Analysis Of Linearly Polarized Modes

Author Ioana Moldovean (Avram)[1], Author B. Ioan Gavril Tarnovan[*,2] Author C. Bogdan Tebrean[*,3]
[1]Author A. The Technical University Of Cluj-Napoca, [2] Author B. The Technical University Of Cluj-Napoca [3] Author C. The Technical University Of Cluj-Napoca
*Corresponding author: 28, Memorandumului str, 400114, Cluj-Napoca, Romania, Ioana.AVRAM@mas.utcluj.ro

Abstract: *This paper presents a study on the propagation modes of electromagnetic waves through a step index fiber optics. Obtaining the propagation modes to get their characterization according to the radial and azimuthal distribution is by modifying the characteristics of the fiber. This study is required for further investigation of states of polarization and analysis of electric field distribution using high frequency conditions.*

Keywords: waveguide, propagation, mode.

1. Introduction

This paper is an analysis of the propagation mode of step index fiber optic.
Currently, the propagation beam method is widely used to study the propagation of light. There are three version of beam propagation method (BPM). The first BMP is based on the fast Fourier transform, the second is based on finite difference method and the third is based on the finite element method. [5][3]
For the analyses described in this paper, a system based on the finite element method has been used - the third method described earlier.
The analysis of Maxwell's equation is a resulting a relation between electric field and magnetic field, which condition the appearance and propagation of an electromagnetic field in the form of electromagnetic waves. The propagation field is shown in figure 1. [6]

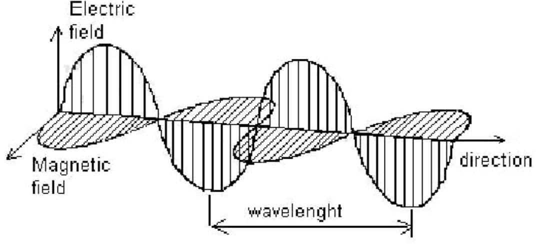

Fig.1 Propagation of the electric and magnetic field [Adapted from 6]

2 Basic equations

To determine the propagation modes of electromagnetic waves the phenomenon of the total refraction at the interference of two mediums with different refractive indices has been used. This phenomenon is governed by Snell's law [6] [7]

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad (1)$$

Where n_1 and n_2 represent the refractive index of the medium in which light propagates, θ_1 and θ_2 represent the angle of incidence, respectively the angle of refraction.
According to the condition at the boundary between core and cladding, the intensity of electric and magnetic field can be determined. [7]
A 3-D optical waveguide has been considered, as figure 1, where x and y are transverse directions and z represents the propagation direction. [4]
The basic equation at the beginning of the analysis is the wave equation. [3]

$$\nabla^2 E + \nabla\left(\frac{\nabla\varepsilon_r}{\varepsilon_r}\cdot E\right) + k^2 E = 0 \qquad (2)$$

Considering a monochromatic wave with the pulsation ω and the constant of propagation β, the phase can be written as:

$$f = \omega t - \beta z \qquad (3)$$

The equation of the electric field becomes

$$\vec{E}(x,y,z,t) = \vec{E}(x,y)e^{j(\omega t-\beta z)} \qquad (4)$$

and the equation of the magnetic field becomes

$$\vec{H}(x,y,z,t) = \vec{H}(x,y)e^{j(\omega t-\beta z)} \qquad (5)[10]$$

The vector wave equations of electric and magnetic fields can be reduced to the Helmholtz's equation, if the relative permeability is constant in the medium. [7]

$$\nabla^2 H + n^2 k_0^2 H = 0 \qquad (6)$$

$$\nabla^2 E + k^2 E = 0 \qquad (7)$$

The Helmholtz's equation for the electric field can be summarized as [7]

$$\nabla_\perp^2 E + (k^2 - \beta^2)E = 0 \qquad (8)$$

or

$$\nabla_\perp^2 E + k_0^2(\varepsilon_r - n_{eff}^2)E = 0 \qquad (9)$$

For the magnetic filed, the equation can be written as follows:

