Numerical Sensitivity Analysis of a complex Glass Forming Process by means of local perturbations

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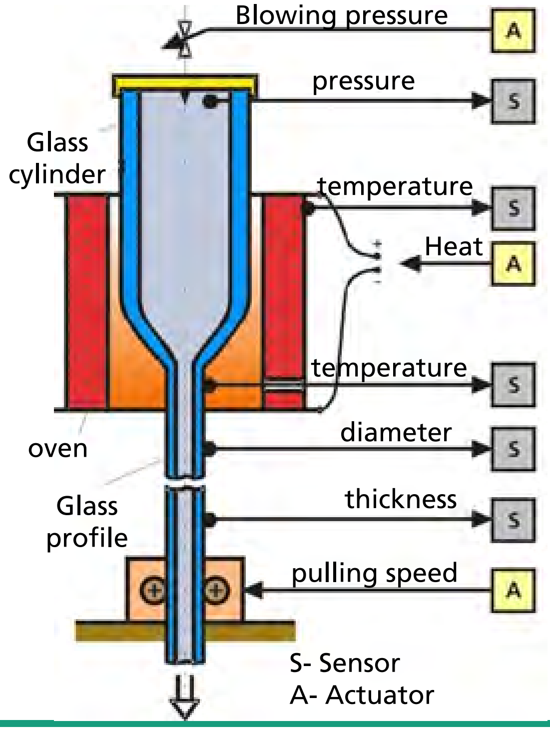
Excerpt from the Proceedings of the 2011 COMSOL Conference in Stuttgart

Motivation: Glass forming Process

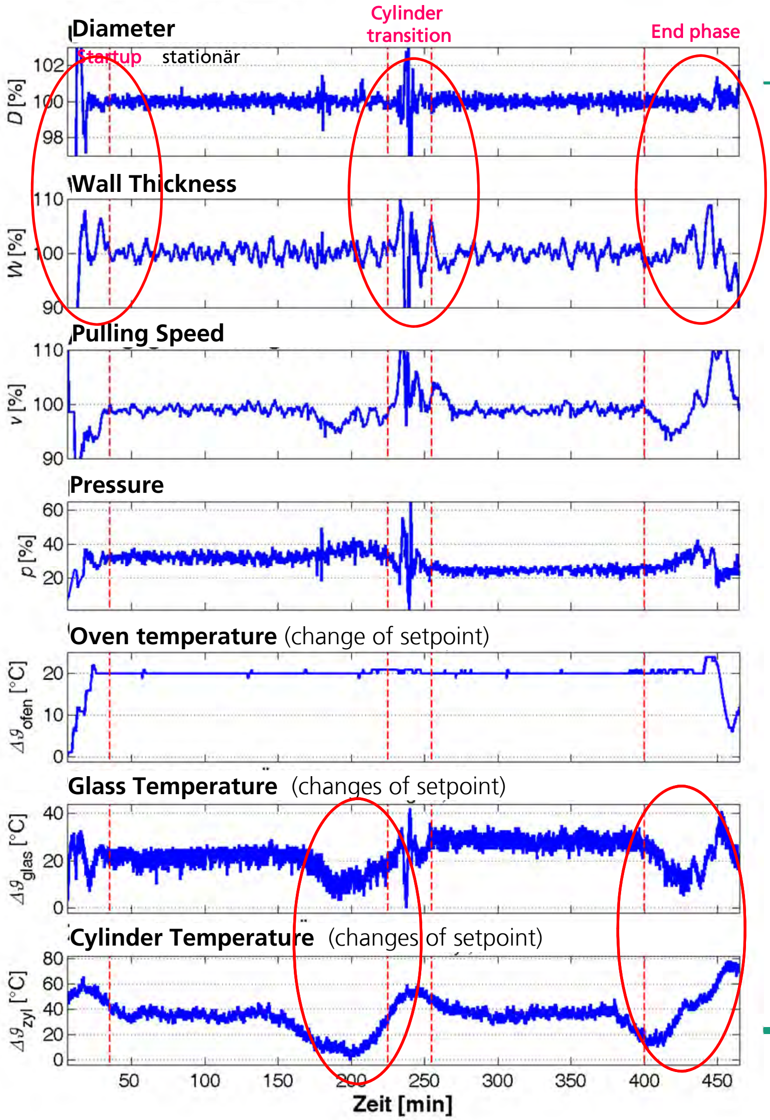
- Industrial glass forming batch process
- Resulting tubes / rods are pre-product for optical fibers
- Very high quality requirements (precise diameter and wall thickness)

