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Study of Hard-and Soft- Magnetorheological Elastomers (MRE's) Actuation Capabilities

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Abstract: In this study, magneto-rheological elastomer (MRE) composite beams made of Barium hexaferrite (BaM) and Iron (Fe) powders combined with a highly-compliant matrix material were simulated using COMSOL's Solid Mechanics and AC/DC modules. The goal of the work was to develop models capable of predicting the actuation behavior of hard- and soft-magnetic MREs.

The BaM provided the hard magnetic behavior while Fe, served as the soft magnetic case. Both composites were cured in the presence of a magnetic field. The dimensions of the beams were Length: 75 mm; Width: 5 mm; and Thickness 2 mm. The beams were fixed at the base; all other surfaces were free. The MRE beam was contained within a simulated air volume modeled as an elastic medium with negligible modulus. The simulation applied a surface current J_z between the left and right surfaces of the entire domain. This gave rise to a magnetic field perpendicular to the poling direction (-x direction in the BaM case) which actuated the cantilevers.

The primary problem COMSOL addresses is the 2-way coupling between elastic deformation and the magnetic field interactions. The authors use actual magnetization and elastic modulus data form experiments to determine the model's material parameters. As prescribed displacements are imposed on the beam, the resulting restoring force R_{xN} (the force that tends to return the sample to its undeformed state) is determined from reactions at the proscribed displacement boundary. The work shows good agreement to data in the literature for BaM MREs.

Keywords: finite element analysis (FEA), magnetorheological elastomers (MRE's), smart materials, magneto elastic coupling, actuation.

1. Introduction

In the last few years, prompted largely by the work of Lord Corporation research group (e.g. Jolly *et al.* [1996]) numerous articles on the magneto-elastic behavior of MRE's have appeared. There are two different phenomenological approaches to the study of MRE's magneto- elasticity: firstly, theories that treat the problem from the viewpoint of continuum mechanics and, secondly, statistical or kinetic theories that attempt to derive magneto-elastic properties from idealized models of the structure of particulate composites. There are several works in the literature that presents the full system of equations suitable for deformable MRE's in an electro-magnetic field; notably Borcea and Bruno [2003] calculated the overall deformation and stress–strain relation using fully coupled magneto-elastic interactions. Dorfmann and Ogden [2003] summarized the equations governing MRE deformation with particular reference to elastomers whose mechanical properties can change rapidly by the application of a magnetic field. Kankanala and Triantafyllidis [2004] illustrated the magneto-elastic coupling phenomena in a cylinder subjected to traction or torsion under the presence of external magnetic fields. Besides these models, Yin *et al*. [2002] and Coquelle *et al.* [2006] have also proposed micro-mechanically based particulate composite concepts.

Although the aforementioned models provide important guidelines to simulate such behavior, most of their solutions are idealized in the sense that they apply only to bodies of infinite extent and are derived for isotropic magneto-elastic materials. Recently, Tuan and Marvalova [2010] and Castañeda and Galipeau [2010] presented constitutive equations that govern the interaction between an anisotropic MRE and a magnetic field. Castañeda and Galipeau study the role of shape anisotropy while Tuan and Marvlova study an incompressible magneto-elastic anisotropic material capable of large deformations. These types of models are important because magnetic anisotropy drives actuation.

In this, and other works, modeling begins by choosing a magnetic vector potential, $\boldsymbol{A}$, as the independent magnetic variable in the constitutive laws .The independent variable, $\boldsymbol{A}$, is then related to higher order dependent variables such as magnetic flux density, $\boldsymbol{B} = \nabla \times \boldsymbol{A}$, for example. Relationships between $\boldsymbol{u}$ the vector displacement field, the independent variable in elasticity theory, and higher order dependent variable such as strain and stress are employed as well (see later sections for details). The resulting boundary-value problem can be presented as a set of differential equations coupling elastic and magnetic behavior that can be solved using a finite-element method. To reflect material behavior under combined external mechanical loads and magnetic fields and to come up with a reasonable and applicable magneto-elastic law is still an important issue in linear and nonlinear magneto-elasticity theory. Moreover, it is important to validate modeling approaches by direct comparison to accurate experimental data.