$$\nabla_\perp^2 H + (k^2 - \beta^2)H = 0 \qquad (10)$$

or

$$\nabla_{\perp}^{2} H + k_0^2(\varepsilon_r - n_{eff}^2)H = 0 \tag{11}$$

Where $$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{12}$$

The optical waveguide has a uniform structure along the z direction. [7] This condition is respected by the following relation:

$$\frac{\partial}{\partial z} = -j\beta \tag{13}$$

where β the propagation constant, and z is the propagation direction.

$$k_0 = \frac{2\pi}{\lambda_0} \tag{14}$$

k_0 represents the free space wave number in vacuum.

The ratio of propagation constant β to the wave number in vacuum k_0 represents the effective refractive index. [7]

$$n_{eff} = \frac{\beta}{k_0} \tag{15}$$

If the λ_0 is the wavelength in vacuum the propagation constant becomes: [7]

$$\beta = \frac{2\pi}{\lambda_0} n_{eff} = \frac{2\pi}{\lambda_{eff}} \tag{16}$$

where $$\lambda_{eff} = \frac{\lambda}{n_{eff}} \tag{17}$$

The propagation constant represents the phase rotation per unit propagation distances, and the effective index n_{eff} represents the ratio of the wavelength in the medium to the wavelength in a vacuum. [7]

The eigenvalue of the equation for the magnetic field H is obtained by derivation of Helmholtz's equation. [2]

$$\nabla \times (n^{-2} \nabla \times H) - k_0^2 H = 0 \tag{18}$$

Where ∇ is the Laplace operator and Helmholtz's equation can be solved for the eigenvalue

$$\lambda = -j\beta \tag{19}$$

β represents the propagation constant along the axis z

The eigenvalue corresponds to the propagation constant itself. [4]

3. The analysis of the propagation of electromagnetic field

To analyze the propagation of electromagnetic field, a simulation in Comsol 4.0 has been made with two optical fibers. The optical fiber has the core of 8 µm, or 50 and 62.5 µm. All three types of fiber have 125 µm coating. The fiber core is made of pure silica, whose refractive index is 1.4457. The cladding is made by silica, with a refractive index of 1.4378. For the phenomenon of total reflection to occur, the coating index must be smaller than the core index.[2]

$$n_2 > n_1 \tag{20}$$

Because the refractive index of the core is higher than the refractive index of the cladding, the optical field is confined to the core. [7]

Obtaining the propagation modes can be done in the Comsol 4.0 application by changing the refractive index of the core, changing the core size, changing the wavelength or setting a specific eigenvalue.

To obtain the propagation mode, on the radial direction no flow of energy should exist. [2] For this to occur, the wave has to be evanescent in the radial direction in the cladding, but not in the core. [2] To obtain this condition, the Comsol package defines the effective refractive index, as the following equation.

$$n_2 < n_{eff} < n_1 \tag{21}$$

To investigate the propagation modes, a section was made through the xy plane of the fiber. The wave will propagate along the fiber with the pulsation ω and propagation constant β [2].

Effective refractive index can be assigned for each propagation modes, according to the phase velocity. [9]

$$V = \frac{2\pi a}{\lambda_0}\sqrt{n_1^2 - n_2^2} = k_0 a \sqrt{n_1^2 - n_2^2} \tag{22}$$

V determines the number of the propagation modes in the waveguide and is related to the propagation of electromagnetic field in the guide. [6]

If V>>1 the propagation mode can be solved with optical geometric calculations, in this case the guides are multimode guides with the following parameters $\Delta = 0.01 \div 0.03$, $d = 20 \div 100$ µm

If $V \cong 1$ the propagation mode is in single mode, and the specific parameters have the values: $\Delta = 0.003 \div 0.01$, $d = 4 \div 10$ µm[11]