Challenges:

- highly nonlinear process
  radiation
  material properties
- strong disturbances
  material inhomogenities
  material transitions



Disturbances of the Process

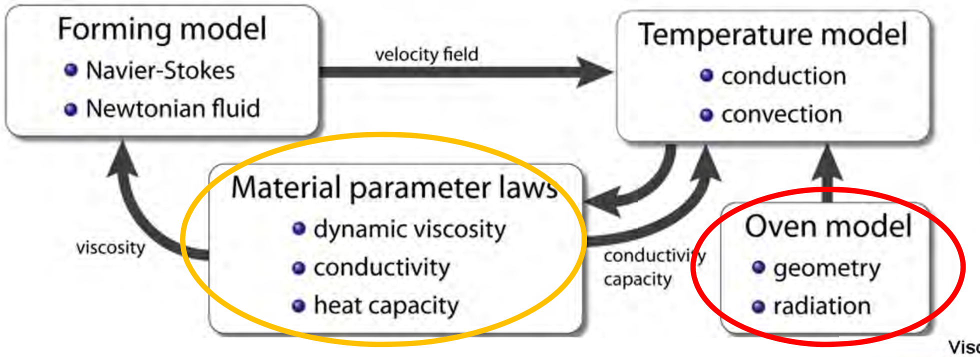


Strong disturbances in Startup phase, end phase and cylinder transition
→ additional radiation

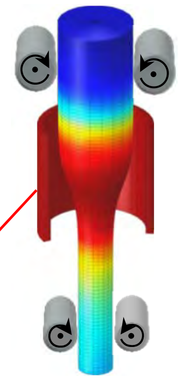
... causes temperature disturbances

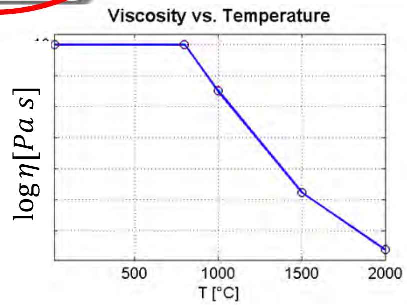
Aim: model based temperature stabilization

Equation for glass forming process
Trouton Model (1D)



- Navier-Stokes equation systems
  - 1 continuity Equation
  - 3 momentum balance (3D)
  - 1 Energy balance





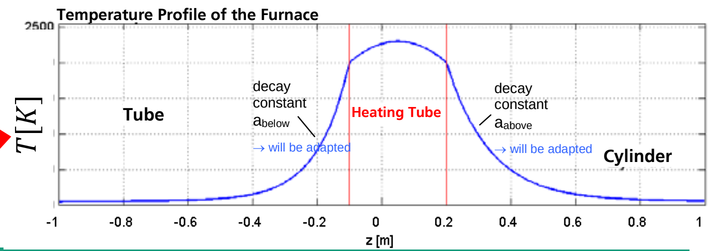
1D Equation for glass-rod

$$\frac{\partial A}{\partial t} + \frac{\partial w A}{\partial z} = 0$$

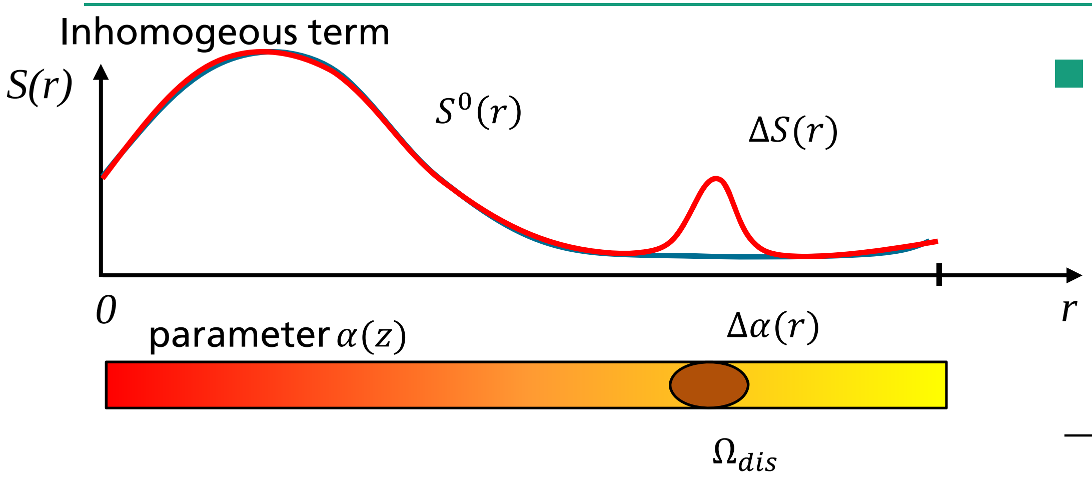
$$\frac{\partial}{\partial z}\left(3\mu(T) A \frac{\partial w}{\partial z}\right) = -\rho g A$$