For this reason, a number of actuation experiments on MRE's driven by a magnetic field exist. Zhou and Jiang [2004] presented the real-time dynamic deformation progress (the vector diagram of the displacement) of MREs and elastomer–ferromagnet composite (EFCs) under the presence of a magnetic field. Von Lockette *et al* [2011] have shown that combined elastic plus magnetic restoring forces in BaM-MRE cantilevers, which are non-zero for non-zero field strengths, increase with tip deflection and field strength. In the same way restoring forces in Fe-MRE beams, which are zero at zero deflection (regardless of field strength), increase with tip deflection and field strength. In addition, experimental work has shown that the magnetic component of the restoring force in BaM cantilevers is relatively independent of displacement.

In this study, we present the simulation of a cantilever beam of finite size subjected to a magnetic field. The constitutive equations are based on generalized forms of Hooke's laws and Maxwell's equations for anisotropic materials that depend on the displacement field $\boldsymbol{u}$ and the magnetic potential vector $\boldsymbol{A}$ as the independent variables. The fundamental problem herein deals with discontinuous changes in physical properties (elastic and magnetic) across innumerable particle-matrix interfaces. These physical properties give rise to magnetic and elastic responses at particle and inter-particle levels that in turn generate forces within an MRE in response to an applied magnetic field or external load.

In order to formulate tractable problems, previous theoretical modeling has been forced to address the issue using simplifying assumptions of material composition and structure, for example roughly spherical soft-magnetic particles either randomly arranged or neatly aligned. This work begins down a computational path, employing simplifying assumptions on behavior that are based on experimentally proven material response which include both isotropy and anisotropy in varying cases. This is an important step since key paradigms of such assumptions, such as the alignment of particles in MREs cured in a magnetic field, are coming under increased scrutiny in MREs with technologically relevant volume fractions and thus are invalidating the basis of previous constitutive models (e.g. Boczkowska [2009]). A finite element modeling approach allows us to analyze the elasto-magnetic behavior (both kinetic and kinematic) numerically while incorporating experimentally determined elastic and magnetic behavior. The computational method then seeks to solve the problem of determining the combined elasto-magnetic behavior of a given geometry under given external mechanical and magnetic loads.

The basic objective of the simulation described in this report was to develop a predictive model of MRE behavior for Hard- and Soft-magnetic MRE's. The objective was accomplished by measuring the physical and rheological properties of actual MRE samples and developing continuum computational models for an MRE particulate composite.. The authors use actual magnetization and elastic modulus data from experiments to determine the model's material parameters.

2. Model Definition

The problem herein is considered in a 2-D plane as illustrated in Figure 1.The model consists of two regions: a cantilever beam (Ω) and the surrounding air region (Φ). The dimensions of the beams were Length: 75 mm; Width: 5 mm; and Thickness 2 mm. Basic mechanical and magnetic properties are established from experimental data and literature values: the Young's modulus $E_{BaM} = 14\ \mathrm{MPa}$ and $E_{Fe} = 30\ \mathrm{MPa}$, Poisson's ratio $\nu = 0.4$, density $\rho = 7870\ \mathrm{Kg/m^3}$, and the magnetic saturation of BaM $M_s = 60\ \mathrm{kA/m}$. For Fe the magnetic properties are defined by an H-B curve calculated form experiments.

In the air region, the magnetic field depends on the vector potential $\boldsymbol{A}$ which is determined from the solution to the problem given appropriate boundary conditions on 1 and 9 (Fig. 1). The direction of the resulting magnetic field in the air region is then parallel with the y-axis. The moving mesh is used for the calculation of the magnetic field values.

