Electromagnetic and Coupled Field Computations: A Perspective

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Outline

- Introduction
- Classification of Electromagnetic Field Problems
- Coupled Field Computations
  - Circuit-Field
  - Magnetic-Thermal
  - Magnetic-Structural
- Case Studies
- Concluding Remarks

Introduction

An Overview of EM Applications

- EM principles form the core of electricity generation, transmission and distribution
- EM, and its computational version, are used to design, test and validate devices across a wide range of sizes – from the smallest micro-electro-mechanical (MEMS) devices to the very large transformers and generators

<u>Low Frequency Devices:</u>

- Transformers
- Electric motors
- Power generators
- EM forming and welding
- EM interference/coupling
- Induction heating devices

<u>High Frequency Devices:</u>

- Antennas
- Waveguides and Resonant cavities
- Magnetic storage and imaging systems
- Optoelectronics and photonics
- Microwave circuits and devices
- Plasma devices

Need for EM Field Computation

- Computation of *magnetic fields* is required in all low frequency and high frequency devices for:
  - Evaluation and improvement of performance parameters at the design stage
  - Reliability enhancement
  - Investigative analysis
- Field computation provides a *non-destructive* technique for testing and evaluation
- In order to *optimize* material costs, in the present-day highly global market, an accurate understanding and analysis of the field distribution is necessary

Low Frequency Devices: Performance Parameters

- Inductances and capacitances
- Insulation design for high voltage applications
- Eddy currents
- Forces in windings and current carrying bars
- Torques in rotating machines
- Mechanical stresses / deformations
- Temperature profiles and hot-spots
- Noise level

High Frequency Devices: Performance Parameters

- S parameters
- Power flow/Poynting vector
- Propagation constants
- Characteristic impedance
- Radiation patterns
- Modal field distributions

Computational Methods

- Difference methods:
  - Finite difference method (FDM)
  - Finite-difference time-domain method (FDTD)
- Variational / Weighted residual approach:
  - Finite element method (FEM)
- Integral methods:
  - Method of moments (MoM)
  - Boundary element method (BEM)
  - Charge simulation method (CSM)

Finite Element Method

- The method has emerged as the forerunner among all the numerical techniques
  - Geometrical complexities can be handled in better ways using FEM
  - Anisotropic, non-uniform and non-linear media can be incorporated
  - Availability of several commercial softwares makes the applicability to real-life problems easier
  - Finite element method can also be used in solving problems involving coupling of electromagnetic fields with circuits and/or other physical fields

Classification of Electromagnetic Field Problems

Basic Governing Equations

- Maxwell's equations: -

$$\nabla \cdot \mathbf{D} = \rho_v$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

- Constitutive relations: -

$$\mathbf{D} = \varepsilon \mathbf{E}$$
$$\mathbf{B} = \mu \mathbf{H}$$
$$\mathbf{J} = \sigma \mathbf{E}$$

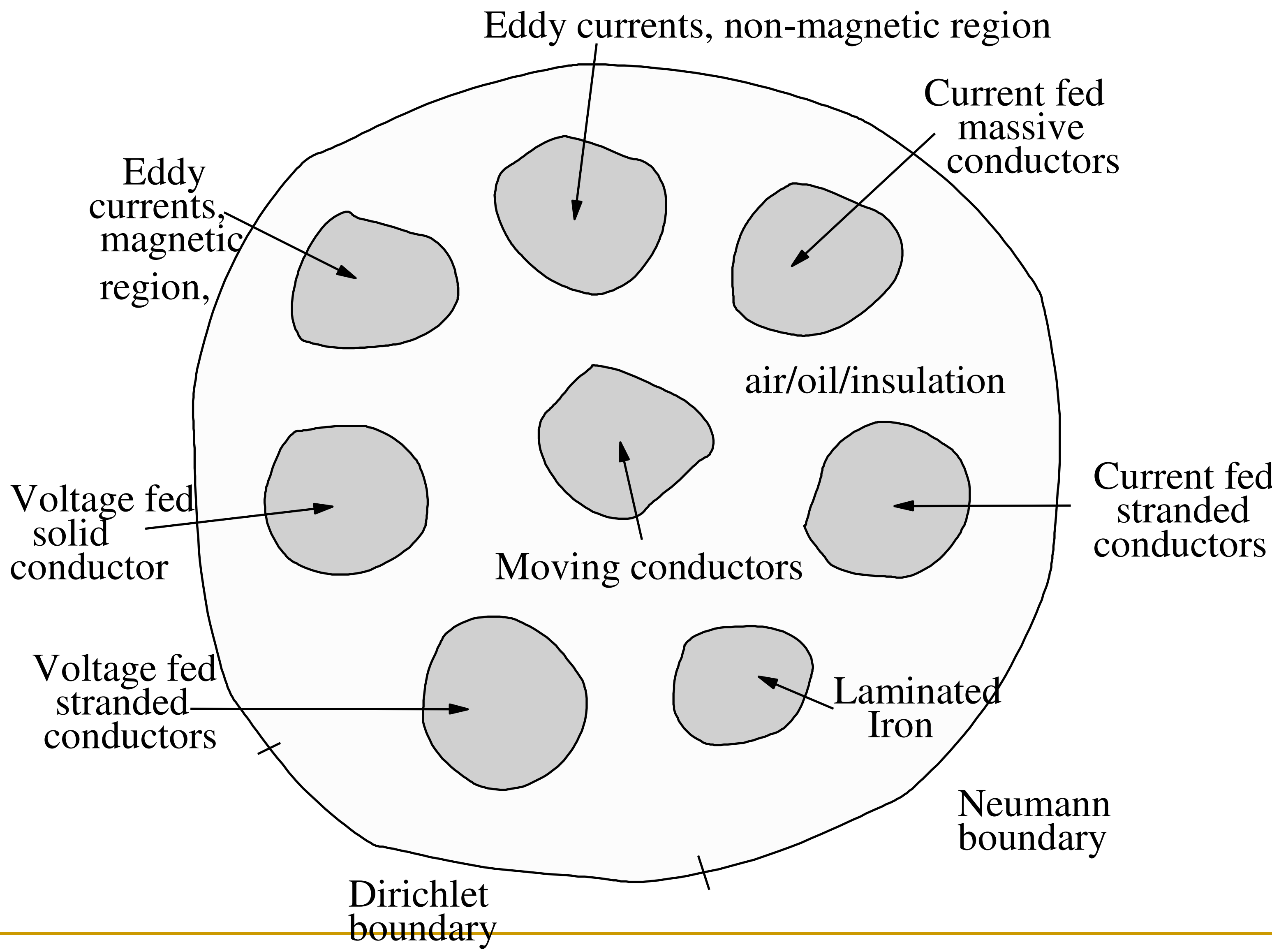
- Continuity equation: -

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}$$

- Lorentz force equation: -

$$\mathbf{F} = Q(\mathbf{E} + v \times \mathbf{B})$$

Typical entities and problem types: Low frequency

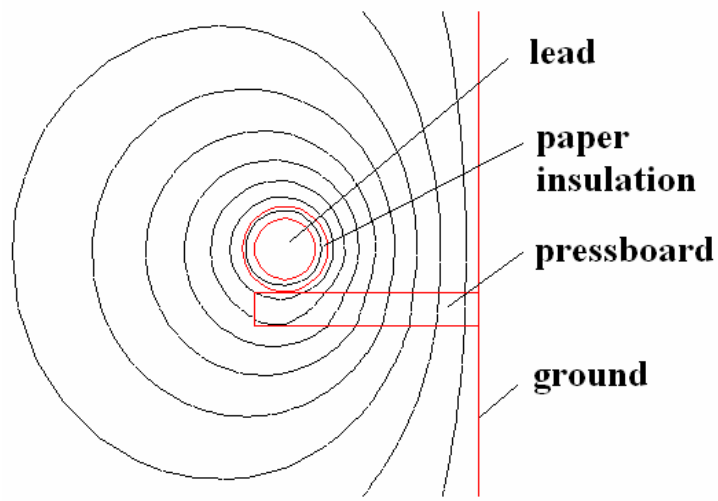


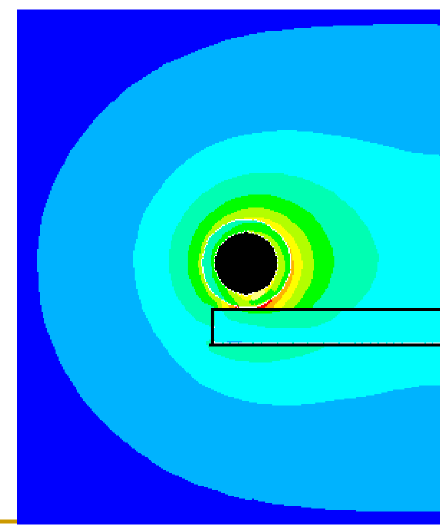
Classification of Electromagnetic Field Problems

- **Electrostatics:**
  - Analysis of the electric field in capacitive or dielectric systems

$$\varepsilon_x \frac{d^2V}{dx^2} + \varepsilon_y \frac{d^2V}{dy^2} = -\rho_v$$

  - Computation of electric field, capacitance, electrostatic forces and torques, etc.