$$A\rho\left(\frac{\partial C_p(T) T}{\partial t} + w\frac{\partial C_p T}{\partial z}\right) = \frac{\partial}{\partial z}\left(A k(T) \frac{\partial T}{\partial z}\right) + 2\pi R S_r$$

$$S_r = \epsilon\sigma\left(T_{oven}^4(z) - T^4(z)\right)$$



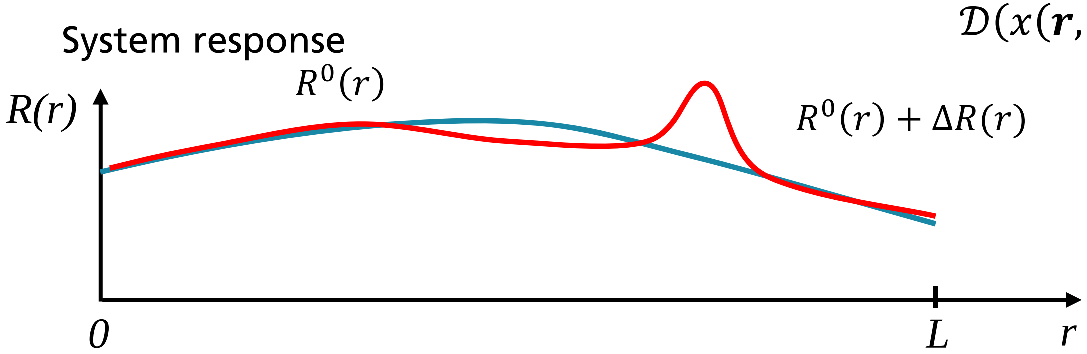
Local perturbation in Partial Differential Equation system



- Given distributed system (IBVP)
  - PDE $\mathcal{D}(x(\boldsymbol{r},t), s(\boldsymbol{r},t)|\boldsymbol{\alpha}) = 0$
  - BC $\mathcal{D}_r\big(x(\boldsymbol{r},t)\big) = b(t)$ for $\boldsymbol{r} \in \partial\Omega$
  - IC $\mathcal{D}_t(x(\boldsymbol{r}, t = 0) = a(\boldsymbol{r})$

→ nominal solution $R^0(x(\boldsymbol{r},t))$

$$\mathcal{D}(x(\boldsymbol{r},t), s(\boldsymbol{r},t) + \Delta s(\boldsymbol{r} \in \Omega_{\text{dis}}, t)|\boldsymbol{\alpha} + \Delta\alpha) = 0$$



The change by local variation compare to nominal solution

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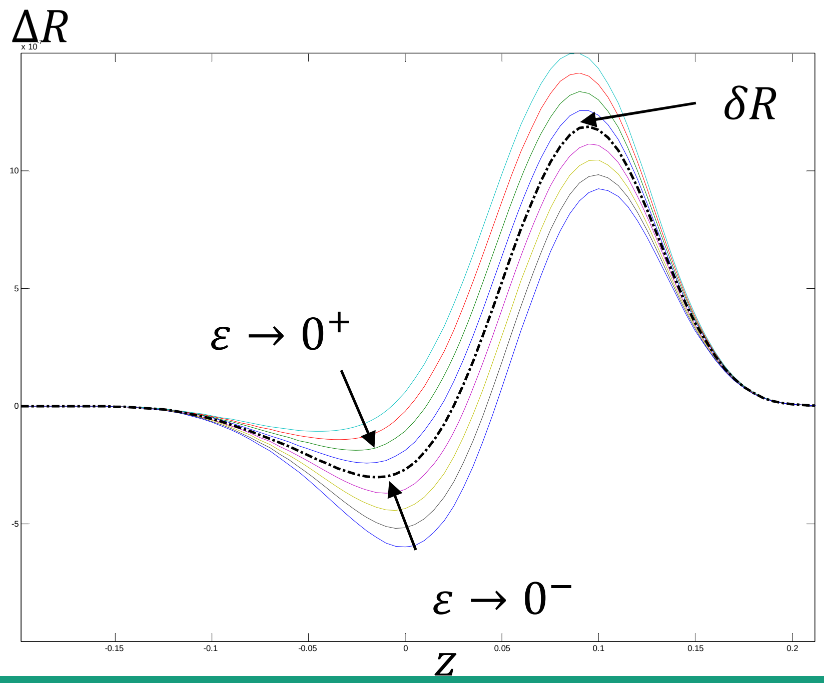
Numerical Sensitivity Analysis

