Modeling of Nonequilibrium Effects in the Gravity Driven Countercurrent Imbibition

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Abstract: One of the main mechanisms in the secondary enhanced oil recovery is countercurrent imbibition[1]. Many mathematical models describe countercurrent imbibition considering local equilibrium. However, Barenblatt proposed a model to describe the effect of nonequilibrium constitutive relations in oil water displacement. This paper implements this model for a gravity driven countercurrent imbibition process. Firstly Darcy's equation was written for vertical displacement of the oil phase and the water phase. In addition, using mass conservation it is possible to formulate Barenblatt's model for two phase gravity driven countercurrent process.

This model allows to simulate countercurrent displacement in a core initially saturated with an oil phase and water phase. The initial condition satisfies gravity - capillary equilibrium in a tube of 1 m length. Moreover, we assume no flow boundary at both ends.

After establishing initial and boundary conditions, initial distribution of local effective saturation, $\eta = S_w + \tau \frac{\partial S_w}{\partial t}$, was calculated from a stationary model using Comsol Multiphysics . The stationary solution was obtained as initial condition for the time dependent model. Simulations were carried out using the situation that tube was turned 180 degree vertically. The result shows the saturation profile as a function of time and at the end a stationary gravity - capillary equilibrium was obtained. However, the solution is not the exact mirror of the initial profile. This is attributed to this fact that a final equilibrium profile can only obtained after a very long time.

Keywords: Countercurrent imbibition, gravity - capillary equilibrium, local effective water saturation, Barenblatt's model.

1 Introduction

Spontaneous imbibition is the process by which the wetting phase fluid enters pores and displaces the non-wetting phase. It is a recovery mechanism in water wet and mixed wet reservoirs[2]. During imbibition, wetting and non-wetting phases can flow in the opposite or the same directions with respect to each other. In the case of the opposite direction, it is called countercurrent imbibition, but when they flow in the same direction, imbibition will be cocurrent [3, 4, 5, 6]. The physics of the process is strongly dependent to the inverse of Bond number, which is defined as the relative strength of the gravitational forces with respect to the capillary forces. For $B_o^{-1} < 1$, spontaneous imbibition is gravity dominated while for the values of $B_o^{-1} > 5$ it is capillary dominated. When B_o^{-1} is in between, it is intermediate and both forces contribute in the imbibition [7, 8]. Moreover, the magnitude of the inverse of Bond number and boundary conditions de-

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termine whether the process is cocurrent or countercurrent[9].

Spontaneous countercurrent imbibition is a challenging oil recovery mechanism in both conventional and fractured oil reservoirs. Commercial reservoir simulators use classical mathematical model to simulate flow of immiscible flow in a porous medium under unsteady state conditions[10]. This traditional theory, was introduced by Muskat and Leverett [11, 12]. It Employs a macroscopic fluid flux to relate the volumetric flux of a phase to its pressure gradient by Darcy's law [13]. Application of Darcy equation in the model requires relative permeabilities as a function of the phase saturation. Moreover, for immiscible multiphase flow in a porous medium, the capillary pressure term is also included in the model. Both relative permeability and capillary pressure functions are determined under steady state condition and they are calculated for the current saturation (S_W). Therefore, it implies that the process is in local equilibrium, which means that phases are locally instantaneously redistributed[13].

From a physical point of view, the process of spontaneous imbibition is an unsteady state process. This implies that the arrangement of fluids in the pore spaces requires some relaxation or redistribution time to achieve an equilibrium condition during simultaneous flow of immiscible phases. Therefore, the classical model is not adequate for the countercurrent imbibition in an oil saturated core. This is also the case when there is a significant saturation change in the porous medium such as in the vicinity of the displacement front. This problem was reported by several studies and suggested that there are significant non equilibrium effects during the spontaneous imbibition[10, 14, 15, 16, 17].

In order to consider the non-equilibrium effects in simultaneous flow of two immiscible fluids in a porous medium, Barenblatt et al. proposed a model of fluid flow in porous media to take this effect into account[10, 17]. The main point in the model is to introduce a relaxation time for redistribution of fluids within the pore space during immiscible displacement, which means that the rearrangement does not occur instantaneously. This implies a transition time for rearrangement of fluids at the pore scale[18, 19]; when this time is comparable with characteristic distribution time, the wetting-fluid relative permeability is higher than that obtained under steady-state condition. However, both the relative permeability of the non-wetting fluid and the capillary pressure are lower than those in steady-state flow[17, 20]. To solve this problem, Barenblatt's model suggests to calculate capillary pressure and relative permeability functions as a function of the effective saturation [18, 20].

