COMSOL MultiphysicsÈ: Time-Lapse Electrical Resistivity Inversion

T. K. Chou[1], M. J. Yi[2], and M. Chouteau[1]

1. Department of Civil, Geological & Mining Engineering, £cole Polytechnique de Montr®al, Montreal, QC, Canada
2. Korea Institute of Geoscience and Mineral Resources, Engineering Geophysics Group, Daejeon City, South Korea

Introduction: Electrical resistivity tomography (ERT) is a non-invasive method for mapping the subsurface resistivity distribution. Time-lapse ERT provides a way to monitor subtle sub-surface changes caused by water flow such as in an infiltration test. Problems arise when the inversion models contain artifacts due to measurement errors, rapid change in soil electrical property during measurement time, etc... A new approach was developed by Kim et al. (2006[1] and 2009[2]) and it was shown to reduce inversion artifacts and provides adequate reconstruction of the true model. The aim of this project is to develop a code using COMSOL Multiphysics and MATLAB, which can invert ERT monitoring data using the latest time-lapse inversion scheme proposed.

Methodology: An approach in interpreting monitoring data is to invert data at each time independently, and the change in ground condition is determined by the difference between the inversion models at successive time. Another approach is to invert the data at a later time while being constrained by the inversion model at the previous time (Loke, 1999[3]). The Kim et al approach incorporates the time constrain $\beta M^T M$ in the standard ERT inversion scheme that simultaneously invert the entire monitoring data set.

$$A = J^T D^T D J + \alpha C^T C + \beta M^T M$$

$$B = J^T D^T (G(X_i) - d_0) + \beta M^T M$$

where *A* and B are variables for the linear equation *Ax=B*. And *x* is the model property vector. The AC/DC module is used for solving the forward electrical resistivity problems $G(X_i)$ and in calculating the Jacobian matrix *J*. LiveLink™ for MATLAB interface is used to exchange information between COMSOL and the inversion algorithm, and to solve the system of linear equations using the preconditioned conjugate gradients method.

Tests: A synthetic time-lapse model consisting of a rectangular shape body with resistivity varying with time is used. The body dimension is 0.5 m in height and 1.5 m in width, and it is centered at coordinate (10 m, -0.75 m). The medium has a resistivity of 300 Ωm, and the body has a resistivity of 300 Ωm, 260 Ωm, 220 Ωm, 180 Ωm and 140 Ωm, for respectively, at observation time 0, 1, 2, 3 and 4.

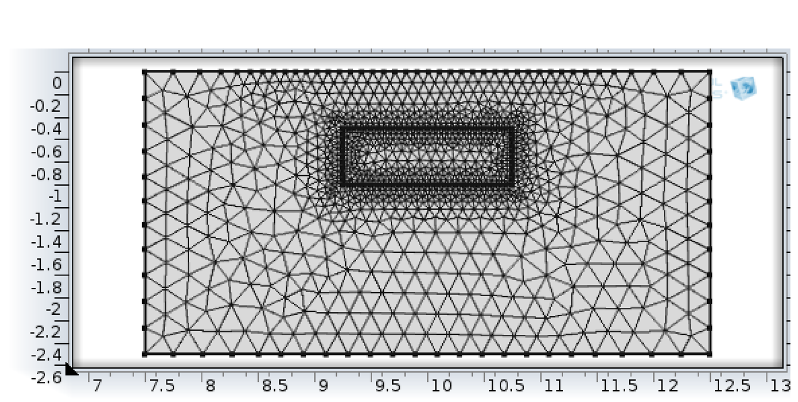

Figure 1. Model meshes (closed-up).

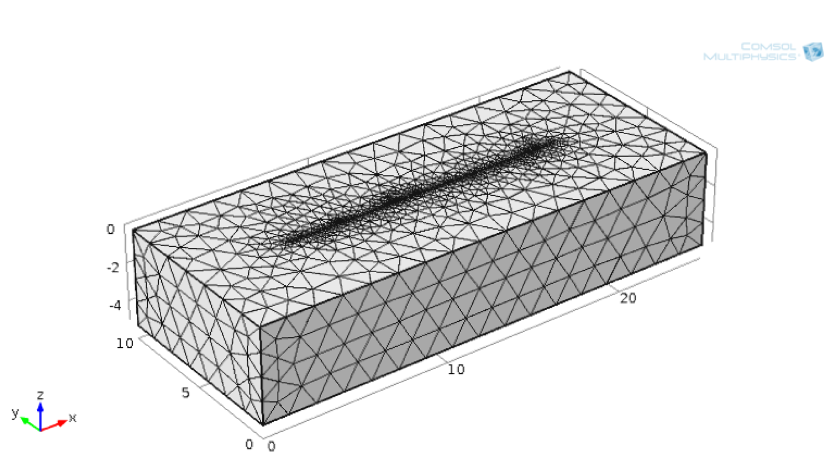

Figure 2. Model meshes.