Sensitivity = Gâteaux variation

$$\delta R(e^0; h) = \lim_{\varepsilon \to 0} \frac{R(e^0 + \varepsilon h) - R(e^0)}{\varepsilon}$$



- Nominal Solution $R(e^0)$, $e^0 = [x^0, \alpha^0]$

for perturbation h

- Solution with perturbation $R(e^0 + h)$,

■ Calculate the term for variation of ε around 0

$$\Delta R = \frac{R(e^0 + \varepsilon h) - R(e^0)}{\varepsilon}$$

■ Find the limit

$$\delta R(e^0; h) = \lim_{\varepsilon \to 0} \frac{R(e^0 + \varepsilon h) - R(e^0)}{\varepsilon}$$

Use of Comsol Multiphysics

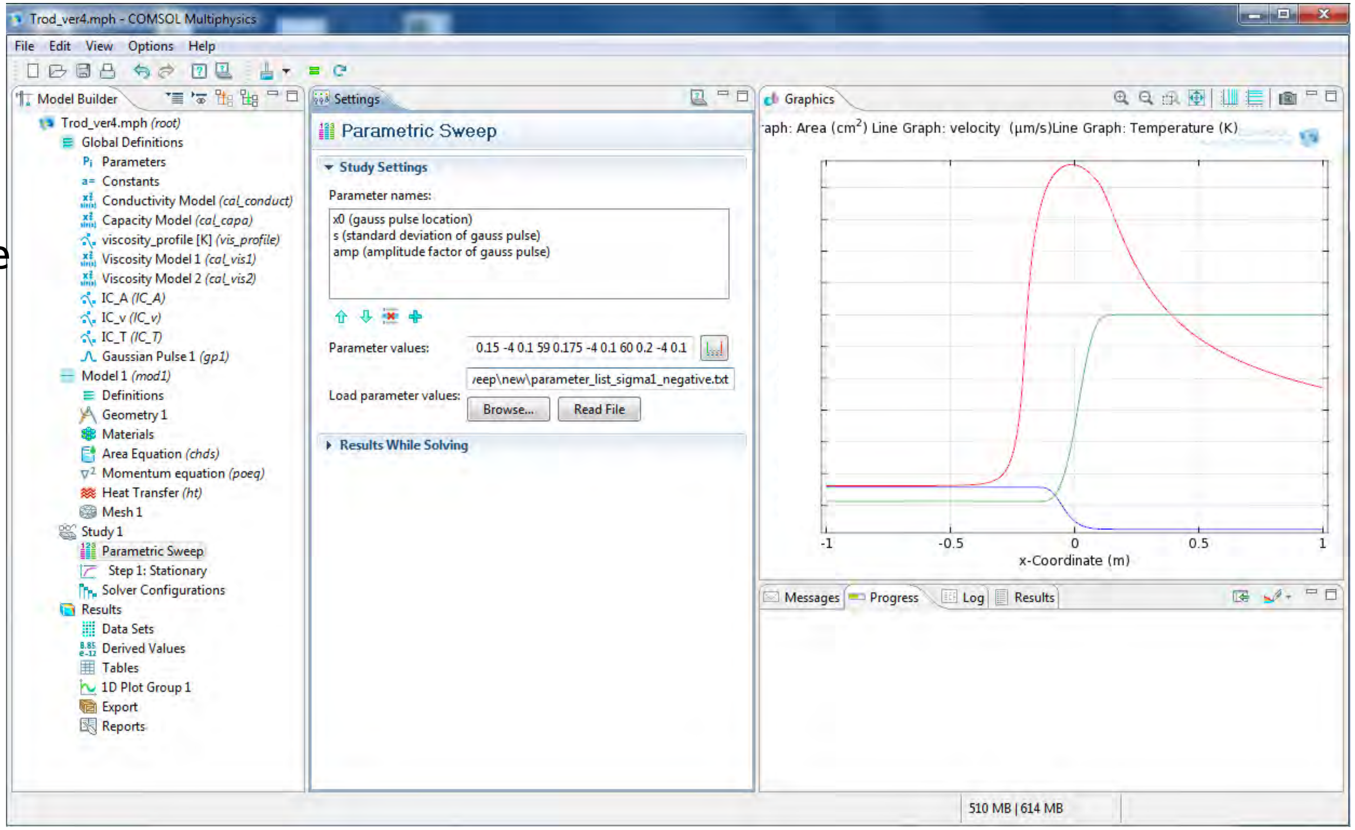
1D Model

3 Multiphysics

- Mass balance
- Momentum balance
- Energy balance

→ calculate $A(z,t), v(z,t), T(z,t)$

- with parameter sweep for variation $\varepsilon h(z)$



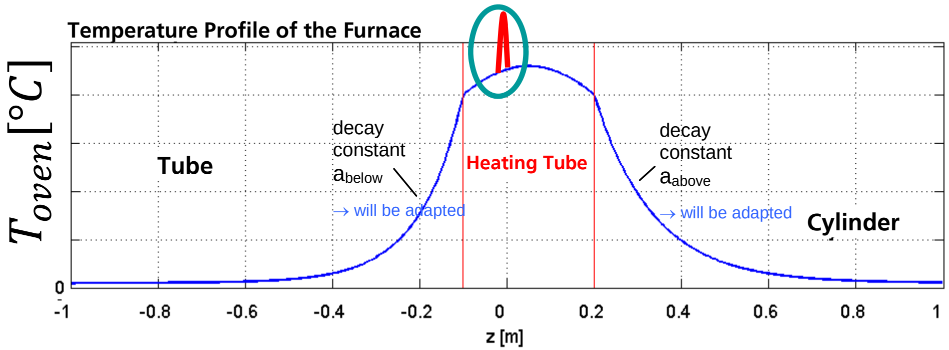
Disturbance analysis in Glass tube drawing process

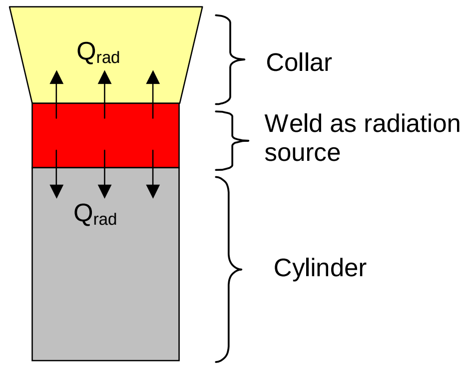
Spatial-description
- disturbance remains place fixed