The difference between the refractive index of the core and the cladding is very small, about 1% [7]. This approximation simplifies the analysis, so the modes obtained are called linearly polarized modes, notated LP_{nm}. [7]. N and m represents the number of radial and azimuthal zeros for each mode. [7]

The optical guides are generally used to transmit pulses, which are dispersed. The value of the dispersion determines the transmission rate of the guide. It is essential that the pulses do not overlap, because of errors in transmission. [8]

4. Comsol modeling

For the electromagnetic wave propagation analysis, we used two optical fibers. The first has 8µm core, the second has 50 µm cores, having a coat of 125 µm. The finite element mesh is shown in figure 2, for both fiber optics.

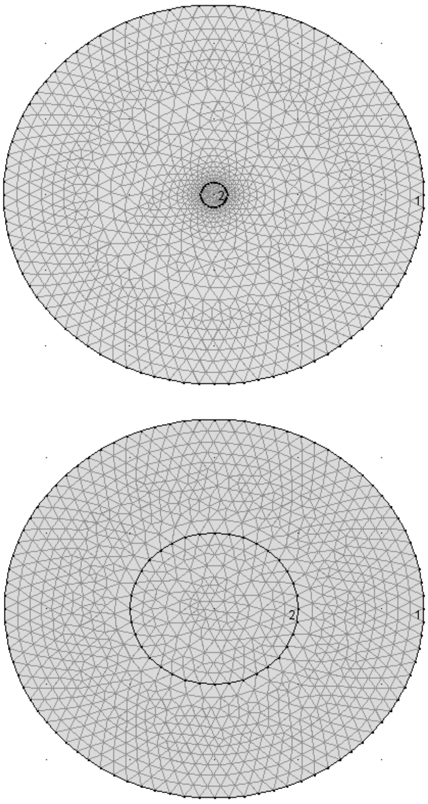

Fig.2 Finite element mesh for single mode and multimode fiber optic

The boundary conditions are:

$$\varepsilon_r = n^2 \tag{23}$$

$$\mu_r = 1 \tag{24}$$

$$\sigma = 0, \tag{25}$$

$$\lambda = -j\beta - \delta_z \tag{26}$$

The classification of linearly polarized modes is made after the radial and azimuthal angle.

The following figures are related to the classification of linearly polarized modes for single mode fiber optic and the distribution of electric and magnetic field

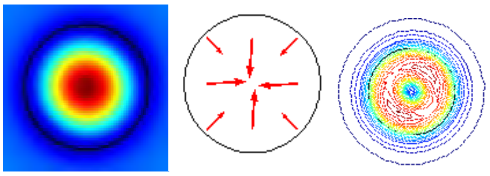

Fig 3. Fundamental mode

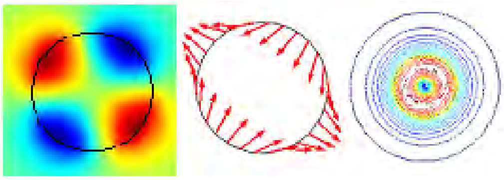

Fig.4 LP_{02} mode

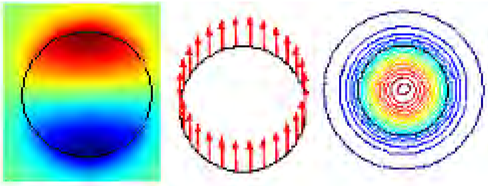

Fig.5 LP_{01} Electric transverse mode

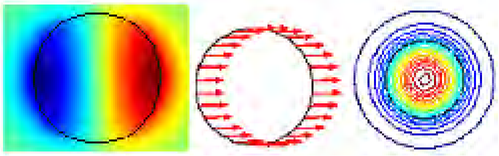

Fig.6 Magnetic transverse mode

Classification and distribution of linearly modes of electric field for multimode fiber optic