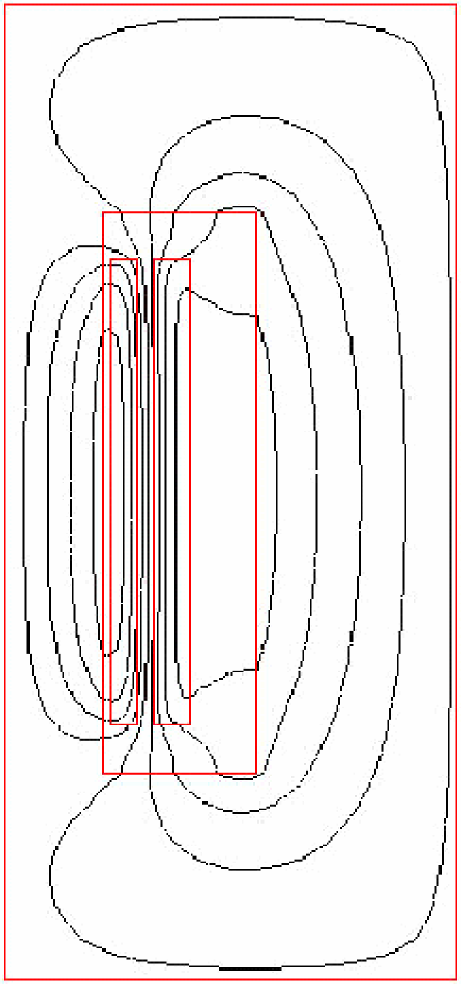




- **Magnetostatics:**
  - This analysis type is used to analyze magnetic field produced by direct electric current, permanent magnet or applied magnetic field

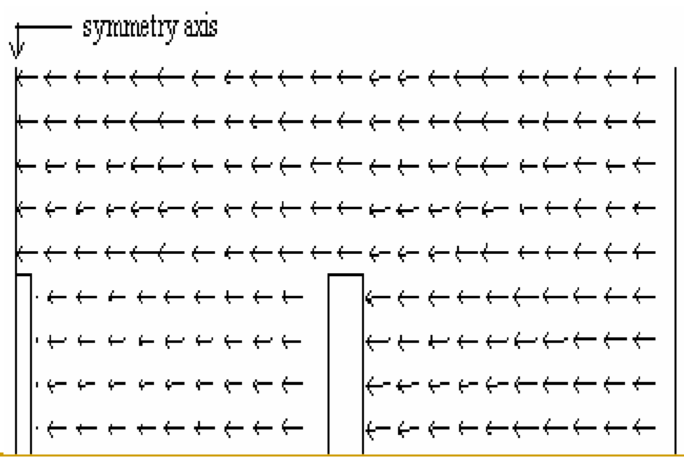
$$\frac{1}{\mu_x}\frac{d^2\mathbf{A}}{dx^2}+\frac{1}{\mu_y}\frac{d^2\mathbf{A}}{dy^2}=-\mathbf{J}_o$$

  - The static analysis is used to compute parameters such as magnetic flux, self and mutual inductances, forces, torques, etc.

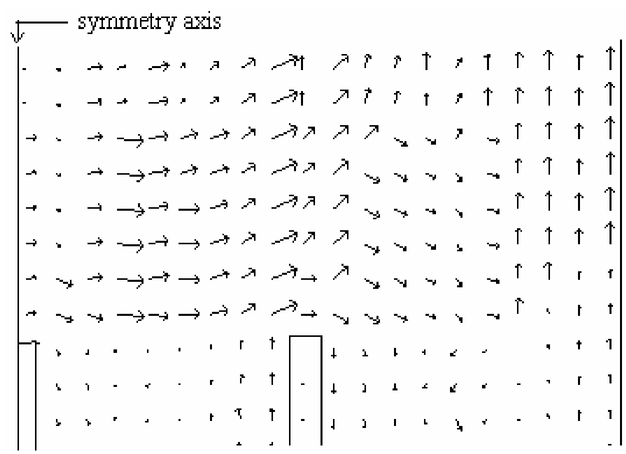


- **Time-harmonic: Diffusion Equation**
  - Harmonic analysis is used for sinusoidal excitations and linear materials
  - The analysis can be carried out for a single frequency or a range of frequencies to compute eddy currents, stray losses, skin effect and proximity effect

$$\frac{1}{\mu_x}\frac{d^2\mathbf{A}}{dx^2}+\frac{1}{\mu_y}\frac{d^2\mathbf{A}}{dy^2}+\frac{1}{\mu_z}\frac{d^2\mathbf{A}}{dy^2}=\sigma\left(\nabla V+j\omega\mathbf{A}\right)$$



Eddy currents in magnetic clamp plate



Eddy currents in non-magnetic clamp plate

- **Time-harmonic: Wave Equation**
  - The wave equations for time harmonic electric and magnetic fields with angular frequency ω are:

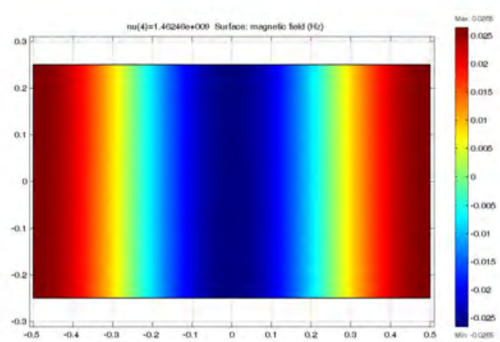
$$\nabla^2 \mathbf{E} + \omega^2 \mu\varepsilon \mathbf{E} = 0$$

$$\nabla^2 \mathbf{H} + \omega^2 \mu\varepsilon \mathbf{H} = 0$$

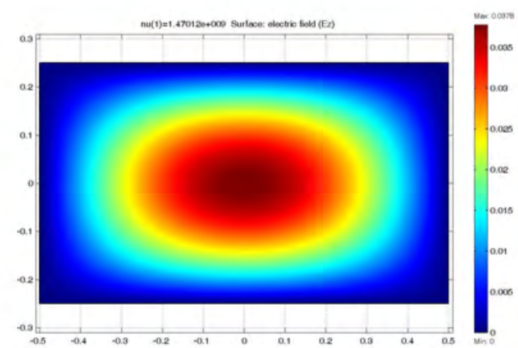
$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Rectangular Waveguides

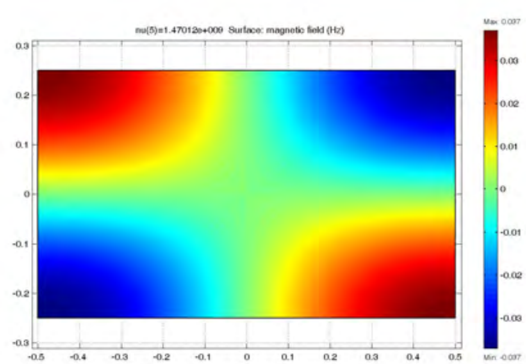
TE20



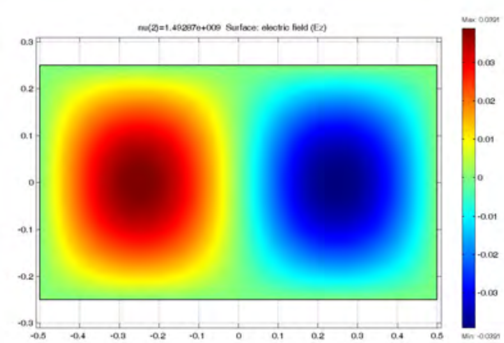
TM11



TE22

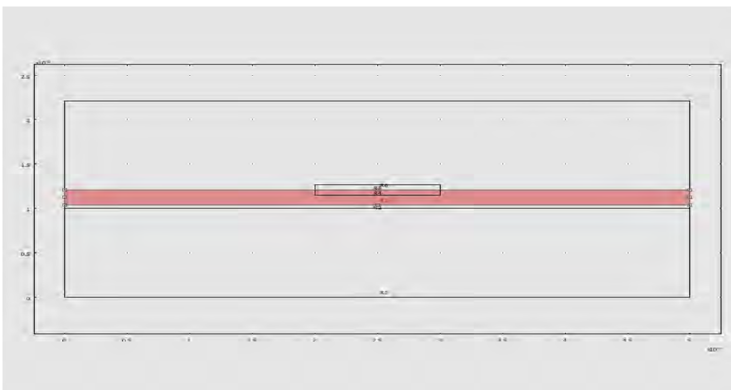


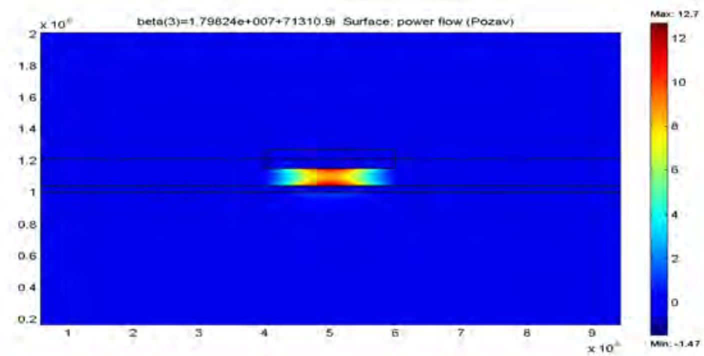
TM21



Eigenvalue FEM analysis of a plasmonic waveguide

- Photonic circuits of nanoscale dimensons: A potential research area
- These could form harmonizing links between nano-scaled electron devices and micro-scaled optical devices