Material-description
- disturbance moves with material

$$h(z(t)) = \Delta q(z(t), t)$$

$$h(z) = \Delta T_{oven}(z)$$





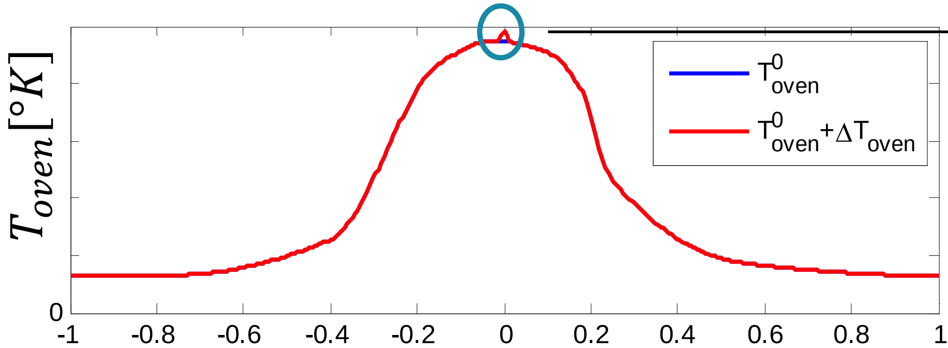
Stationary disturbance scenario

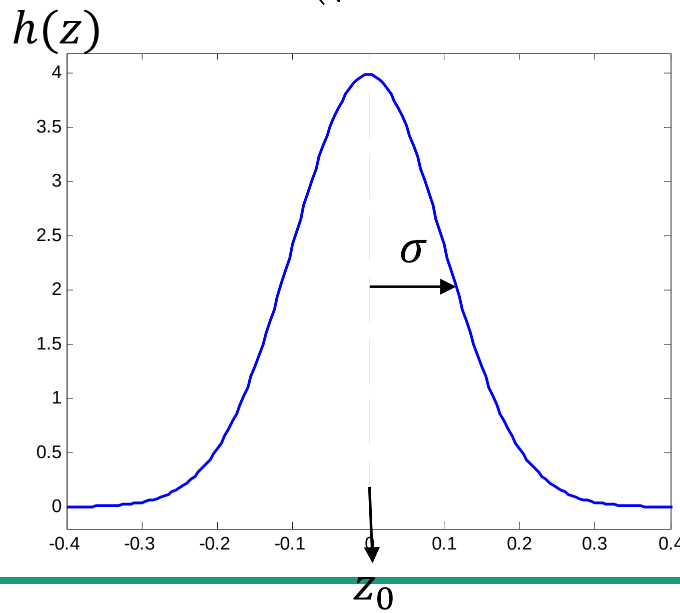
- fixed place disturbance

$$A\rho\left(\frac{\partial C_p(T)T}{\partial t} + w\frac{\partial C_p T}{\partial