Modal Analysis of Functionally Graded Metal-Ceramic Composite Plates

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Abstract: Metal-matrix composites (MMCs) enhance the strength achievable with metal alloys alone though the reinforcing effect of dispersed hard particulates in a continuous metal matrix. Functionally-graded MMCs expand the range of possibilities through detailed spatial control of the concentration of particulates. This variation can be readily produced in practice using any of the various additive manufacturing processes now available. The determination of the modes of vibration of FG-MMC plates is important in practice in order to prevent undesired resonances in structural components. This paper describes the application of the finite element method COMSOL Multiphysics for the determination of the modes of vibration of Aluminum A356 alloy-ZrO2 MMC FG square plates.

Keywords: Functionally-graded materials, composite materials, metal-ceramic composite, modal analysis, additive manufacturing.

1. Introduction

Metal-matrix composites are multi-phase mixtures consisting of fine hard particles (reinforcement) dispersed in a ductile matrix (metal). Since the properties of the composite are generally better that those of the components, these materials have found wide applicability [1]. A functionally graded material (FGM) is a two phase composite where the spatial distribution of the volume fraction of reinforcement is carefully controlled so as to optimize the functional mechanical response of the component [2]. FGMs differ from traditional ply laminate composites in two ways. Whereas in laminates the material properties change discontinuously at each ply-ply interface [3] in FGMs the properties vary continuously. And while individual plies in laminates almost always exhibit strong anisotropy, FGMs are isotropic at any given point although the values of the elastic constants vary with position. FGMs accomplish this through a gradual, controlled change in the volume fraction of the materials which make up the material along a specified direction in the component.

Functionally graded metal matrix composites combine the reinforcement effect of dispersed hard particles with the functional effect of controlled spatial distribution of reinforcing particles. These new materials are now being produced by various additive manufacturing processes such as spray deposition or 3D printing and the range of potential applications is huge. However, for these applications to materialize, solid, fundamental understanding of their behavior must be developed. This paper described recent work at RPI designed to obtain insight into applicability of the finite element method in COMSOL Multiphysics to the understanding of the vibration response of FG-MMC plates. The mechanical response of FGM plates has been the subject of extensive analytical research work over the past decade [4]-[7] which provides an excellent baseline for comparison with the results of finite element approximation.

2. Description of the System

The purpose of this project was to perform modal analysis of a functionally graded metal matrix composite plate. The metal was Aluminum alloy (A356) and the reinforcement was Zirconia (ZrO_2). A356 is a widely used aluminum alloy in automotive applications. It is usually produced by permanent mold casting and subsequently heat treated to yield optimal properties [8]. The material properties used in this project are summarized in Table 1.

Table 1: Material Properties

Material	ρ (kg/m^3)	E (Pa)	ν (-)
A356	2670	7.24e9	0.33
ZrO_2	5575	1.75e11	0.27

The dimensions of the plate were 1 m by 1 m by 0.05 m. The plate was assumed simply supported. The volume fraction of ZrO2 across the thickness of the plate is assumed given by the following expression

$$V_2 = V_b + (V_t - V_b) \cdot \left(\frac{1}{2} + \frac{z}{H}\right)^p$$

where V_b and V_t are the volume fractions of ZrO_2 at the lower and upper surfaces of the plate H is the plate thickness, z is the distance measured from the center of the plate and p is a controllable parameter. The volume fraction of Al across the thickness is given by

$$V_1 = 1 - V_2$$

Note that the subscripts 1 and 2 refer to Al and ZrO_2, respectively.

3. Use of COMSOL Multiphysics

The Mori-Tanaka method [9] was employed to estimate the material properties of the functionally graded plate as functions of the distance through the thickness. The bulk modulus for FGM is given by

$$K_{FGM} = K_1 + \frac{(K_2 - K_1) \cdot V_2}{\left(1 + \frac{(1 - V_2) \cdot (K_2 - K_1)}{K_1 + \left(\frac{4}{3}\right) \cdot \mu_1}\right)}$$

The shear modulus for FGM is given by

$$\mu_{FGM} = \mu_1 + \frac{(\mu_2 - \mu_1) \cdot V_2}{\left(1 + \frac{(1 - V_2) \cdot (\mu_2 - \mu_1)}{\mu_1 + f_1}\right)}$$

where

$$f_1 = \frac{\mu_1 \cdot (9 \cdot K_1 + 8 \cdot \mu_1)}{6 \cdot (K_1 + 2 \cdot \mu_1)}$$

The density for FGM is given by

$$\rho_{FGM} = \rho_1 \cdot V_1 + \rho_2 \cdot V_2$$

From the above the Poisson Ratio and elastic modulus of the FGM are given by

$$\nu_{FGM} = \frac{1}{2 \cdot \left(1 + \frac{\mu_{FGM}}{\lambda}\right)}$$

$$E_{FGM} = 3\,(1 - 2\,\nu_{FGM})\,K$$

where

$$\lambda = K - 2\,\mu_{FGM}/3$$

All the above equations were entered directly into COMSOL Multiphysics by first creating a parameter list under Global Definitions and then the functions in Analytic form under Model Definitions. The required material properties (Young's modulus, Poisson's ratio and density) were written as functions of position under Materials. The simply supported condition was simulated by restricting the displacement of the plate normal to its plane along the four lower edges (the frequencies of the rigid displacement modes were always found to be negligible). The default normal element size was selected under Meshing with a physics controlled mesh. The above methodology significantly facilitates the use of the model for the investigation of other material combinations, volume fraction distributions and plate geometries.