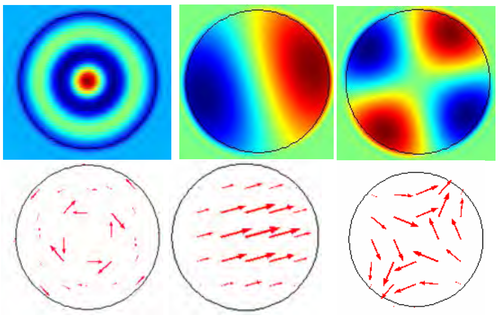

Fig.7 LP_{01}, LP_{11}, LP_{21}

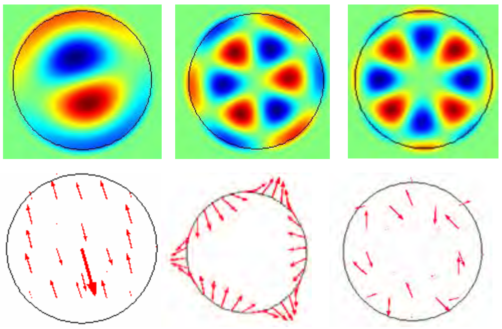

Fig.8 LP_{61}, LP_{71}, LP_{81}

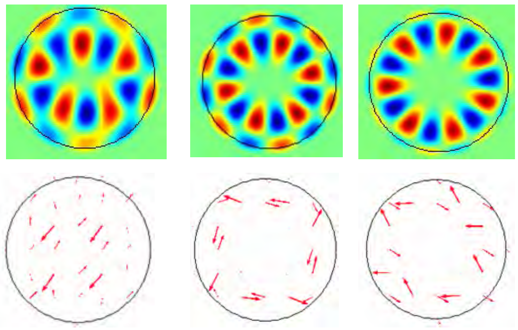

Fig.9 LP_{12}, LP_{32}, LP_{42}

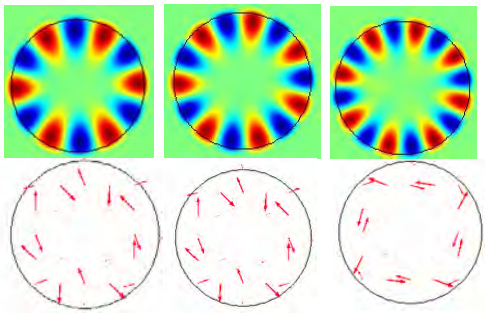

Fig.10 LP_{52}, LP_{62}, LP_{72}

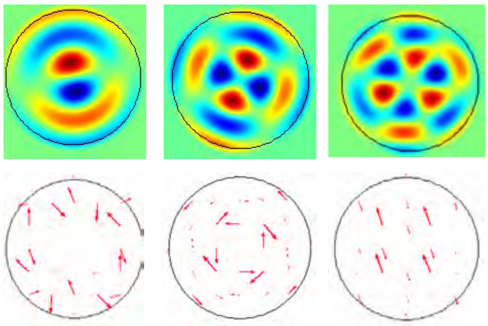

Fig.11LP_{13}, LP_{23}, LP_{23}

Another study was done by simulating the wave propagation through an optical fiber using a cross section. A two-steps study was developed using an implemented Comsol application . The first is *Mode analysis* and the second step is *Eigenvalue*. To obtain one fundamentally mode through the whole section a wavelength of 2 µm has been used, the desired number of modes being 20 and the eigenvalue is 35.

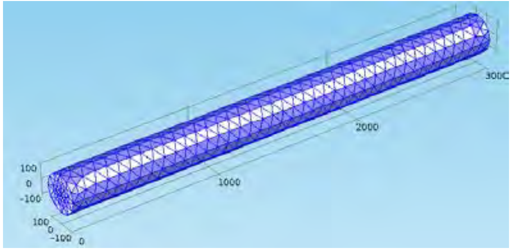

Fig. 12 The 3-D optical fiber

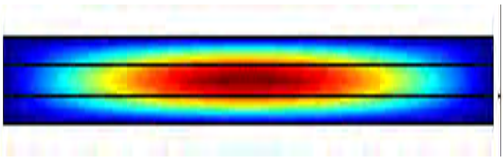

Fig.13 The electrical field distribution through the optical fiber. One fundamental mode

Changing the characteristics of the fiber can be obtained another linearly polarized modes and also may get a different distribution along the fiber of fundamental mode.