z}\right) = \frac{\partial}{\partial z}\left(Ak(T)\frac{\partial T}{\partial z}\right) + C\left(T_{oven}^4(z) - T^4(z)\right)$$

$$\Delta T_{oven}(z) = \varepsilon h(z) = \varepsilon\left(\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{z-z_0}{\sigma}\right)^2\right)\right)$$

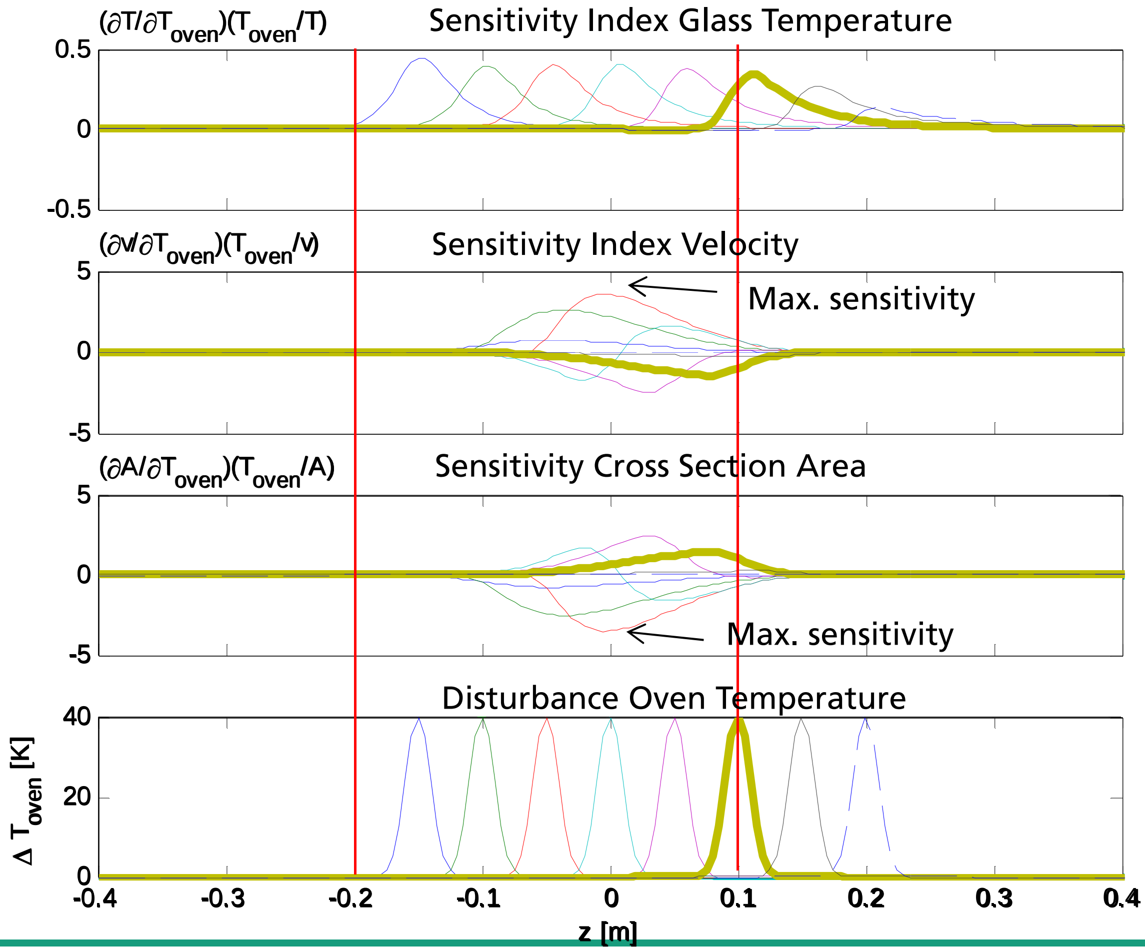
$$T_{oven}(z) = T_{oven}^0(z) + \Delta T_{oven}(z)$$