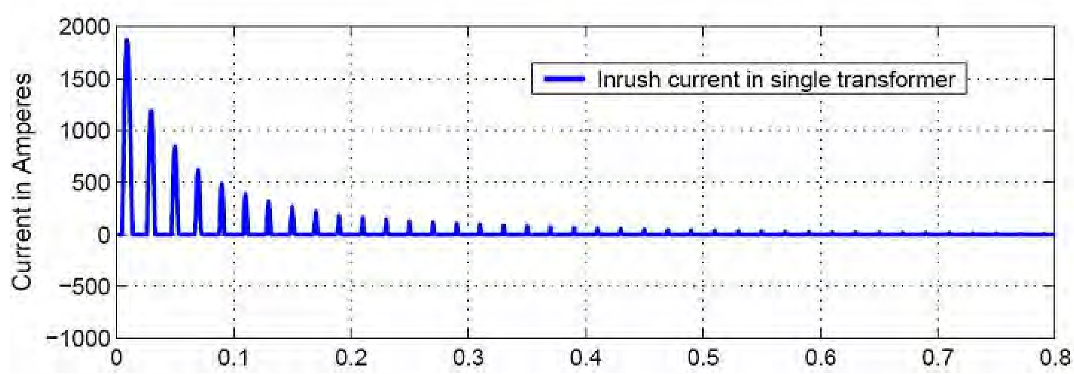
- FEM analysis*: Metal-dielectric-slotted metal structure
- A good confinement and a propagation length of 10 μm is observed

* Courtesy: Ms. Padmaja, Research Scholar, EE Dept, IIT Bombay

- **Transient:**
  - Transient magnetic analysis is a technique for calculating magnetic fields that vary over time, such as those caused by surges in voltage or current or pulsed external fields

$$\frac{1}{\mu}\frac{d^2\mathbf{A}}{dx^2} + \frac{1}{\mu}\frac{d^2\mathbf{A}}{dy^2} = \sigma\left(\nabla V + \frac{\partial \mathbf{A}}{\partial t}\right)$$

  - Performance parameters such as inrush current, eddy currents and forces can be computed when electrical machines are subjected to transient stresses.



Potentials Used in Computational Electromagnetics

- **Electric scalar potential (*V*):**
  - Used in electrostatic formulations: Insulation design in high voltage equipment
  - Analysis of frequency-dependent performance of dielectrics

$$\nabla \cdot \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) = 0$$

If the effects of magnetic field can be neglected,

$$\nabla \times \mathbf{E} = 0 \;\Rightarrow\; \mathbf{E} = -\nabla V$$

$$-\nabla \cdot \left(\sigma \nabla V \right) - \nabla \cdot \left(\frac{\partial}{\partial t} \left(\varepsilon \nabla V \right) \right) = 0$$

$$-\nabla \cdot \left(\left(\sigma + j\omega\varepsilon \right) \nabla V \right) = 0$$

$$-\nabla \cdot \left(\left(\sigma_{eff} + j\omega\varepsilon' \right) \nabla V \right) = 0 \qquad \sigma_{eff} = \sigma + \omega\varepsilon'' = \sigma + \omega\varepsilon' \tan\delta$$

- **Magnetic vector potential (A):**
  - In the case of 2-D models, the formulation based on magnetic vector potential (MVP) is generally used
    - The number of unknowns at any point reduce from 2 (Bx, By) to one (Az), with the current in z-direction

$$\varphi = \iint_S \mathbf{B} \cdot d\mathbf{S} = \oint_L \mathbf{A} \cdot d\mathbf{l}$$

Flux passing through any two points 1 and 2 $\rightarrow \phi = A_1 - A_2$ (2-D case)

  - However in 3-D models, the MVP formulation has three degrees of freedom per node, Ax, Ay and Az, making it computationally unattractive
  - MVP formulation can be used for static, time-harmonic or transient magnetic analyses
  - If there are eddy current regions, additionally the electric scalar potential needs to be considered (**A**-V, **A** formulation)

- **Magnetic scalar potential (Ω):**
  - In 3-D problems, **A** has three unknowns at every point
  - The scalar potential formulation has only one degree of freedom per node
  - The magnetic scalar potential based formulation is therefore suitable for 3-D magnetostatic problems
  - The scalar potential cannot be used for current carrying regions and/or in any part which surrounds such regions
  - *Reduced scalar potential* is used to circumvent above restrictions
  - *Reduced Scalar Potential* (Ω_r) or *Total Scalar Potential* (Ω) is selected as variable depending on presence and absence of current carrying domains, respectively: Hybrid formulation

- **Electric Vector Potential (T)**
  - Used for solving eddy current problems

    $$\nabla \times \mathbf{T} = \mathbf{J}$$

    $$\mathbf{H} - \mathbf{T} = -\nabla\Omega \qquad \Rightarrow \qquad \mathbf{H} = \mathbf{T} - \nabla\Omega$$

  - Compare with: $\mathbf{E} = -\partial\mathbf{A}/\partial t - \nabla V$.
  - **T** represents induced eddy currents like the term $-\partial\mathbf{A}/\partial t$ does in the **A**-based formulation
  - The formulation is advantageous for the analysis of eddy currents in laminated structures

- **Nodal Vs Vector (Edge) Elements**
  - Nodal formulation: Popularly used for low frequency computations
    - Simpler and easy to implement
    - ‘Scalar’ FEM
  - Edge elements: Better suited for high frequency computations
    - Degrees of freedom are associated with edges
    - Continuity of tangential components of field vectors is ensured
    - Spurious modes/solutions are avoided as the divergence condition is satisfied
    - They are better in handling singularities