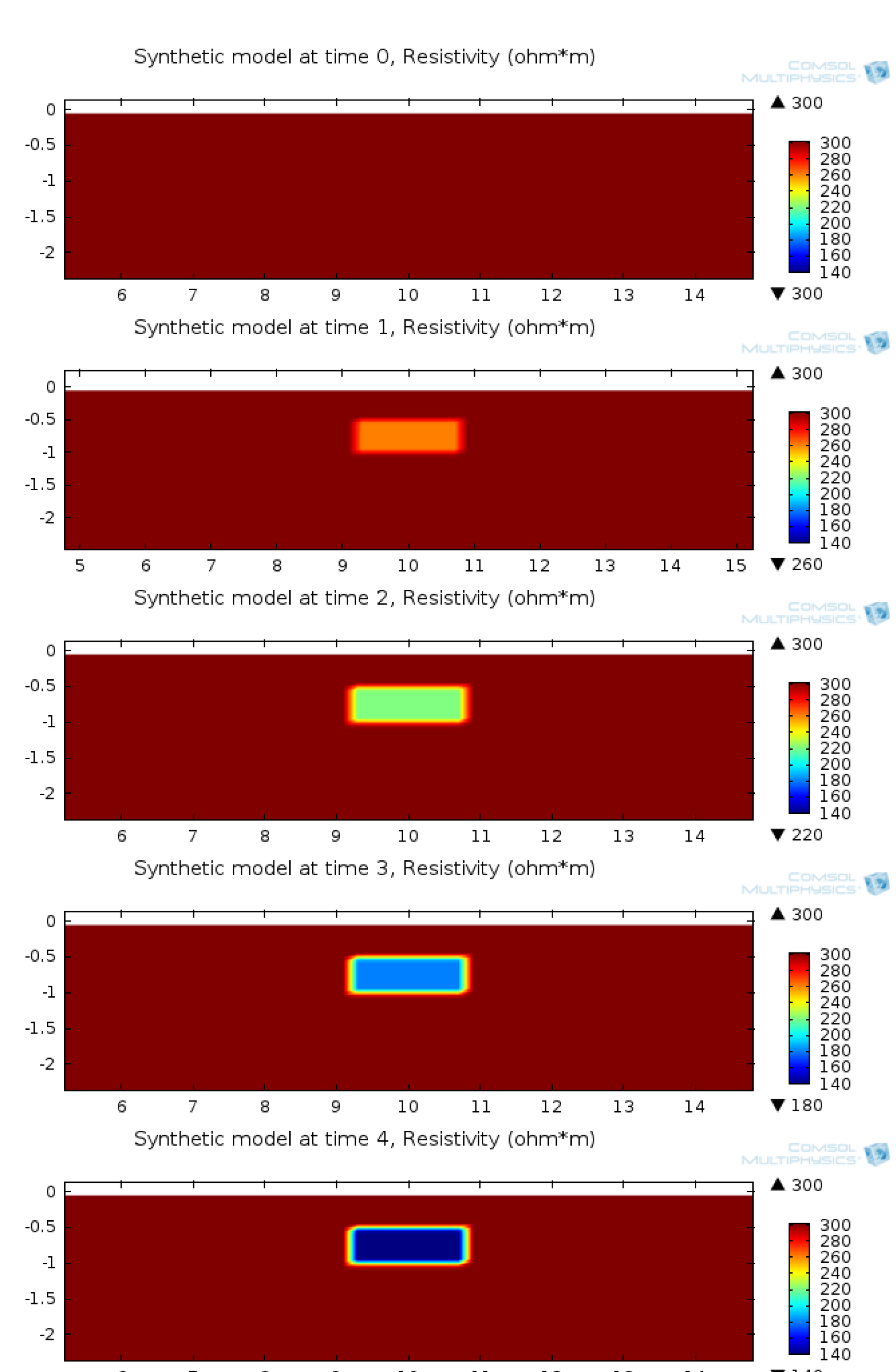

Figure3. Synthetic models.