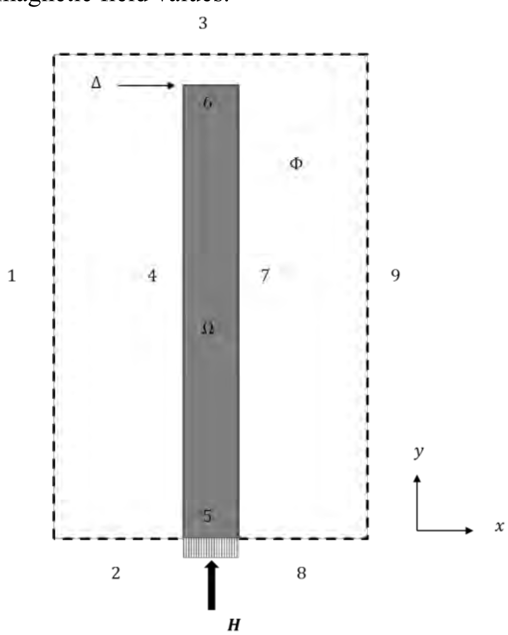

Figure 1. Geometry of the studied 2-D problem. Omega represents the beam domain in grey) which is fixed at the base and Gamma the air domain (dashed line. Boundaries are numbered.

The mesh movement inside the internal subdomain and at its boundaries is determined by the displacements of the deformed beam. The primary problem COMSOL addresses is the 2-way coupling between the elastic deformation and the magnetic field interactions.

Boundary Number	**Elastic Boundary Condition**	**Magnetic Boundary Condition**
1	Free	SC (-)
2	Fixed	PMC
3	Free	PMC
4	BL	FC
5	Fixed	FC
6	PD, BL	FC
7	BL	FC
8	Fixed	PMC
9	Free	SC (+)

Table 1. Boundary conditions on the propose model where SC is the surface current, PMC the perfect magnetic conductor, FC the force calculation, BL the boundary load, and PD the prescribe displacement condition.

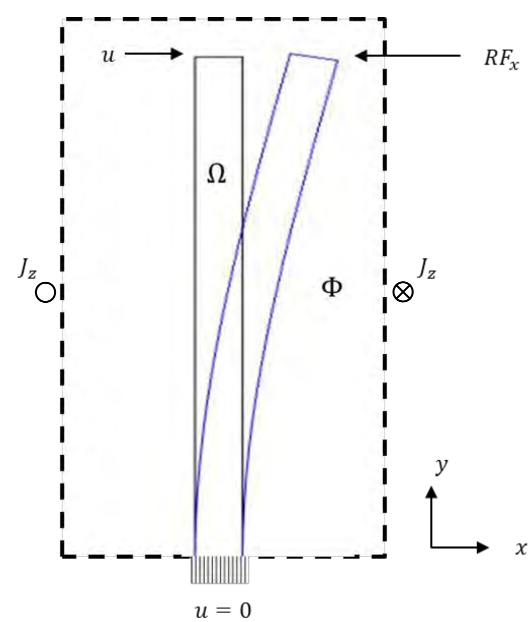

Figure 2. Schematic of the finite element model showing the direction of: the applied current J_z(the open circle is out and circle with x is into the board), prescribed displacement u, and reaction force RF_x.

As the tip of the beam undergoes blocked free deformation through the application of a magnetic field or prescribed displacement via applied boundary conditions, a reaction force that tends to bring the beam back to an equilibrium position is created. Hence, the accurate prediction and analysis of this reaction force is important in predicting the actuation capabilities of the MRE's. Therefore, the analysis and computation of the x-direction reaction force on boundary 6 (Fig. 1) is of primary interest.

3. Governing Equations

Herein the magneto-elastic behavior of the MRE's is analyzed using the continuum-mechanics approach in which the appropriate mechanical deformation equations are coupled with electromagnetic equations. Let us consider a differential volume element in static equilibrium within the cantilever beam acted on by an arbitrary body force. A body force is any externally applied force that acts on each element of volume of the continuum, thus, a force per unit volume $\boldsymbol{F}_v$. Applying Newton's first law of motion, we can obtain the set of differential equations that govern the stress $\boldsymbol{\sigma}$ distribution within the beam, given in tensor notation by

$$-\nabla \cdot \boldsymbol{\sigma} = \boldsymbol{F}_v \qquad 1$$

Next we employ generalized Hooke's Law

$$\boldsymbol{\sigma} = \boldsymbol{C\varepsilon} \qquad 2$$

where $\boldsymbol{\sigma}$ is stress tensor, $\boldsymbol{C}$ is the stiffness matrix defined in 2D plane strain.