In most cases, dominant forces in countercurrent imbibition in conventional reservoirs and fractured reservoirs are capillary forces. While in some cases like spontaneous imbibition of ultra-low IFT process in porous media, the process may be gravity dominant because the inverse of Bond number is smaller than 1.

In the proposed model by Barenblatt et al., the gravity term has not been taken into account. In this work, the model was carried out with gravity term to simulate a gravity dominated countercurrent imbibition process. Firstly, Darcy's equation was written for vertical immiscible water-oil displacement. Afterwards, using mass conservation for the two phase fluid flow, Barenblatt's model was formulated for gravity dominated countercurrent imbibition. Then countercurrent imbibition process was simulated in a saturated core, which the initial condition satisfied gravity - capillary equilibrium in a core of 1 m length[21, 22]. We assumed no flow boundary at the sides and both ends of the core. After establishing the initial and boundary conditions, the initial distribution of the local effective saturation was determined from the stationary model using COMSOL MULTIPHYSICS. Afterwards, the solution of the stationary model was applied as initial condition for the time dependent model. Simulations were carried out using that at t=0, the core was turned 180 degree such that the core is still vertical, but in the opposite direction.

2 Theory

2.1 Mass Conservation Equation

The governing equation of spontaneous countercurrent imbibition in porous media reads

$$\varphi \partial_t S_w + \nabla u_w = 0, \tag{1}$$

where the velocity field is calculated from Darcy's law

$$u_w = -k\lambda_w(S_w)\frac{\partial}{\partial z}(P_w - \rho_w g z), \tag{2}$$

and the phase mobility is defined as

$$\lambda(S_w) = \frac{k_l(S_w)}{\mu_l}. \tag{3}$$

The total velocity is equal to zero

$$\nabla . u_t = \nabla . \left(u_w + u_o\right) = 0, \tag{4}$$

the velocity of the oil phase is

$$u_t - u_w = \lambda_o(S_w)\,\frac{\partial}{\partial z}(P_c - \Delta\rho g z) + \lambda_o(S_w)\frac{\partial}{\partial z}(P_w - \rho_w g z), \tag{5}$$

then using Darcy velocity, we can write

$$u_w = f_w(S_w)\lambda_o(S_w)\,\sigma\,\sqrt{\frac{\varphi}{k}}\,\times \left(\frac{\partial J(S_w)}{\partial z} - \Delta\rho g \chi(z)\right), \tag{6}$$

which the fractional flow of the water phase is

$$f_w(S_w) = \frac{\lambda_w(S_w)}{\lambda_w(S_w) + \lambda_o(S_w)}. \tag{7}$$

Combination of the Eq. (1) with the Eq. (6) leads to

$$a^2\nabla\left(\nabla\int_0^{S_w} f_w(u)k_{ro}(u)J'(u)\mathrm{d}u\right) - b^2\nabla\left(f_w(S_w)k_{ro}(S_w)\chi(z)\right) = -\partial_t S_w, \tag{8}$$

where a^2 and b^2 are

$$a^2 = \frac{\sigma}{\mu_o}\sqrt{\frac{k}{\varphi}} \tag{9}$$

$$b^2 = \frac{k\Delta\rho g}{\varphi\mu_o}. \tag{10}$$

2.2 Implementation of gravity term in the Barenblatt's Model

Barenblatt et al. considered the effect of the non-equilibrium condition in the flow of immiscible fluid through porous medium by introducing an effective saturation parameter. With dimensional analysis, they suggested a simple relationship between local effective saturation and actual saturation[10]

$$\eta_w = S_w + \tau\frac{\partial S_w}{\partial t}. \tag{11}$$

Assuming a constant relaxation time, τ, and application of this Eq. in the Eq. (8) leads to

$$\eta_w - S_w + \tau\nabla\left(a^2\nabla\phi(\eta_w) - b^2\psi(\eta_w)\right) = 0, \tag{12}$$

here ϕ and ψ are defined by

$$\phi\left(\eta_w\right) = \int_0^{\eta_w} f_w(u)k_{ro}(u)J'(u)du, \tag{13}$$

and

$$\psi\left(\eta_w\right) = f_w(\eta_w)k_{ro}(\eta_w)\chi(z). \tag{14}$$

Finally, a differentiating towards time and combining the result with the equation (12) gives

$$\partial_t \eta_w + a^2 \partial_z \left(\partial_z \phi(\eta_w) + \tau \partial_t \phi(\eta_w) \right) - b^2 \partial_z \left(\psi(\eta_w) + \tau \partial_t \psi(\eta_w) \right) = 0. \quad (15)$$