Sensitivity Index

$$SI = \frac{\Delta R/R}{\Delta h/h} = \frac{\partial R}{\partial h}\frac{h}{R}$$



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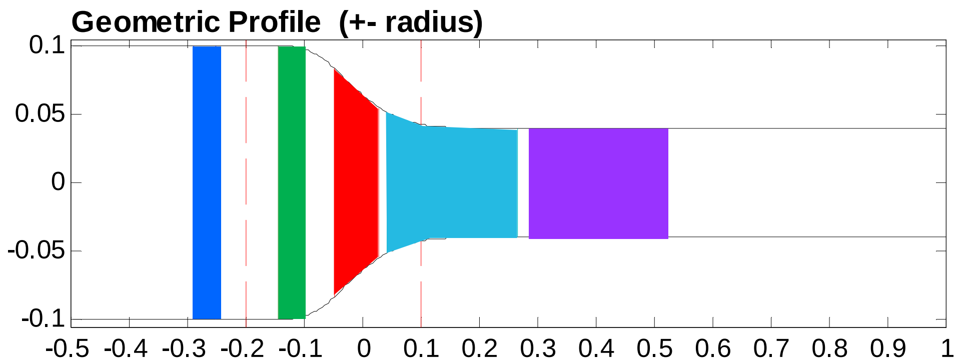
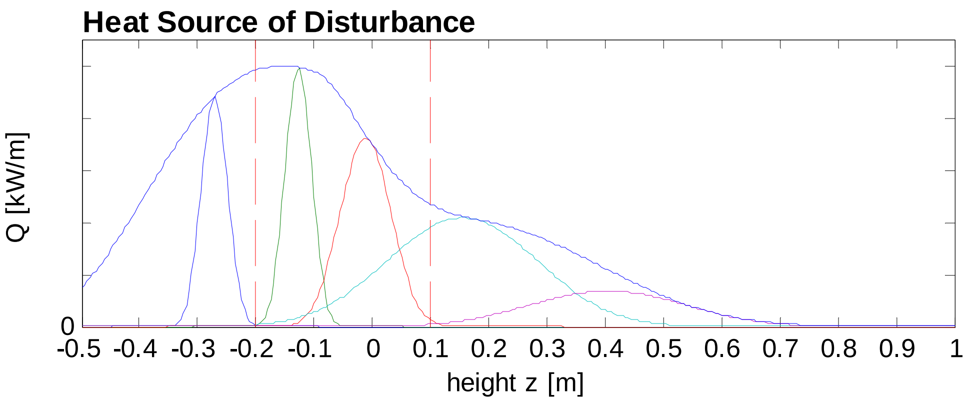
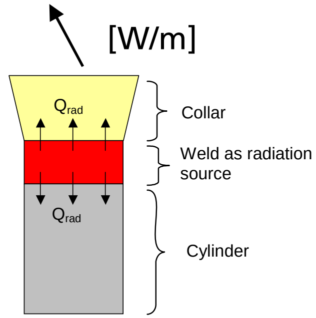
Transient disturbance scenario (Moving welding point)





$$A\rho\left(\frac{\partial C_p(T)T}{\partial t} + w\frac{\partial C_p T}{\partial z}\right) - \frac{\partial}{\partial z}\left(Ak(T)\frac{\partial T}{\partial z}\right) =$$
$$2\pi R\epsilon\sigma\left(T_{oven}^4(z) - T^4(z)\right) + Q_{dis}(z,t)$$