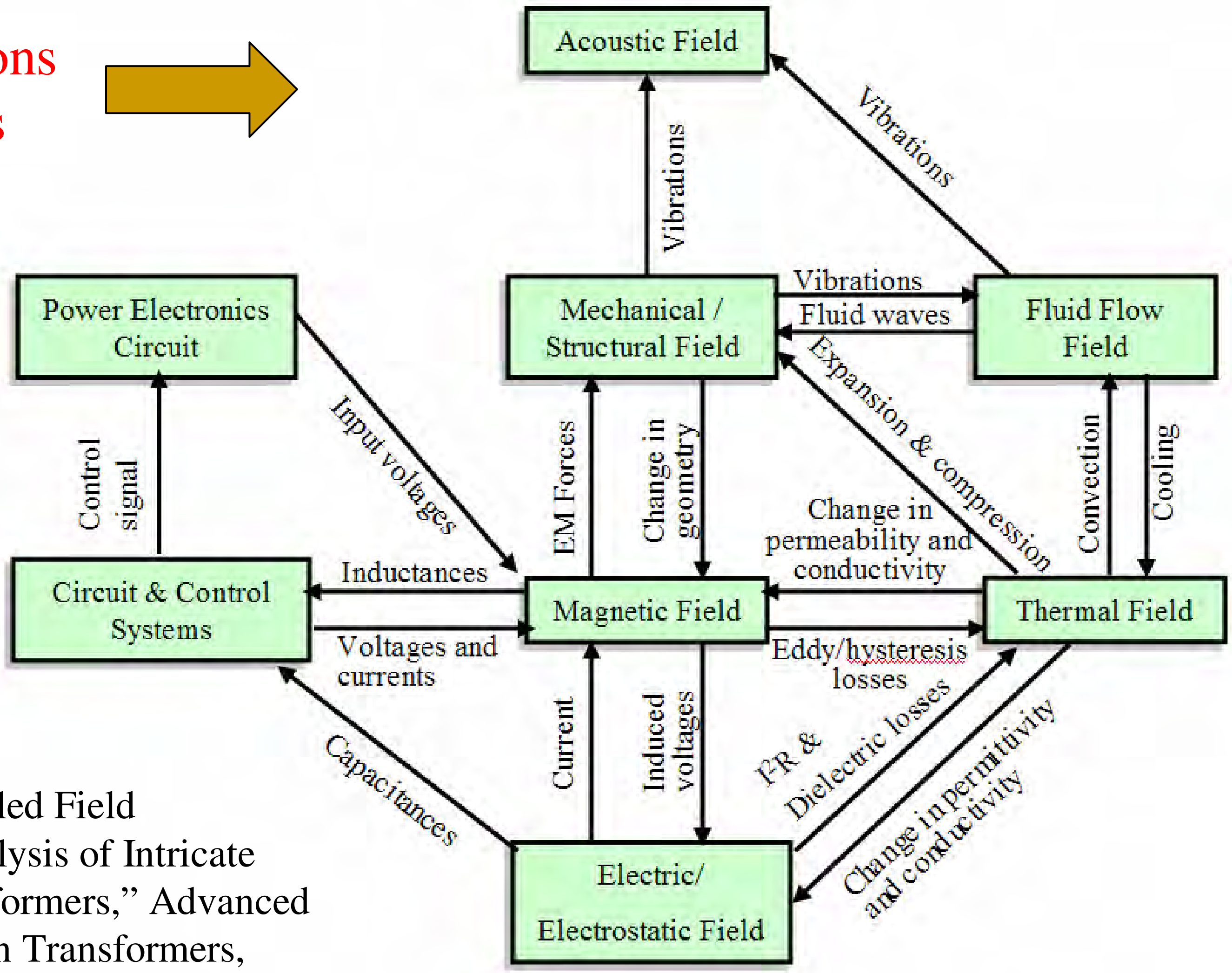
Coupled Field Computations

About Coupled Fields

- **Classification:**
  - Weakly coupled
  - Strongly coupled
- **Weak or indirect coupling:**
  - Solution of one field acts as load to another field
  - It is flexible, modular and easy approach
- **Strong or direct coupling:**
  - Coupled field equations are solved simultaneously
  - The approach is used when field interactions are highly nonlinear and the coupled fields have comparable time constants.

Coupled Systems: Real-Life Design Problems

Coupled interactions in Transformers



Ref: S. V. Kulkarni, "Coupled Field Computations for Analysis of Intricate Phenomena in Transformers," Advanced Research Workshop on Transformers, Baiona, Spain, October, 28-31, 2007, pp. 172-186

Coupled Field Computations

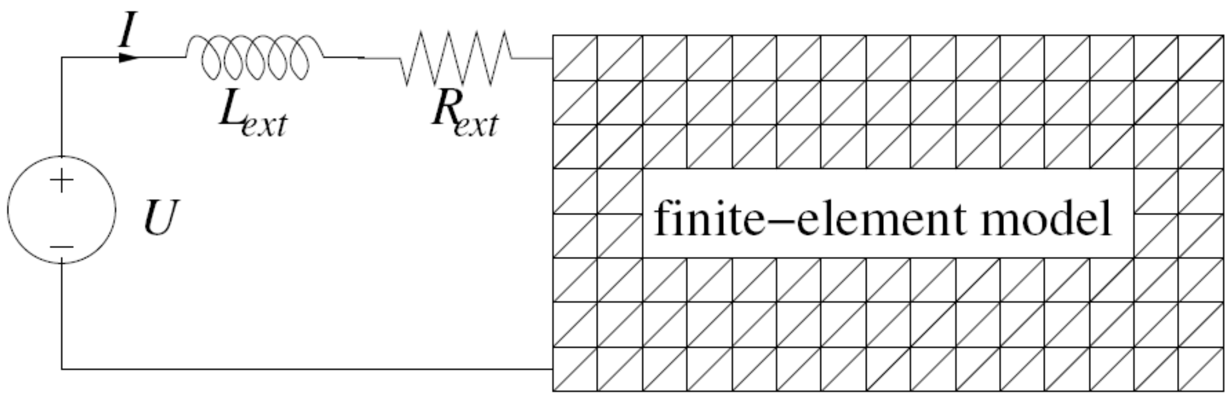
Circuit – Field

Field-Circuit Coupling

Electromagnetic model:

$$\frac{\partial}{\partial x}\left(\upsilon\frac{\partial A_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\upsilon\frac{\partial A_z}{\partial y}\right)=-J_z$$

Circuit coupling:



Conductor Models

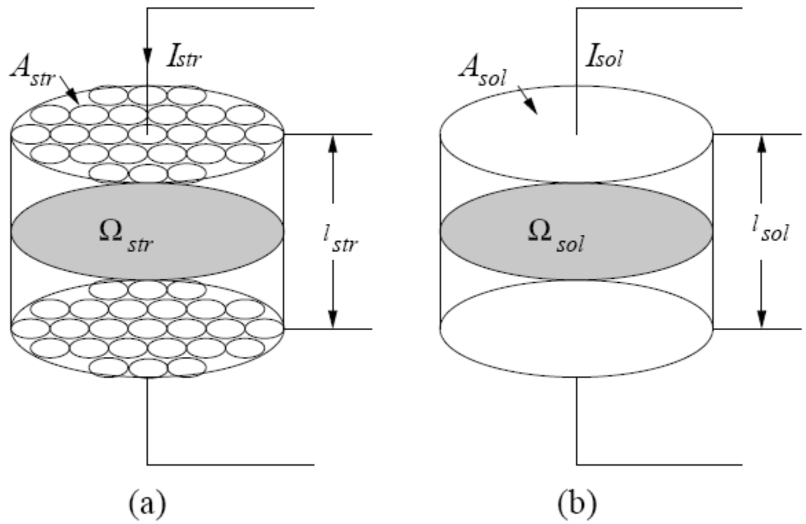


Figure 3.2: (a) Stranded (b) solid conductor models

Stranded conductor:

$$J_{str} = \frac{N_{str} I_{str}}{A_{str}}$$

Solid (massive) conductor:

$$J_{sol} = \sigma \frac{V_{sol}}{l_{sol}} - \sigma \frac{\partial A_z}{\partial t}$$

Field-Circuit Coupling

FEM formulation for stranded conductor

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial y}\right)=-J_z \qquad J_z=\frac{N_{str}\,I_{str}}{A_{str}}$$

Finite element discretization leads to:

$$[\mathbf{K}]\{\mathbf{A}\}+[\mathbf{P}]\{\mathbf{I}\}=0$$

where,

$$[\mathbf{K}]=\sum_{\Omega}\upsilon\iint_{\Delta_e}\left(\frac{\partial N_e^T}{\partial x}\frac{\partial N_e}{\partial x}+\frac{\partial N_e^T}{\partial y}\frac{\partial N_e}{\partial y}\right)dxdy$$

$$[\mathbf{P}]=\sum_{\Omega}\frac{N_{str}}{A_{str}}\iint_{\Delta_e}N_e^T\,dxdy$$

Field-Circuit Coupling

Circuit equations can be written as:

$$\{\mathbf{U}\} = \left\{\frac{d\mathbf{\Phi}}{dt}\right\} + [\mathbf{R}]\{\mathbf{I}\} + [\mathbf{L}]\left\{\frac{d\mathbf{I}}{dt}\right\}$$

$$\frac{d\phi}{dt} = \frac{L_{str}}{A_{str}} \int_{\Omega} \frac{\partial A_z}{\partial t}\, d\Omega$$

$$\{\mathbf{U}\} = [\mathbf{G}]\left\{\frac{d\mathbf{A}}{dt}\right\} + [\mathbf{R}]\{\mathbf{I}\} + [\mathbf{L}]\left\{\frac{d\mathbf{I}}{dt}\right\}$$

Global system of equations is given as:

$$\begin{bmatrix} [\mathbf{0}] & [\mathbf{0}] \\ [\mathbf{G}] & [\mathbf{L}] \end{bmatrix} \begin{Bmatrix} \{\dot{\mathbf{A}}\} \\ \{\dot{\mathbf{I}}\} \end{Bmatrix} + \begin{bmatrix} [\mathbf{K}] & [\mathbf{P}] \\ [\mathbf{0}] & [\mathbf{R}] \end{bmatrix} \begin{Bmatrix} \{\mathbf{A}\} \\ \{\mathbf{I}\} \end{Bmatrix} = \begin{Bmatrix} \{\mathbf{0}\} \\ \{\mathbf{U}\} \end{Bmatrix}$$

Coupled Field Formulations: Solid Conductor

- **Field Equation:** $$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial y}\right)=-\sigma\frac{V}{l_{sol}}+\sigma\frac{\partial A_z}{\partial t}$$

- **Internal Current Equation:** $$I=G_{sol}\,V-\int_{\Omega}\sigma\frac{\partial A_z}{\partial t}\,d\Omega$$