This equation represents the model of countercurrent imbibition process, which includes both capillary and gravity forces. But, it needs an initial distribution of the effective local saturation; It is determined from

$$\eta_w^0(z) + \tau \partial_z \left(a^2 \phi(\eta_w) - b^2 \psi(\eta_w) \right) = S_w^0(z). \quad (16)$$

3 Model Description

In order to simulate the gravity driven countercurrent imbibition process in a cylindrical saturated tube with Barenblatt's model, we assumed a tube, which is saturated with the oil phase and the water phase under gravity - capillary equilibrium. To increase the gravitational forces, we simulate a case that we put the core in a centrifuge, Then, we increased the gravity acceleration by a factor of 20. Using this term we established the initial water saturation as

$$S_{w0} = S_{wc} + \left(\frac{1}{2} - S_{wc} \right) \times \left(\gamma \sigma \sqrt{\frac{\varphi}{k}} / \left(\Delta \rho \bar{g} z \right) \right)^{\lambda}. \quad (17)$$

Afterwards, we used COMSOL MULTIPHYSICS to simulate the process and determine the saturation profile in different time steps in a saturated tube after we turned the tube 180 degrees vertically. We used a weak form PDE model to simulate this process. To force the displacement to a countercurrent imbibition, both ends and sides were taken as no flow boundary. Because the inverse of Bond number is smaller than one, $B_o^{-1} = 0.42$, the process is gravity dominated. Therefore, the non-equilibrium model, which includes the gravity term is suitable to simulate this process. Furthermore, the process in the cylindrical core is symmetric and we reduce it to a 1D problem.

3.1 Model parameters

Model parameters are presented in Table 1

Table 1 - Model Parameters

Parameter	Value	Unit
ϕ	0.3	$[m^3/m^3]$
k	1×10^{-12}	$[m^2]$
λ	0.5	[1]
γ	0.5	[1]
μ_w	0.001	$[Pa.s]$
μ_o	0.001	$[Pa.s]$
ρ_w	1000	$[kg/m^3]$
ρ_o	800	$[kg/m^3]$
M	1	[1]
Height	1	$[m]$

Moreover, below functions have been used for the water and the oil relative permeabilities functions and the Leverett J-function in the model

$$k_{rw}(S_w) = S_w \quad (18)$$

$$k_{ro}(S_w) = 1 - S_w \quad (19)$$

$$J(S_w) = 1 - S_w, \quad (20)$$

linear permeabilities are considered to grasp the unstable nature of the displacement process optimally.

4 Results and Discussion

Fig. 1 shows the initial water saturation profile, S_w, in the core.
Fig. 2 depicts the distribution of the effective water saturation with redistribution time of $\tau = 50$ s. The final distribution of the effective water saturation, η_w, is not exactly a mirror-like of the initial distribution. This indicates that a final equilibrium profile is only obtained after a very long time.

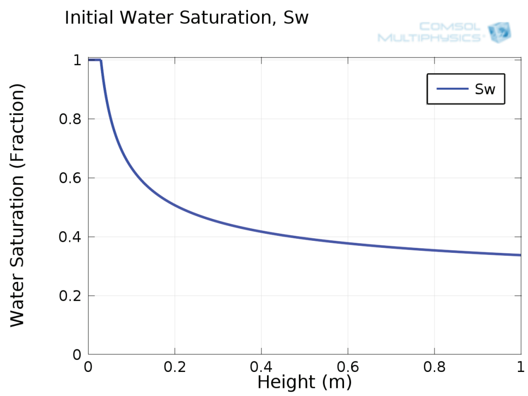

Figure 1 - Initial distribution of water saturation, S_w, under gravity - capillary equilibrium

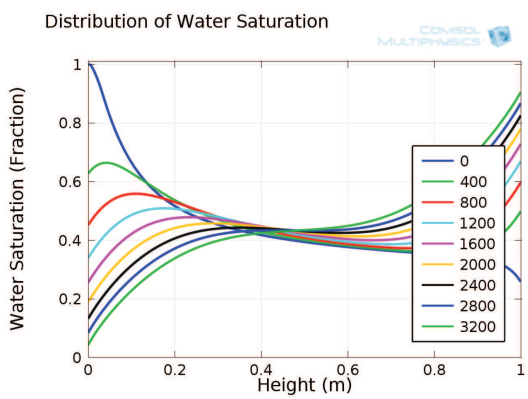

Figure 2 - Initial distribution of the effective water saturation, η_w, with $\tau = 100$ s

The solution of the model using the equilibrium scheme is shown in Fig. 3. Similar to the effective water saturation, this figure shows a different distribution of water saturation, S_w, rather than its mirror-like distribution.