$$\int_{z_l}^{z_r} Q_{dist}\,dz = Q_0 = const.$$

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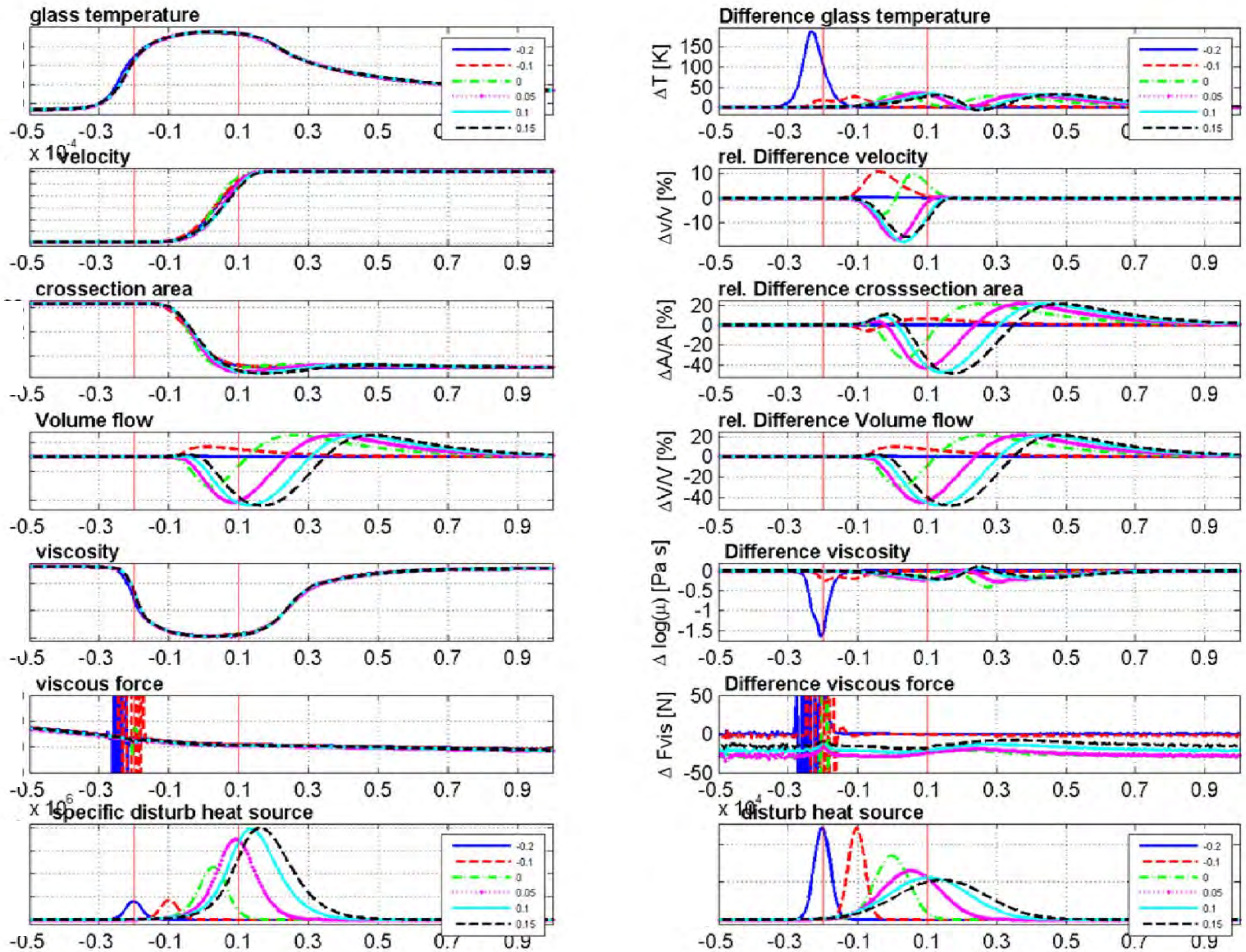
Simulation of moving welding point (snapshots)



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Conclusion and Future work

- Approach: Method to compute the Local sensitivity δR of system response R with regarding to variation h
- 2 scenarios of disturbance
  - Stationary disturbance (spatial fixed)
    - Improved process comprehension
  - Transient disturbance (material fixed)
    - Close to reality process behavior
- Future works:
  - Process optimization -> Find optimal control strategies to minimize of the welding point effect
  - Parameter estimation (Disturbance, Oven Profile)