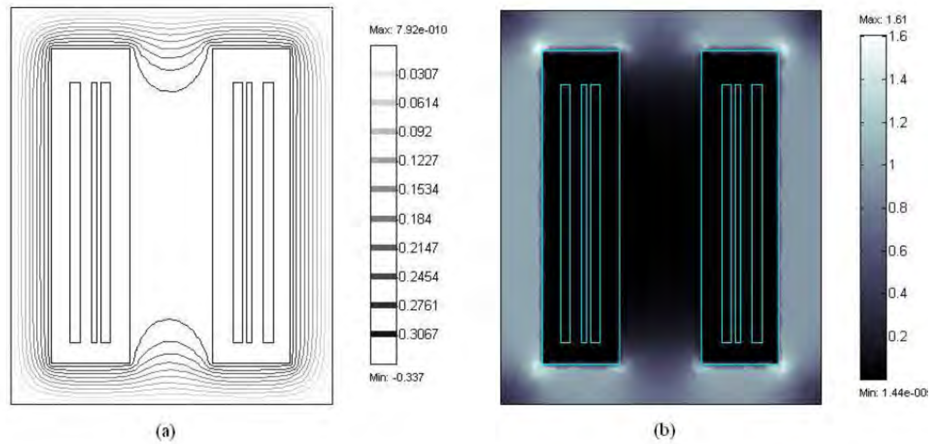
- **Circuit Equation:** $$\{\mathbf{U}\}=\{\mathbf{V}\}+[\mathbf{R}]\{\mathbf{I}\}+[\mathbf{L}]\left\{\frac{d\,\mathbf{I}}{dt}\right\}$$

$$\begin{bmatrix}[\mathbf{Q}] & [\mathbf{0}] & [\mathbf{0}]\\ [\mathbf{B}]^T & [\mathbf{0}] & [\mathbf{0}]\\ [\mathbf{0}] & [\mathbf{0}] & -[\mathbf{M}]\end{bmatrix}\frac{d}{dt}\begin{Bmatrix}\{\mathbf{A}\}\\ \{\mathbf{V}\}\\ \{\mathbf{I}\}\end{Bmatrix}+\begin{bmatrix}[\mathbf{C}] & [\mathbf{B}] & [\mathbf{0}]\\ [\mathbf{0}] & [\mathbf{S}] & [\mathbf{P}]\\ [\mathbf{0}] & [\mathbf{P}]^T & -[\mathbf{R}]\end{bmatrix}\begin{Bmatrix}\{\mathbf{A}\}\\ \{\mathbf{V}\}\\ \{\mathbf{I}\}\end{Bmatrix}=\begin{Bmatrix}\{\mathbf{0}\}\\ \{\mathbf{0}\}\\ \{\mathbf{U}\}\end{Bmatrix}$$

1. Half-Turn Effect

Single-phase three-limb transformer

	Measured	FEM
Flux density in end limbs (T)	**1.04**	**0.93**
Extra core loss due to the half-turn effect (kW)	**4.2**	**3.9**



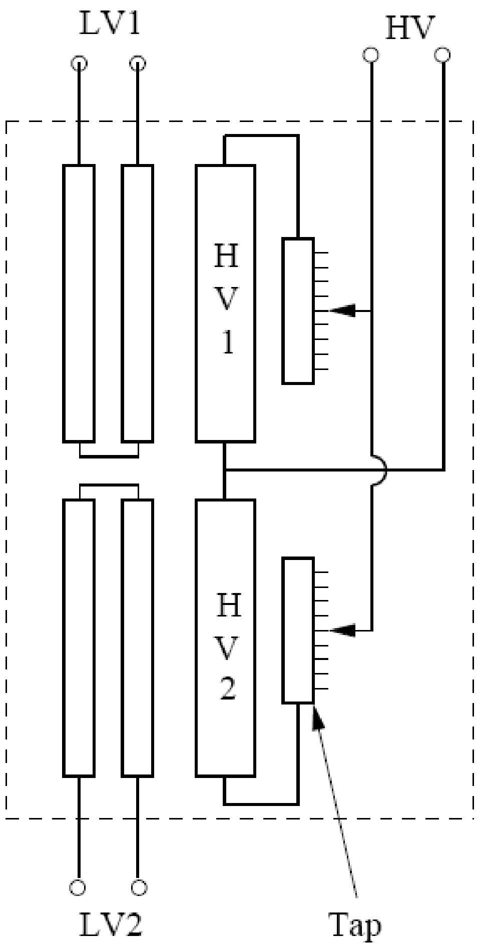
(a) Flux lines (b) flux density plots with half-turn

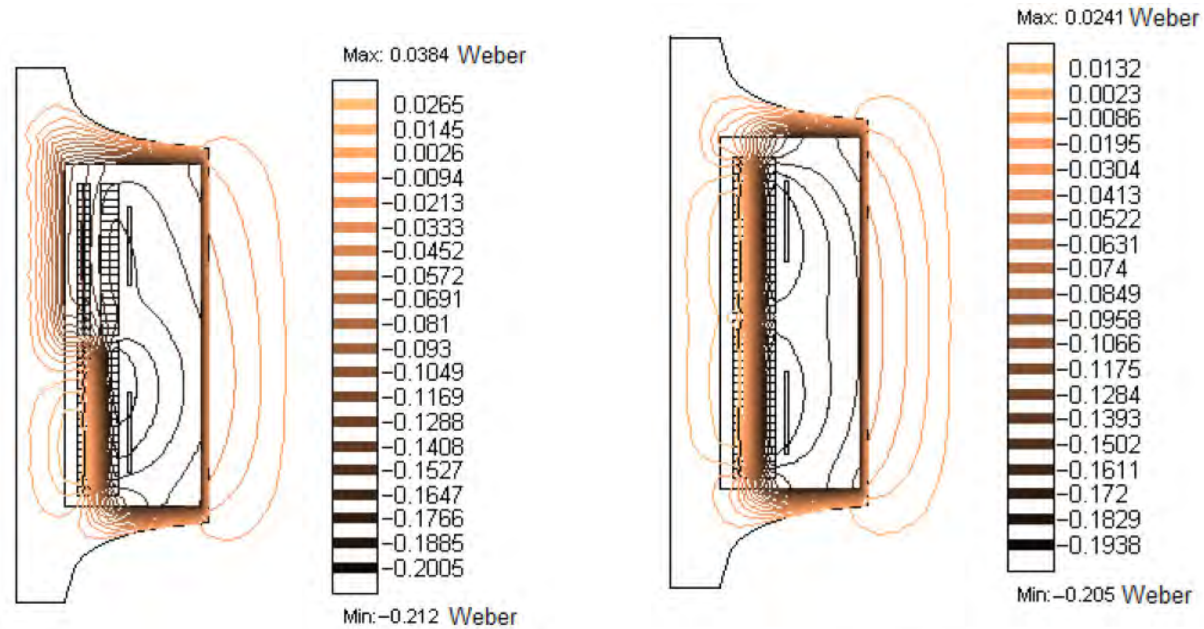
Three-phase five-limb transformer

	Flux density (T) for unbalanced currents in windings		
	Balanced	10% unbalance	20% unbalance
Without half-turn	**0.02**	**0.035**	**0.045**
With half-turn	**0.04**	**0.108**	**0.25**

Ref: G. B. Kumbhar, S. V. Kulkarni, and V. S. Joshi, "Analysis of half-turn effect in power transformers using nonlinear-transient FE formulation," *IEEE Trans. Power Delivery,* vol. 22, no. 1, Jan 2007, pp. 195-200.

2. Forces in Split-Winding Transformer





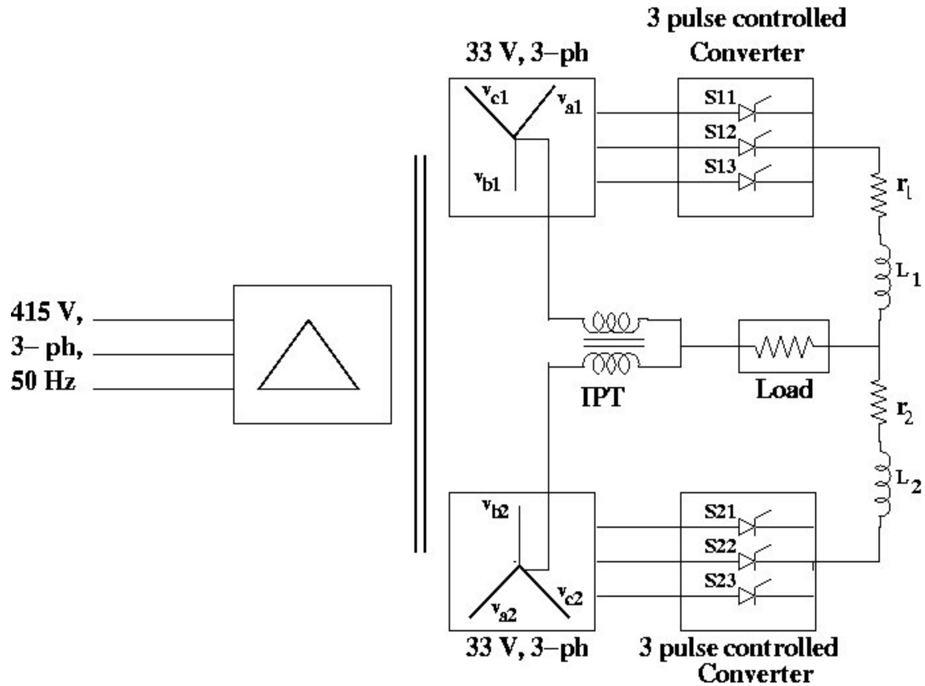
(a) (b)