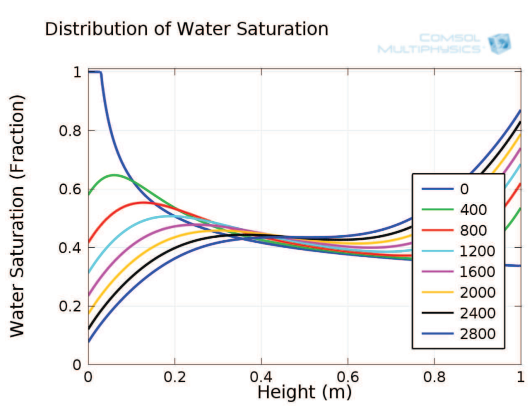

Figure 3 - Initial and final distribution of water saturation, S_w, considering equilibrium scheme with $\tau = 0$ s

Fig. 4 depicts the comparison between the initial effective water saturation using different relaxation times in the model, i.e., $\tau = 0, 50, 100, 200$ s. Initial saturation profiles have been obtained from the solution of the stationary model. This figure indicates that in the middle of the core ($0.4 < H < 0.8$) saturation is constant and does not depend on the relaxation time.

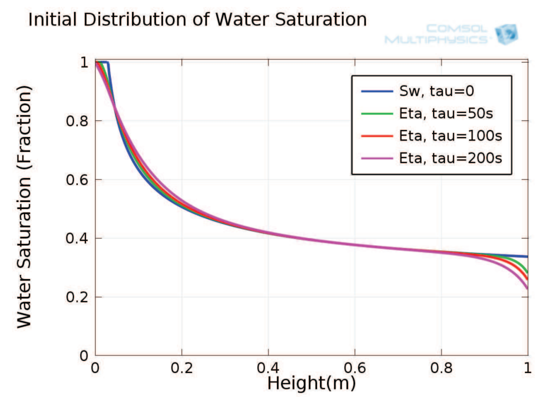

Figure 4 - Initial distribution of the effective water saturation with different relaxation times of 0, 50, 100 and 200 s

The final distribution of the effective water saturation with different relaxation time is shown in the Fig. 5.

Figs. 4 and 5 show that the effective water saturation in the top of the core decreases more rapid for higher relaxation times, rather than lower relaxation times. While at the bottom of the core, the rate of saturation change is higher for the lower redistribution times.

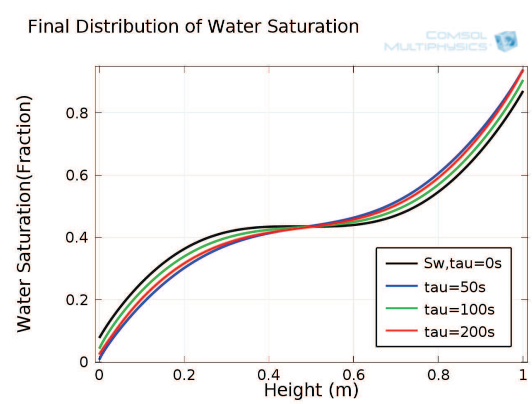

Figure 5 - Final distribution of the effective water saturation, η_w, with different relaxation times of 0, 50, 100 and 200 s

5 Conclusion

In this paper the mathematical model of Barenblatt that includes the non-equilibrium effects for countercurrent imbibition was carried out including a gravity term to simulate a gravity dominated countercurrent imbibition process in a cylindrical tube. A weak form PDE model was used to simulate the problem with COMSOL MULTIPHYSICS. The result shows the saturation profile as a function of time and again a stationary gravity - capillary equilibrium was obtained. However, the solution is not the exact mirror of the initial profile. This is attributed to the fact that a final equilibrium profile is only obtained after a very long time. This model may be helpful for the simulation of other types of gravity dominated countercurrent imbibition in porous media.

6 Nomenclature

a^2 = diffusivity coefficient, [L^2/t]
b^2 = gravity diffusion, [L/t]
f_w = water fractional flow function, [-]
J = Leverett J-function, [-]
k = absolute permeability, [L^2]
k_{ro} = relative permeability to oil, [-]
k_{rw} = relative permeability to water, [-]
H = height of the core, [L]
S_w = actual water saturation, [-]
S_w^0 = initial actual water saturation, [-]
t = time, [t]
u_t = total flux, [L/t]
u_o = oil flux, [L/t]
u_w = water flux, [L/t]
σ = oil-water interfacial tension, [m/t^2]
η_w = effective water saturation,[-]
η_w^0 = initial effective water saturation, [-]
M = the ration μ_o/μ_w
μ_o = oil viscosity, [m/Lt]
μ_w = water viscosity, [m/Lt]
τ = redistribution time, [t]
ϕ = dimensionless function
ψ = dimensionless function

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