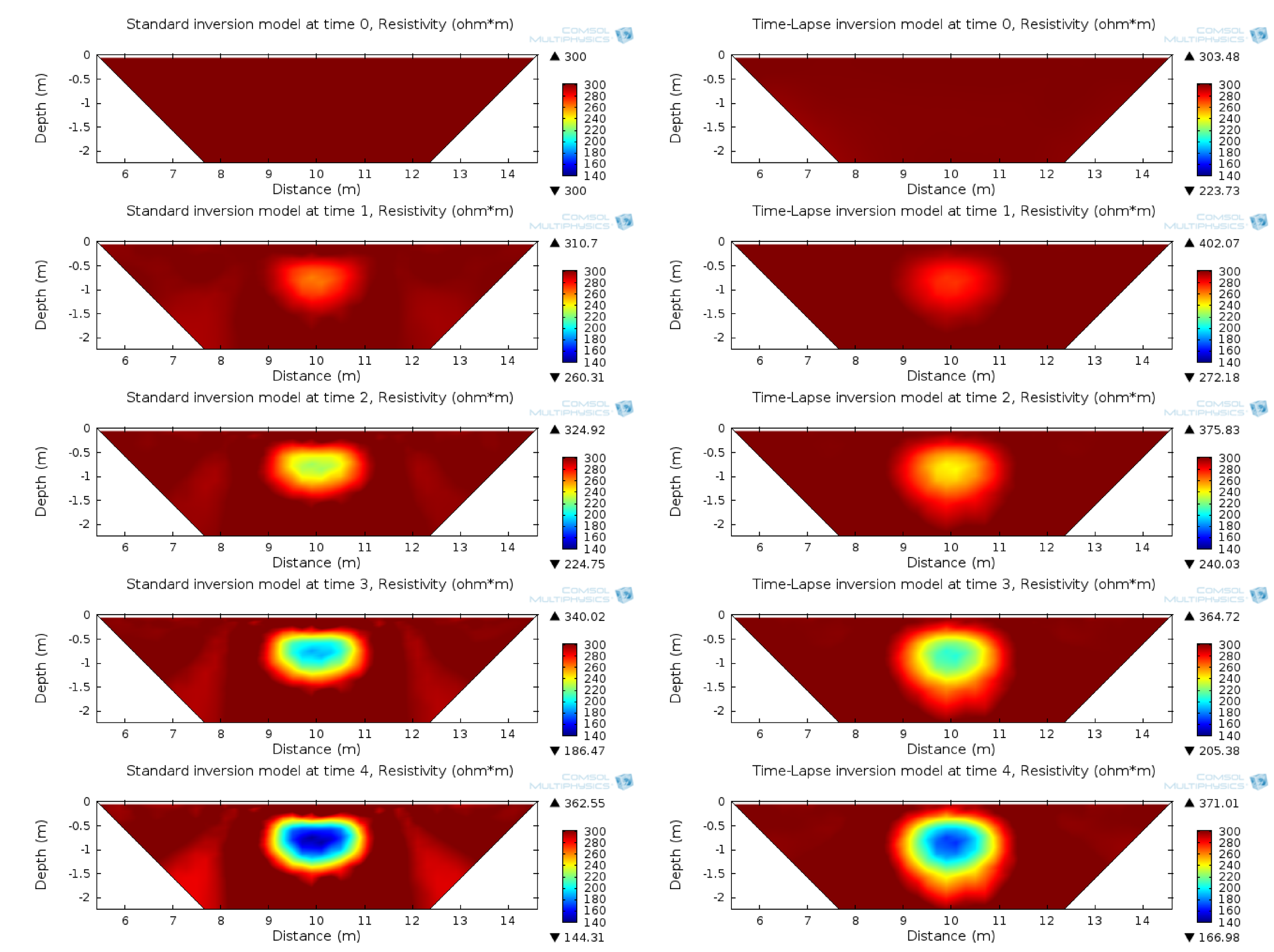

Figure 4. Resistivity models for (left) Standard and (right) Time-lapse inversions.