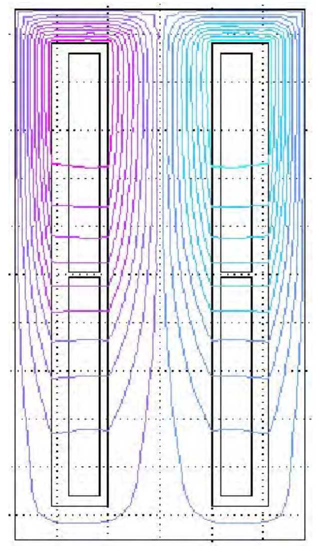
Contour lines of magnetic vector potential

(a) One winding short circuited (b) Both windings short circuited

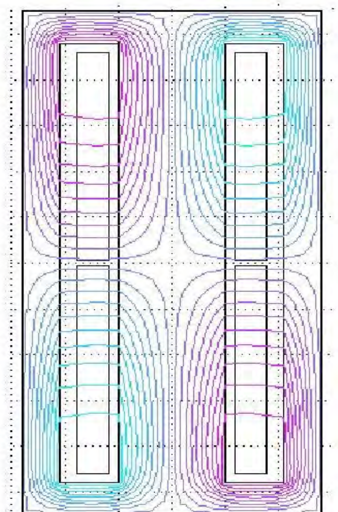
Ref: G. B. Kumbhar, S. V. Kulkarni, and V. S. Joshi, "Analysis of short circuit performance of split-winding transformer using coupled field-circuit approach," *IEEE Trans. Power Delivery,* vol. 22, no. 2, April 2007, pp. 936-943.

3. Coupled Circuit – Field Analysis: Interphase Transformer (IPT)





Unbalanced case



Balanced case

Flux in the IPT Core

Ref: R. S. Bhide, G. B. Kumbhar, S. V. Kulkarni, and J. P. Koria, "Coupled circuit-field formulation for analysis of parallel operation of converters with interphase transformer," *Electric Power Systems Research*. Vol. 78, Issue 1, January 2008, pp. 158-164.

Coupled Field Computations

Magnetic – Thermal

Magnetic-Thermal Coupling

- The 2-D transient magnetic and thermal equations are

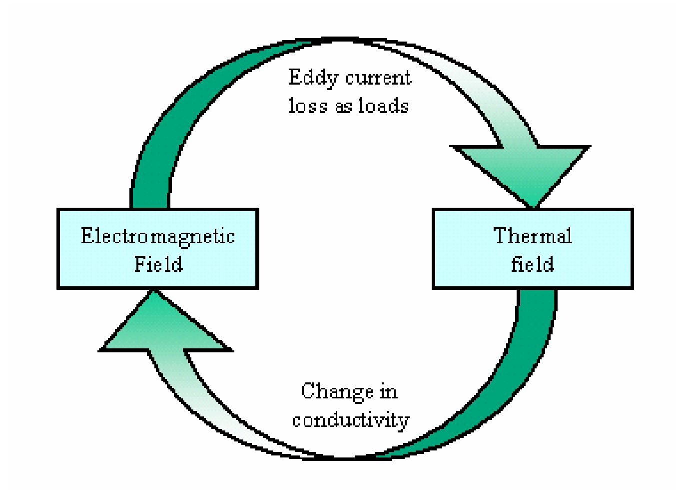
$$\nabla \cdot \left(\frac{1}{\mu} \nabla (A_z) \right) = -\sigma(T)\frac{V}{l} + \sigma(T)\frac{\partial A_z}{\partial t}$$

$$\nabla \cdot \left(k \nabla (T) \right) = -q(A_z, T) + mc\frac{\partial T}{\partial t}$$

These are coupled by the following relations

$$\sigma(T) = \frac{\sigma_{ref}}{\left(1 + \alpha(T - T_{ref})\right)}$$

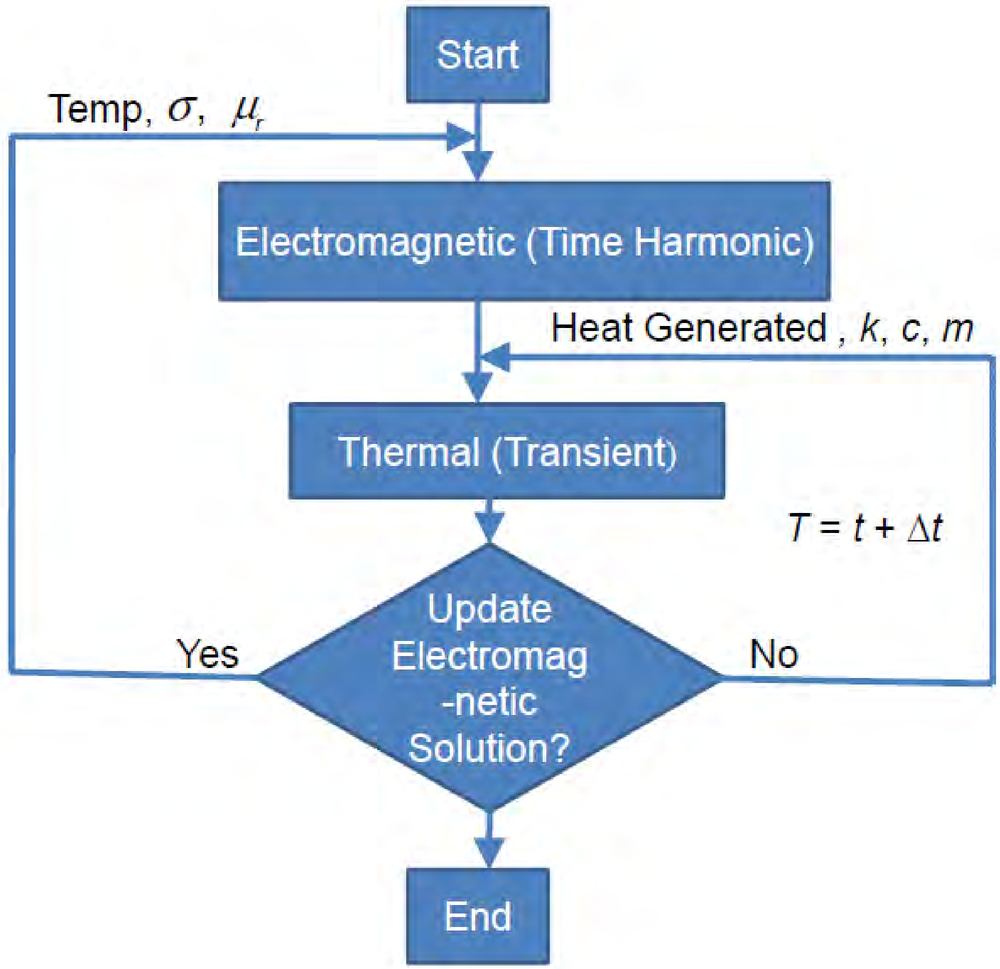
$$q(A_z, T) = \frac{1}{\Omega_e} \int_{\Omega_e} \sigma(T) \left(-\frac{V}{L} + \frac{\partial A_z}{\partial t} \right)^2 d\Omega$$



- Convection boundary conditions

$$k \nabla (T) \cdot n + h_c (T - T_a) = 0$$

Magnetic-Thermal Coupling



Formulation

- Magnetic equations for solid conductors (g=-V/L)

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu_0}\frac{\partial A_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu_0}\frac{\partial A_z}{\partial y}\right)=\sigma\left(g+j\omega A_z\right)$$

$$I=\int_{\Omega_c}\sigma\left(g+j\omega A_z\right)d\Omega_c$$

- In matrix form

$$\begin{bmatrix}\mathbf{K}+j\omega\mathbf{V} & \mathbf{W}\\ j\omega\mathbf{W}^T & \mathbf{G}'\end{bmatrix}\begin{Bmatrix}\mathbf{A}\\ \mathbf{g}\end{Bmatrix}=\begin{Bmatrix}0\\ \mathbf{I}\end{Bmatrix}$$

- Elemental loss

$$Q_e=\int_{\Omega_e}\sigma\left(g+j\omega A_z\right)\left(g+j\omega A_z\right)^* d\Omega_e$$

