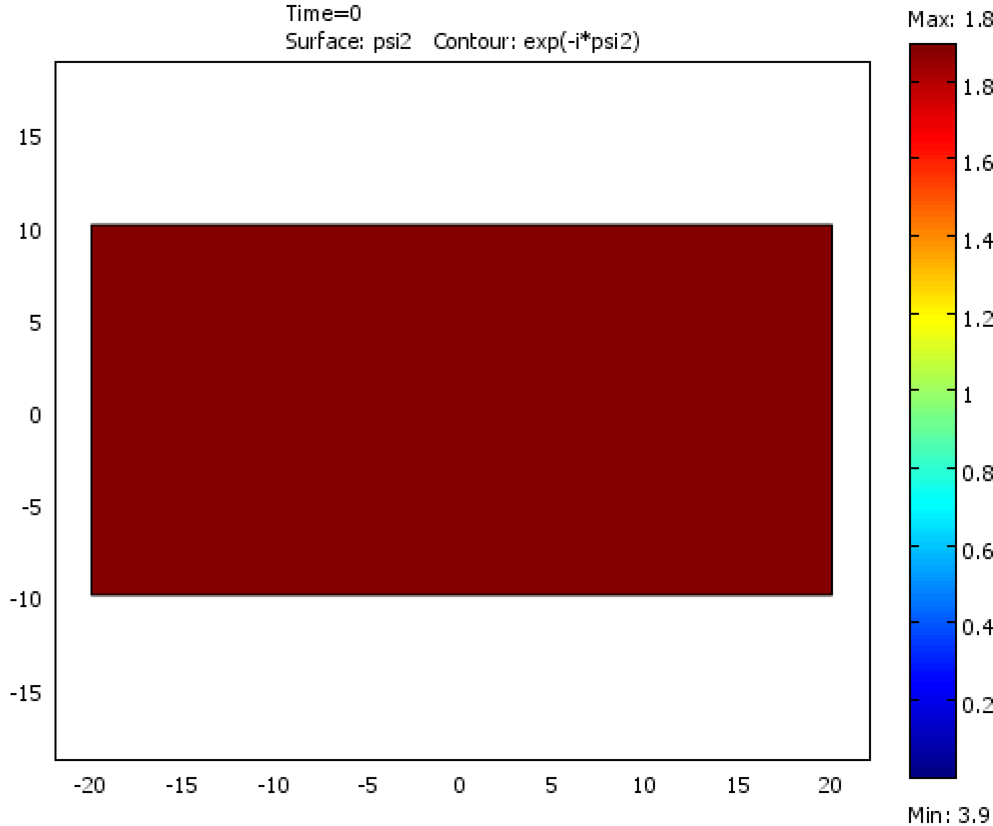
Keywords: TDGL, Ginzburg-Landau theory, London theory, vortices

1. Introduction

In 1994 was published a new phenomenon, the appearance of macro-turbulence in type II superconductors. The authors showed that during the remagnetization process of high-Tc superconductors, in some temperature range, the vortices are condensed into drops of increased density which then move along the sample. They believed that this turbulence appears only when a noticeable creep takes place.[1] The turbulent relaxation in the vortex lattice was studied by Koblischka et al. [2]. They observed the same phenomenon, when applying a reversed external magnetic field to a remaining state, along the front of penetrating antivortices, vortices droplets are formed which can scape from the flux front and move along the sample. They concluded that this phenomenon is due the heat released in the vortex-antivortex annihilation process. In 2009, I. V. Voloshin et al.[3] studied the macro-turbulent instability in a YBCO single crystal. In their experiment the turbulence was manifested as advance of the magnetization-reversal front into the interior of the sample. In all works above the turbulence is caused by the meeting of vortex

nomenon is produced by the non-homogeneous distribution of the vortices along the vortices region, and the repulsion interaction between vortices.

The main characteristic of superconductors is their diamagnetic response for applied magnetic field, keeping the whole interior of the sample superconducting, where the magnetic induction is $B = 0$ and the order parameter of the superconducting state is $|\Psi|^2 = 1$, it is the well know Meissner state. The superconductors break down their superconductivity when is applied over them a determined magnetic field, called as critical magnetic field. There are two different ways of this breakdown, and it divides the superconductors in two types. In type I the magnetic field became uniform across the entire region. In the type II superconductors, the penetration of the magnetic field is quantified with a magnetic flux $hc/2e$. The rule followed is, if the Ginzburg-Landau constant, $\kappa < 1/\sqrt{2}$ it is type I, and if $\kappa > 1/\sqrt{2}$ it is type II. This value is determined by the study of surface energy.[4], [5] For mesoscopic type II superconductors the penetration of magnetic field occur and it is possible to enumerate the number of vortices into



Conclusion

Using COMSOL we can solve the non-linear time dependent Ginzburg-Landau equations.

Using COMSOL we could understanding the magnetic field penetration into mesoscopic type II superconductors with a slit (or any other mechanic deffect)

Using COMSOL we could show a macroturbulent effect in type II superconductors

Thank you !!!