4. Results

Figure 1 shows the variation in the volume of ZrO2 across the thickness of the plate for values of the parameter p in the range from 0.01 to 100. It is clear that a wide range of material characteristics can be obtained in this system using the same two components. From a plate that is mostly ZrO_2 with an A356 underside coating (p = 0.01) to a plate with linearly varying ZrO_2 content (p=1) to a plate that is mostly metal with a thin ZrO_2 coating on the upper side (p=100). This ability to create a virtually infinite array of structural components by combining only two materials and manipulating their relative contents by means of the single parameter p is a very interesting feature of FGMs.

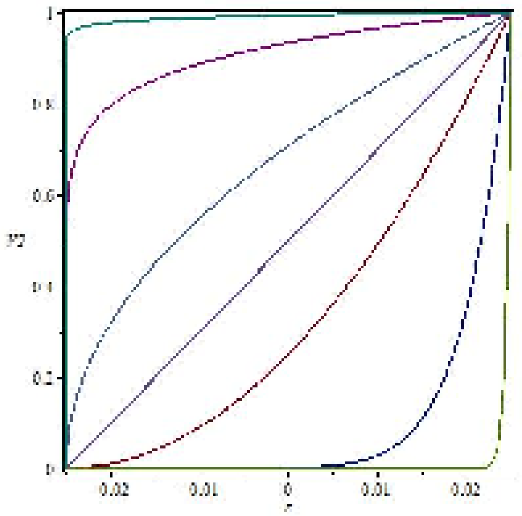

Figure 1. Variation in the volume fraction of ZrO2 across the thickness of the plate for values of the parameter p in the range from 0.001 to 100.

As an initial test of the model the modes of a plate made up entirely of A356 alloy were computed. The results were compared to the exact values [10]. The error on the first frequency was 2% and was considered satisfactory.

As an example of the results obtained from the COMSOL Multiphysics model of the FG plate, figures 2 through 6 show the computed mode shapes for the first five natural frequencies of vibration of the plate for the case when p = 10.

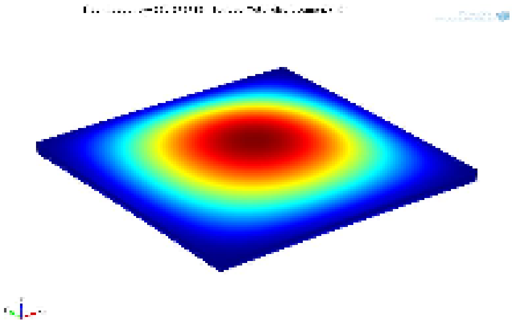

Figure 2. Mode 1 (251.67 Hz)

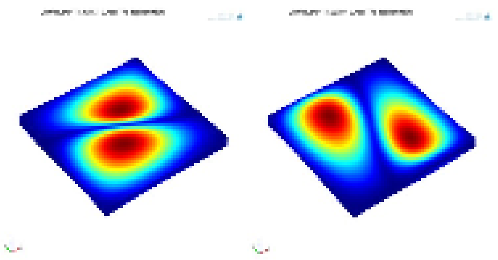

Figure 3 a,b. Mode 2 (626.404, 626.825 Hz)

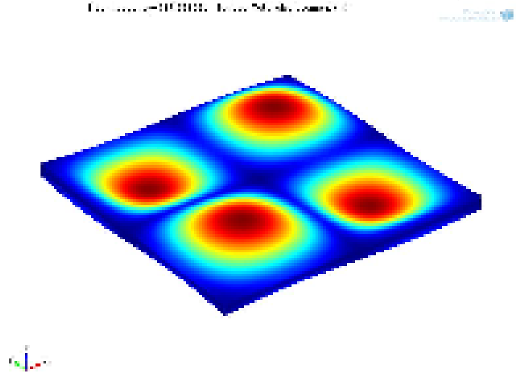

Figure 4. Mode 3 (987.25 Hz)

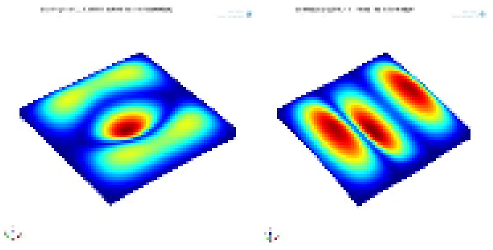

Figure 5. Mode 4 (1239.03, 1239.41 Hz)

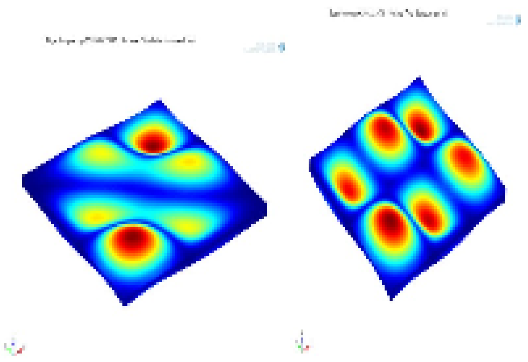

Figure 6. Mode 5 (1585.00, 1586.11 Hz)

A comparison of the computed first frequency value with the predicted value calculated according to the method in [6] gave a difference of about 2%.

The model was used to compute natural frequencies of vibration for various plate thicknesses, values of the parameter p and for several other material systems. A similar model was created using a different commercial finite element program to carry out the same computations and the results were in good agreement with those calculated using COMSOL Multiphysics. Detailed information about these efforts is available in the Rensselaer Master's Project Reports produced by Mr. Saunders and Mr. Pendley [11], [12].

5. Conclusions

It is possible to use COMSOL Multiphysics to create mathematical models for the computation of frequencies of vibration of functionally graded metal ceramic composites. Model construction is fairly easy and computation and visualization of the results are straightforward. The results of the finite element approximations obtained were found sufficiently accurate for engineering use.

6. References

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