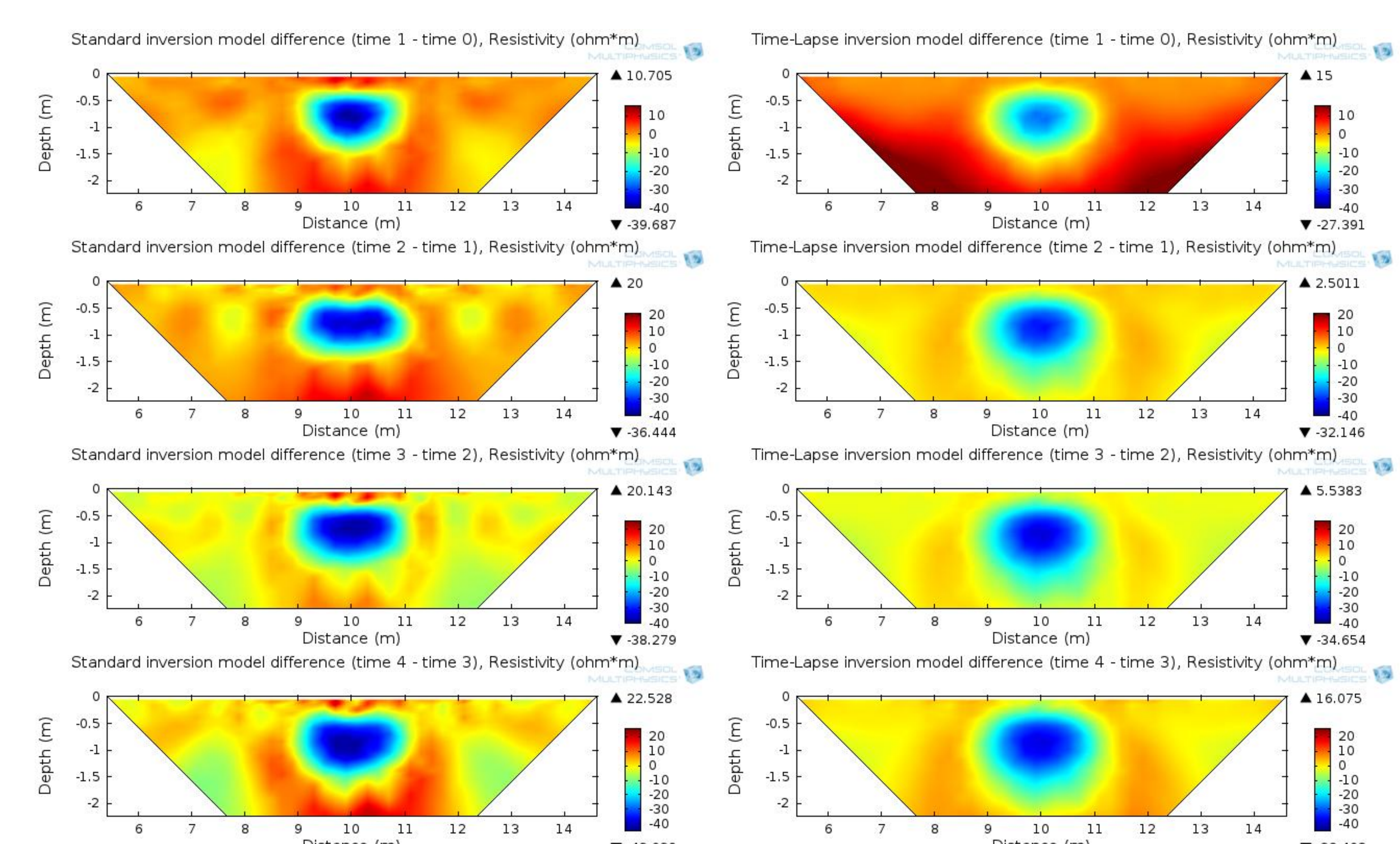

Figure 5. Models difference for (left) Standard and (right) Time-lapse inversions.

Discussion and Conclusion: A greater reduction in artifact produced during the inversion is observed when time-lapse inversion scheme is used (Figures 4 and 5). The location and shape of the varying resistive body is approximately the same as the model difference produced by standard independent inversion. The developed time-lapse inversion code was compared with the code developed by Kim et al. The percentage error between the physical property of the inversion models and the true model is calculated and compared (Table 1). The model reconstruction produced by Kim et al time-lapse inversion code are, to some extent, better due to an improvement in the algorithm by utilizing L2-norm for data and space domain, and L1-norm for the time domain (Kim et al, 2012[4]).

Table 1. Model reconstruction error (%) for different time-lapse inversion.

Time-Lapse	Percent Error (%)				
Inv. done by	t0	t1	t2	t3	t4
Chou et al	0.36	2.29	2.82	3.86	5.46
Kim et al	1.29	1.49	1.89	2.42	3.13

References:

1. Kim, J.-H., Park, S.-G., Yi, M.-J., and Kim, J.-G., 4-D inversion of geophysical monitoring data acquired over dynamically changing 4-D earth model, Proceedings of 8th SEGJ International Symposium, Kyoto Univ., Kyoto, Japan, 89-94, (2006)
2. Kim, J.-H., Yi, M.-J., Park, S.-G., and Kim, J.-G., 4-D inversion of DC resistivity monitoring data acquired over a dynamically changing earth model, Journal of Applied Geophysics, 68, 522-535, (2009)
3. Loke, M.H., Time-lapse resistivity imaging inversion. Proceedings of the 5th Meeting of the Environmental and Engineering Geophysical Society European Section. Budapest, Hungary, (1999)
4. Kim, J.-H., Supper, R., Tsourlos, P. and Yi, M.-J., 4D Inversion of Resistivity Monitoring Data through Lp Norm Minimizations, Extended Abstract of Near Surface Geoscience 2012 – 18th European Meeting of Environmental and Engineering Geophysics, (2012)