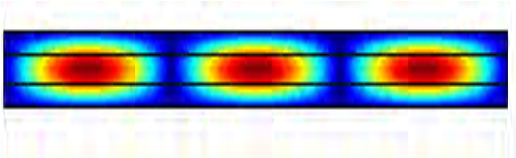

Fig.14. The electrical field distribution. Three fundamental modes.

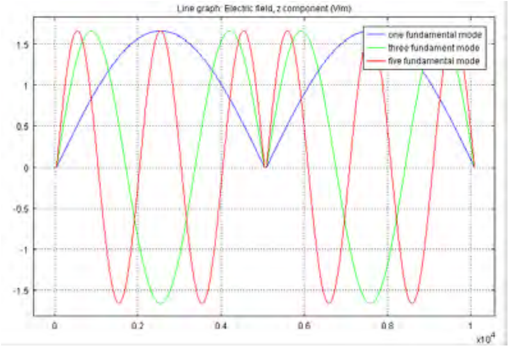

Fig.15 Line graph for electric field distribution along the z axis

The propagation of electromagnetic wave was achieved by a cross section of a curved optical fiber. The fiber has a 50 µm core and 125 µm cladding. The refractive indexes are the same, 1.4457 for the core and 1.4378 for the cladding. This study contain the same steps as the last study, with the difference that the eigenvalue was established 2 and the wavelength 1.55 µm

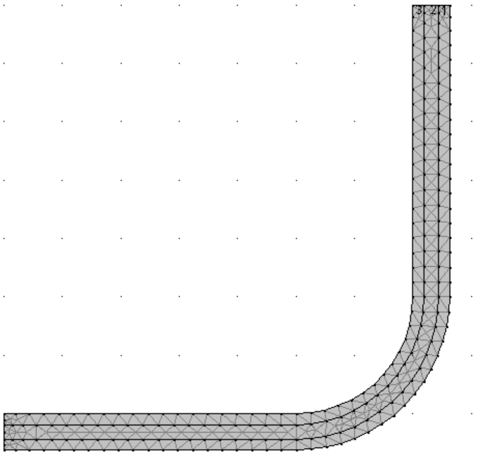

Fig. 16 The mesh for the multimode curved optical fiber

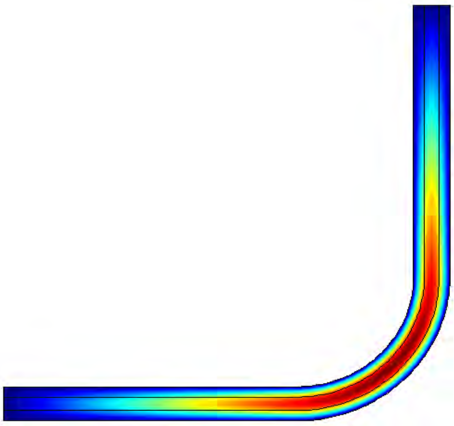

Fig. 17 The electric field distribution thought the curved optical fiber. One fundamental linearly polarized mode

In this figure the linearly polarized mode is the fundamental mode. If the effective refractive index or the eigenvalue was changed through the optical fiber more fundamental modes will appear.