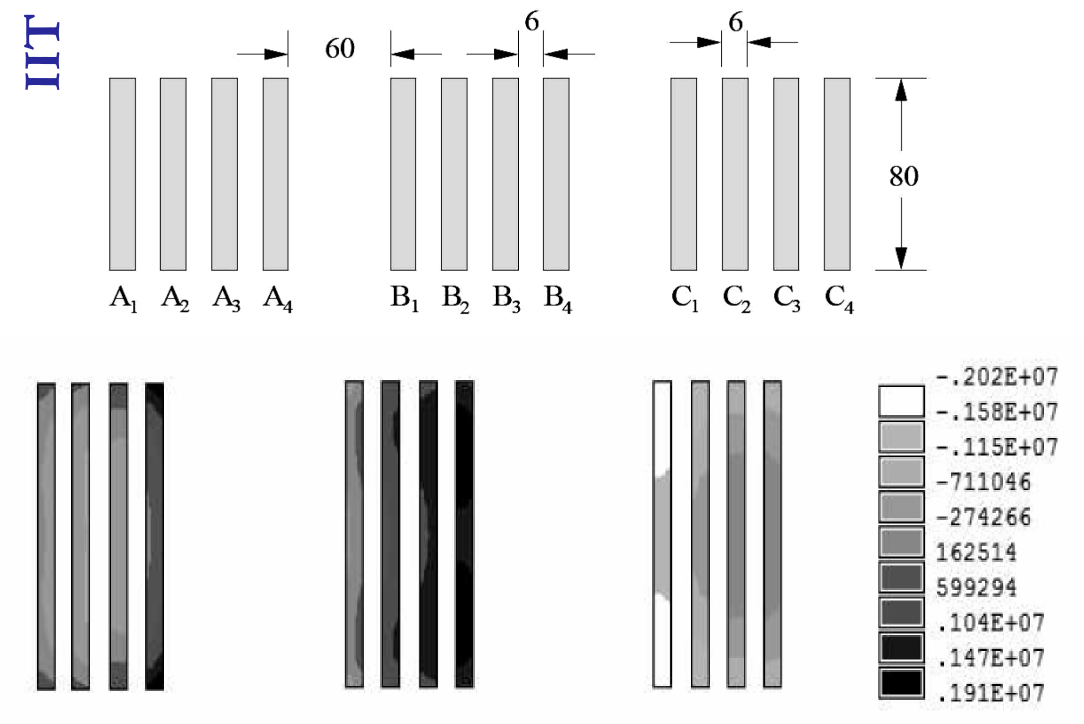
$$\left[\mathbf{K}\right]=\sum_{\Omega}\upsilon\iint_{\Delta_e}\left(\frac{\partial N_e^T}{\partial x}\frac{\partial N_e}{\partial x}+\frac{\partial N_e^T}{\partial y}\frac{\partial N_e}{\partial y}\right)dxdy$$

$$\left[\mathbf{V}\right]=\sum_{\Omega}\sigma\iint_{\Delta_e}N_e^T N_e\,dx\,dy$$

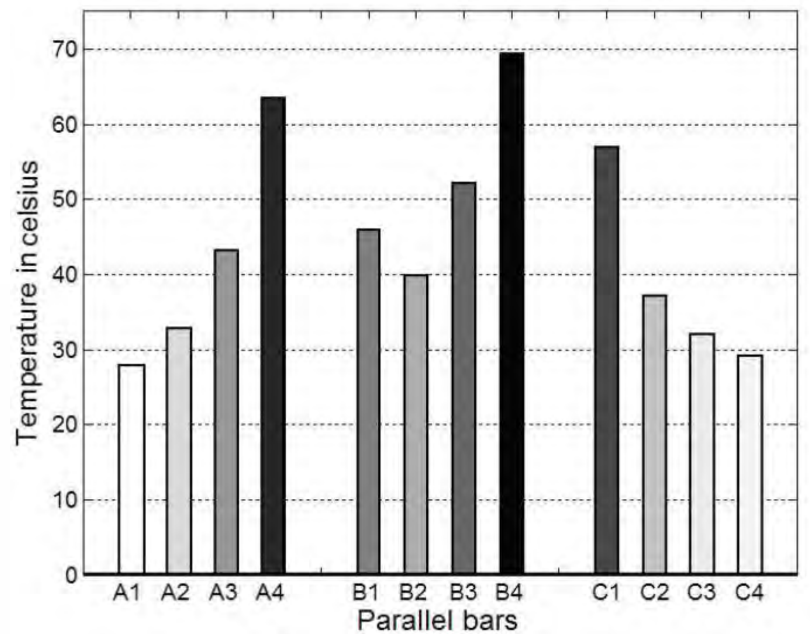
$$\left[\mathbf{W}\right]=\sum_{\Omega}\sigma\iint_{\Delta_e}N_e^T\,dxdy$$