Given tensor notation for strain-displacement,

$$\varepsilon = \frac{1}{2}[(\nabla u)^T + \nabla u] \qquad 3$$

we substitute 2-3 into 1 yielding the governing equation in terms of the independent displacement variables, u_i,

$$\boldsymbol{F}_v + \nabla \cdot \left\{ \boldsymbol{C}\left[\frac{1}{2}[(\nabla \boldsymbol{u})^T + \nabla \boldsymbol{u}]\right]\right\} = 0 \qquad 4$$

This set of equations encompasses what COMSOL solves for the elastic aspect of the elasto-magnetic problem. On the other hand, Maxwell's equations represent the governing equation of electro-magnetic phenomena. In this case,

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad 5$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{J} \qquad 6$$

where $\boldsymbol{E}$ is the electric field intensity, $\boldsymbol{B}$ the magnetic flux density, $\boldsymbol{H}$ the magnetic field intensity, $\boldsymbol{D}$ the displacement current density, and $\boldsymbol{J}$ the electric current density.

First, we formulate the vector potential $\boldsymbol{A}$ defined by

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \qquad 7$$

Next, assuming a quasistatic model, all time derivatives are zero, specifically

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \frac{\partial(\boldsymbol{A})}{\partial t} = 0 \quad \text{and} \quad \frac{\partial \boldsymbol{D}}{\partial t} = 0 \qquad 8$$

which, from eq. (5) yields

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} = 0 \qquad 9$$

Together with the material law

$$\boldsymbol{J} = \sigma \boldsymbol{E} + \boldsymbol{J_e} \qquad 10$$

where σ is the material electric conductivity, $\boldsymbol{J_e}$, the applied current density, and μ the material's permeability we can reduce our remaining Maxwell equation to

$$\nabla \times \boldsymbol{H} = \boldsymbol{J_e} \qquad 11$$

The three materials in question, the air medium, the Fe-MRE, and the BaM-MRE, have three different $\boldsymbol{B} - \boldsymbol{H}$ relationships**.** The general constitutive equation for the magnetic response of the air medium is given by

$$\boldsymbol{B} = \mu_0 \mu_r \boldsymbol{H} \qquad 12$$

yielding the governing equation for the air medium domain in terms of the independent variable, $\boldsymbol{A}$

$$\nabla \times \left(\frac{1}{\mu_0\, \mu_r} \nabla \times \boldsymbol{A}\right) = \boldsymbol{J_e} \qquad 13$$

where μ_r, is the relative permeability of the material (unity in this case) and $\mu_0 = 4\pi x 10^{-7}\; NA^{-2}$ is the permeability of free space.

The general constitutive equation for BaM , a hard magnetic material, can be expressed by

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}) \qquad 14$$

Therefore, the governing equation for a BaM-MRE domain is given by

$$\nabla \times \left(\frac{1}{\mu_0} \boldsymbol{B} - \boldsymbol{M}\right) = \boldsymbol{J_e} \qquad 15$$

which in terms of the dependent variable yields

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \boldsymbol{A} - \boldsymbol{M}\right) = \boldsymbol{J_e} \qquad 16$$

Magnetization values for the anisotropic BaM material will be found from experimental results.