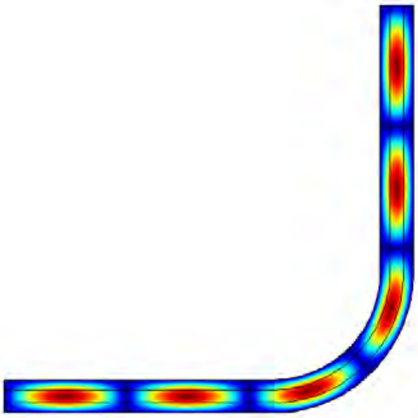

Fig. 18 The electric field distribution thought the curved optical fiber. Six fundamental linearly polarized modes

Simulation of the optical fiber junction was made at a wavelength of 2.2 µm. The study used consists of the step mode analyses and the step eigenvalue. The searching of eigenvalues was around 2.

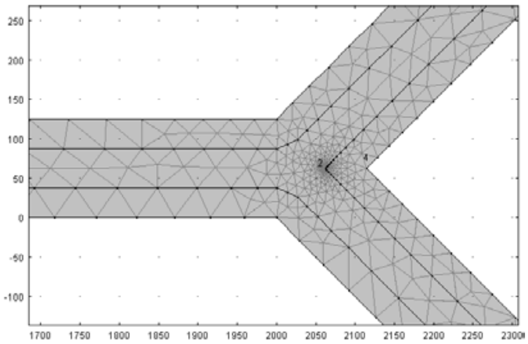

Fig.19. The mesh of optical fiber junction

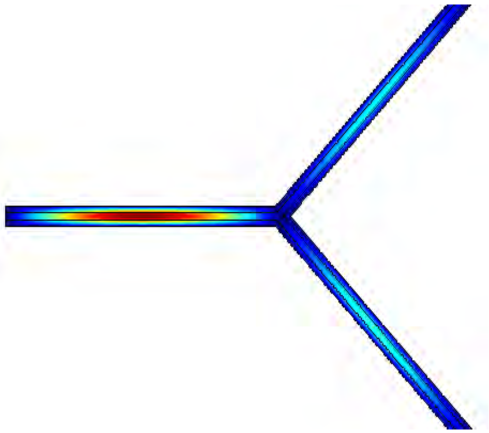

Fig 20. First step of electrical field distribution along z axis

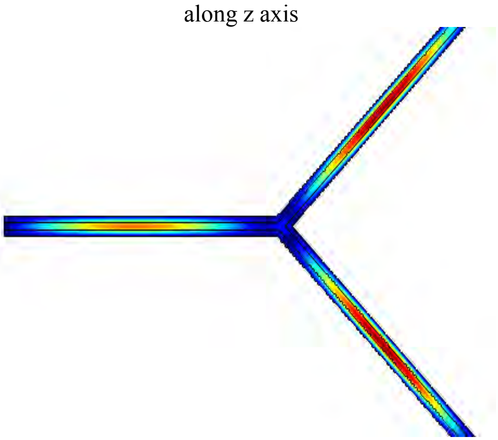

Fig.21 The second step of electric field distribution along the axis z

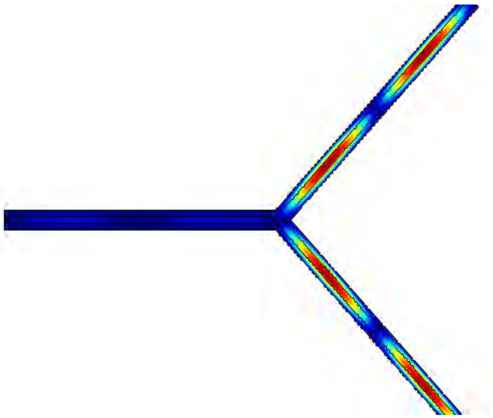

Fig.22 The third step of propagation along the z axis

Following the simulations it can be seen that the distribution and the intensity of electric field is identical on both sides for the fundamental mode. Changing the characteristics of the fiber changes the linearly polarized modes and the symmetrical distribution along the fiber.

5. Conclusions

According to the simulation through the single-mode fiber, the wave is transmitted in one way, without the appearance of modal noise. Through the multimode fiber can pass more light waves, but each with its particular linearly polarized mode.
These simulations will be developed to simulate Faraday Effect.

6. References

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