G’ is the diagonal matrix of the conductance of the bars

1. High Current Terminations



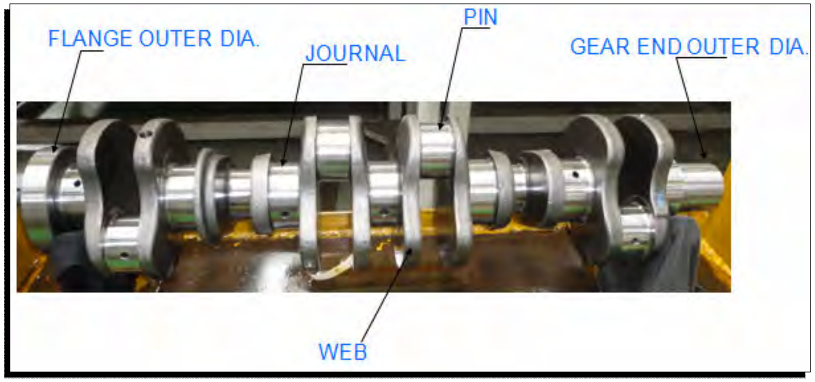
Unequal Current Distribution

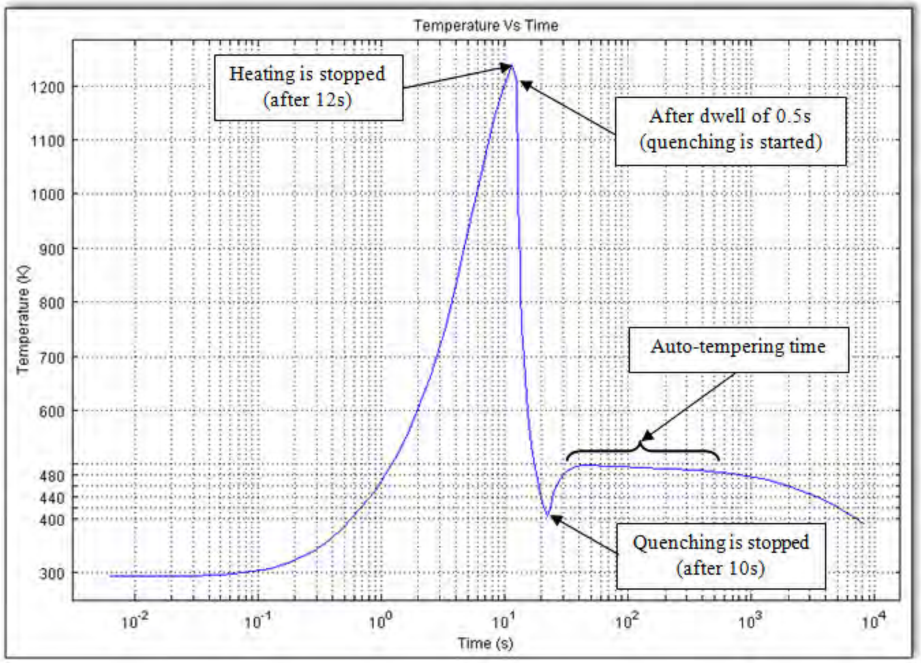


Non-uniform Temperature distribution

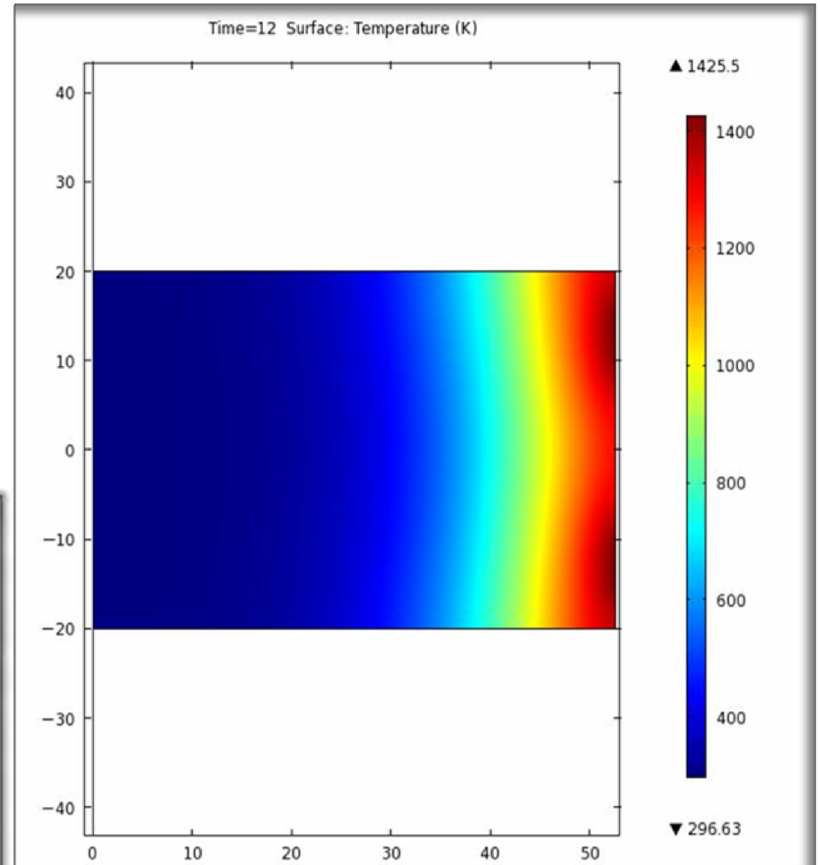
Ref: G.B. Kumbhar, S.V. Kulkarni, R. Escarela-Perez, and E. Campero-Littlewood, "Applications of coupled field formulations to electrical machinery," COMPEL Journal, Vol. 26, Issue 2, 2007, pp. 489-523.

2. Induction Hardening





Auto-tempering process



Temperature distribution

Ref: Sanjay Patil, *Simulation of auto-tempering of induction hardened crackshaft steel*, M.Tech Thesis, MEMS Dept, IIT Bombay, 2011.

Coupled Field Computations

Magnetic – Structural

Coupled Field Formulations: Magnetic-Structural

- **Coupled Equations:**

$$\begin{bmatrix} [K] & [C] \\ [N] & [M] \end{bmatrix} \begin{Bmatrix} \{\mathbf{A}\} \\ \{\mathbf{X}\} \end{Bmatrix} = \begin{Bmatrix} \{\mathbf{J}\} \\ \{\mathbf{F}_{ext}\} \end{Bmatrix}$$

K and M are magnetic and mechanical stiffness matrices respectively. A and X are nodal values of magnetic vector potential and displacements.

- **The formulation with suitable modifications can be used for:**

  - Analysis of core noise: Magnetostriction phenomenon

  - Computation of noise due to winding vibrations (J x B Force)

  - Analysis of winding deformations due to short circuit forces

  - Design of high current carrying bars in large rectifier and furnace duty applications

- [N] and [C] are the coupling matrices, and $\{\mathbf{J}\}$ and $\{\mathbf{F}_{ext}\}$ are the column vectors representing the magnetic and mechanical source terms, respectively.

-The term [N] represents the effect of magnetic parameters on mechanical displacements, whereas [C] represents the effect of mechanical displacements on magnetic parameters.

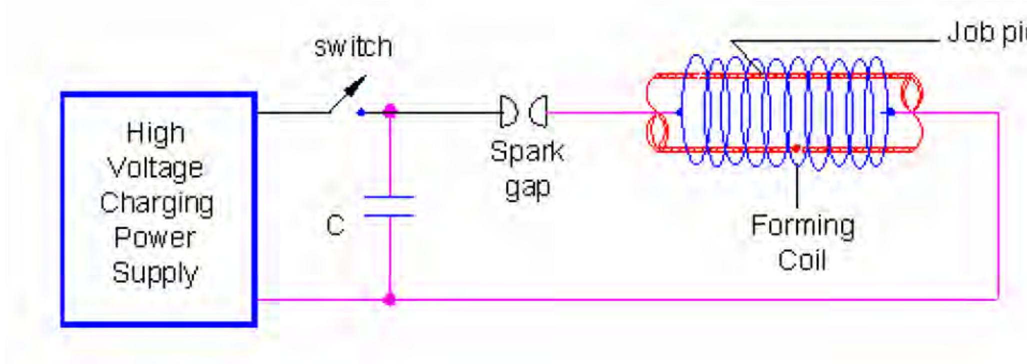
- It can be proved that the total magnetic force ($\mathbf{F}_{mag}$) can be represented by $-[N]\{\mathbf{A}\}$

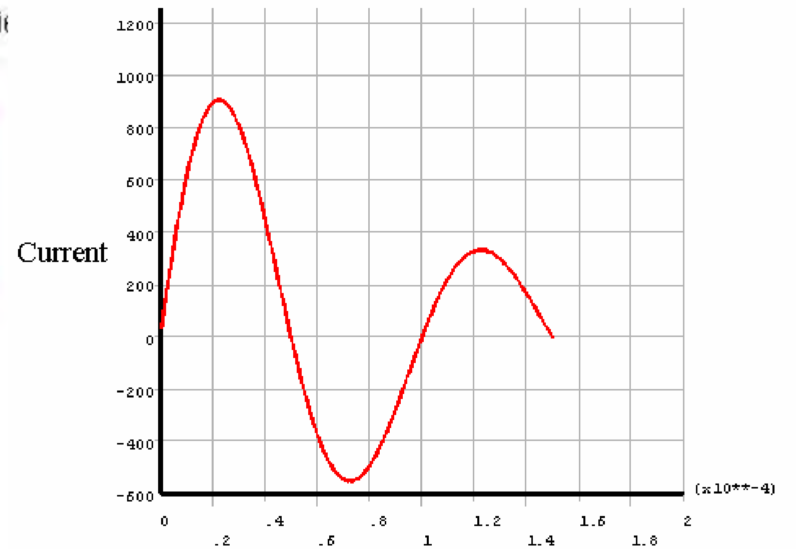
-If the effects of displacements on magnetic fields are not appreciable, [C] can be neglected and the magnetic forces affecting displacements can be added to the mechanical (external) forces:

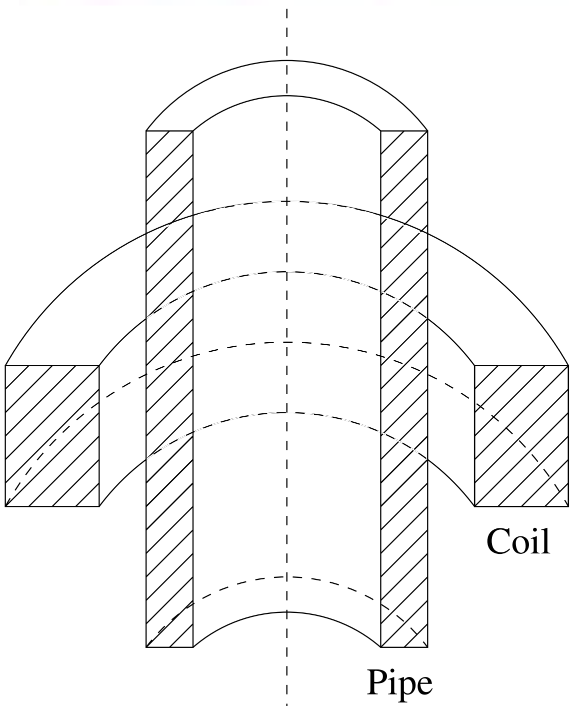
$$\begin{bmatrix} [K] & [0] \\ [0] & [M] \end{bmatrix} \begin{Bmatrix} \{\mathbf{A}\} \\ \{\mathbf{X}\} \end{Bmatrix} = \begin{Bmatrix} \{\mathbf{J}\} \\ \{\mathbf{F}_{ext} + \mathbf{F}_{mag}\} \end{Bmatrix}$$

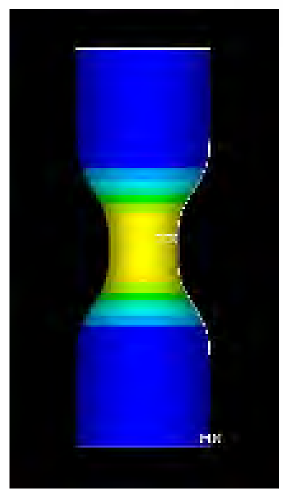
-The magnetostriction phenomenon can also be considered in a weakly coupled scheme by adding the corresponding vector $\{\mathbf{F}ms\}$ to the force terms

Electromagnetic Forming









Simulation

Verification

Concluding Remarks

Current and Emerging Trends

- Competence in 3-D analysis is essential
- Coupled field computations (circuit-field, magnetic-thermal, magnetic-structural) will be increasingly used
- Coupled EM-thermal-structural analysis is not uncommon these days
- Hybrid numerical techniques are being used for complex problems
- Other trends
  - Parallel computing
  - Real time FEM
  - Meshless methods
  - Wavelet based FEM

Thank You !