For Fe-MREs the gradual alignment of the magnetic domains within the material causes an increase in $\boldsymbol{B}$ as $\boldsymbol{H}$ is gradually increased. The constitutive relation is not a simple linear function and thus requires a general definition,

$$|\boldsymbol{H}| = f(|\boldsymbol{B}|) \qquad 17$$

where the function $f(|\boldsymbol{B}|)$ will be found from experimental results. Finally, substituting (17) into (11) we get

$$\nabla \times f(|\nabla \times \boldsymbol{A}|) = \boldsymbol{J_e} \qquad 18$$

3.1 Model Boundary Conditions

The basic idea behind our proposed system is to allow the mechanical structure to bend freely (as expected for BaM composites) in the presence of a magnetic field or to impose a tip deflection and measure the required external load via the x-component of the reaction forces. The dimensions of the beams were Length: 75 mm; Width: 5 mm; and Thickness 2 mm. The beam was contained within a simulated air volume modeled as an elastic medium with negligible modulus. The bending results in a nonlinear relationship between the beam tip deflection and the resulting restoring force on its surface.

Prescribed displacements, *u*, are imposed on boundary 6; the resulting restoring force RF_x is determined from reactions at the proscribed displacement boundary. A surface current $\boldsymbol{J_z}$ is applied between the left and right surfaces of the entire air domain (boundaries 1 and 9). This gives rise to a magnetic field aligned with +*y*, which is perpendicular to the poling direction (the BaM was poled in the -x direction), that actuates the cantilevers (see Figure 2).

A parametric study was developed to determine the influence of the applied magnetic field on the deflection of the beam. Since the applied magnetic field plays a major role in determining the beam deflection, the study defines one parameter: the applied surface current, $\boldsymbol{J_z}$, which gives rise to a magnetic field and is defined by a start value 0, an end value 1240, and the step of the range 310. Then for each prescribe displacement imposed on the tip of the beam, it is possible to calculate the resulting restoring force for a range of magnetic field values and displacements.

3.1.1 Mechanical Boundary Conditions (MBC's)

The MBC's are formulated as prescribed displacement $u = u_0$, prescribed constraints where $\boldsymbol{u} = 0$, and boundary load specify on domain Ω. This boundary load is defined by the Maxwell surface stress tensor (included to account for the stress due to the electromagnetic force induced by the magnetic field) on the surface Ω. The calculated Maxwell surface tensor is imposed as a surface traction on the boundary of the MRE (boundaries 4-7) and is the basis of the elasto-magnetic coupling.

3.1.2 Electromagnetic Boundary Conditions (EBC's)

The EBC's are formulated as prescribed surface current $\boldsymbol{J_{s0}}$, magnetic insulation on boundary 5 to impose symmetry for the magnetic field, and perfect magnetic conductor condition on the upper and lower surface of Φ (i.e. boundaries 2, 3, and 8) so that the prescribed current is not allowed to flow out of the air domain.

4. Mesh

Free Quad elements were used to mesh the beam and Free Triangular elements to mesh the remaining domain. The quality of the triangular mesh was set by the triangulation method: Advancing front. The sizes of the elements were specified using the predefined sizes presented in table 2.

Domain	Element	Size [mm]
Beam	Extra Fine	1.6
Air	Extremly Fine	0.8

Table 2. Element type and size of the used mesh.

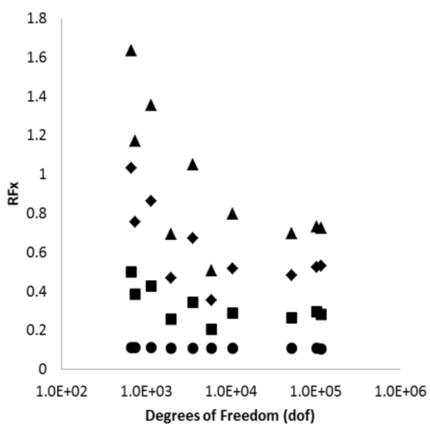

Figure 3. Convergence study of the reaction force RFx vs. degrees of freedom (DOF) at specific magnetic field densities.

Determination of an appropriate level of mesh refinement started with a coarse divided mesh, which was gradually reduced in size while the *x*-reaction force was used as a metric of mesh convergence. Element size reduction was terminated when the results were found to asymptotically converge (see Figure 3). The resulting mesh consisted of 2620 elements and 28340 degrees of freedom and is presented in Figure 4.

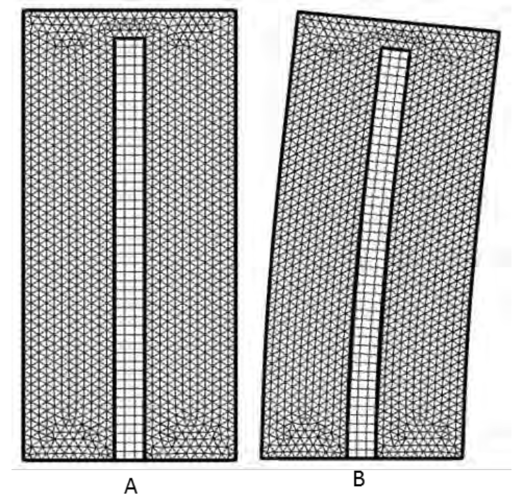

Figure 4. (A) Resulting and (B) deformed mesh, the converged mesh consists of 2620 elements and 28340 degrees of freedom. Tip Displacement is 4 mm.

5. Results and Discussion

In this study, a magneto-elastic finite element formulation is presented. Two problems have been considered using COMSOL as case studies. In one problem, we model an MRE beam either made of nominally 30% v/v 40-micrometer Barium hexaferrite (BaM) particles, which provides the hard magnetic behavior or 325 mesh Iron (Fe) also at 30% v/v, which serves as a soft magnetic case. Both are combined with a compliant elastomer matrix. Initially we model the static deformation of the MRE's under combined elasto-magnetic behavior.

For the case of the BaM MRE the deformation obtained from the simulation shows good qualitative agreement with experimental.

Secondly, reaction forces to the blocked bending deformation of the beam under the influence of static magnetic field are analyzed. The results obtained from the simulation are compared with those reported in the literature (see Figure 6) and show very good agreement.

The BaM composite behavior, as reproduced by the simulation, is shown in Figure 6, the composite reaction force is non-zero for non-zero field strengths and as expected, they increase with tip deflection and field strength.

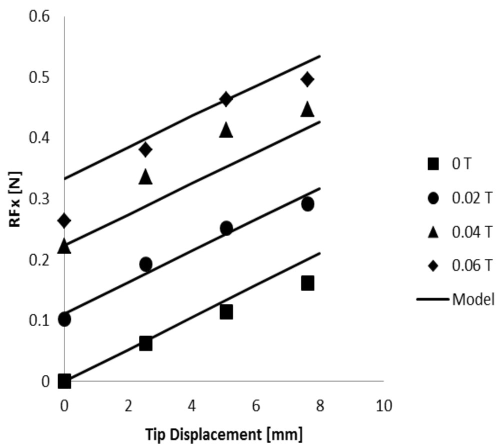

Figure 6. *x-direction* reaction force vs. tip displacement for BaM beam MRE composite. Data (in symbols) taken from Von Lockette *et al*. [2011] is compared to FEM model predictions (lines).

This result is again in agreement with experimental observation [Von Lockette 2011]. Nevertheless, in experimental results a non-linear behavior is noticeable as the field is increased and tip displacement gets larger.

6. Conclusion

Based on the comparison between the propose model and experimental data we conclude that the model can be useful to predict the behavior of hard-MRE's specially those comprise of BaM particles.

On the contrary, the Fe composite behavior, as reproduced by the simulation, is not in agreement with experimental data. The authors' believe that the problem is inherent to the formulation of the Maxwell stress tensor. Once the model becomes asymmetric when it bends, it develops an erroneous net force with +x direction. . This error does not go away even at millions of DOF.

A suggested fix is to use a combination of sensitivity analysis, deformed geometry, and AC/DC modules to get a better solution of the magnetic force tensor. However, one cannot couple this with solid mechanics (e.g. assign this traction to the MRE surface). Consequently, the Fe-MRE model is not effective. The support technicians at COMSOL are aware of this issue.

7. References

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