

# COMSOL CONFERENCE 2019 BOSTON

## Phase Field Modelling of Gas Migration in Bentonite Based Barrier Materials

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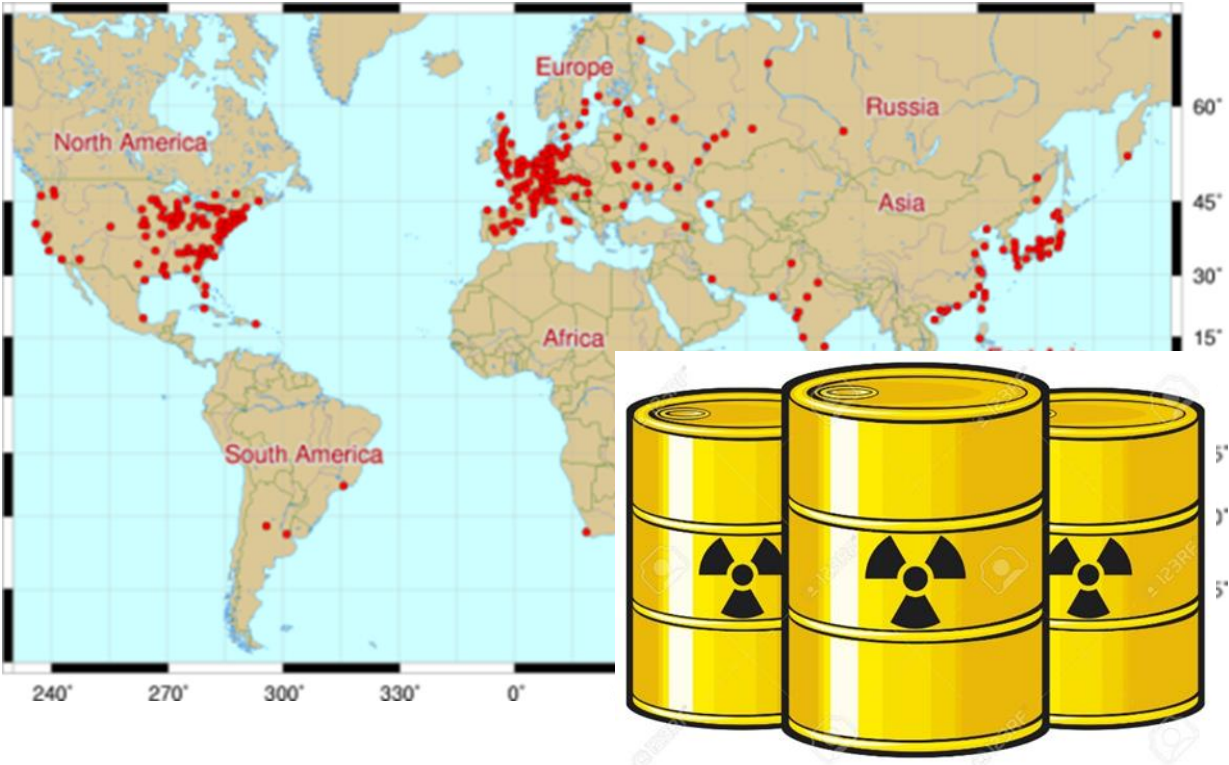
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# OUTLINE

- 1** Introduction
- 2** Conceptual Coupled HM-PF Model
- 3** Numerical Implementation
- 4** Simulation Results
- 5** Conclusions

# 1 Introduction



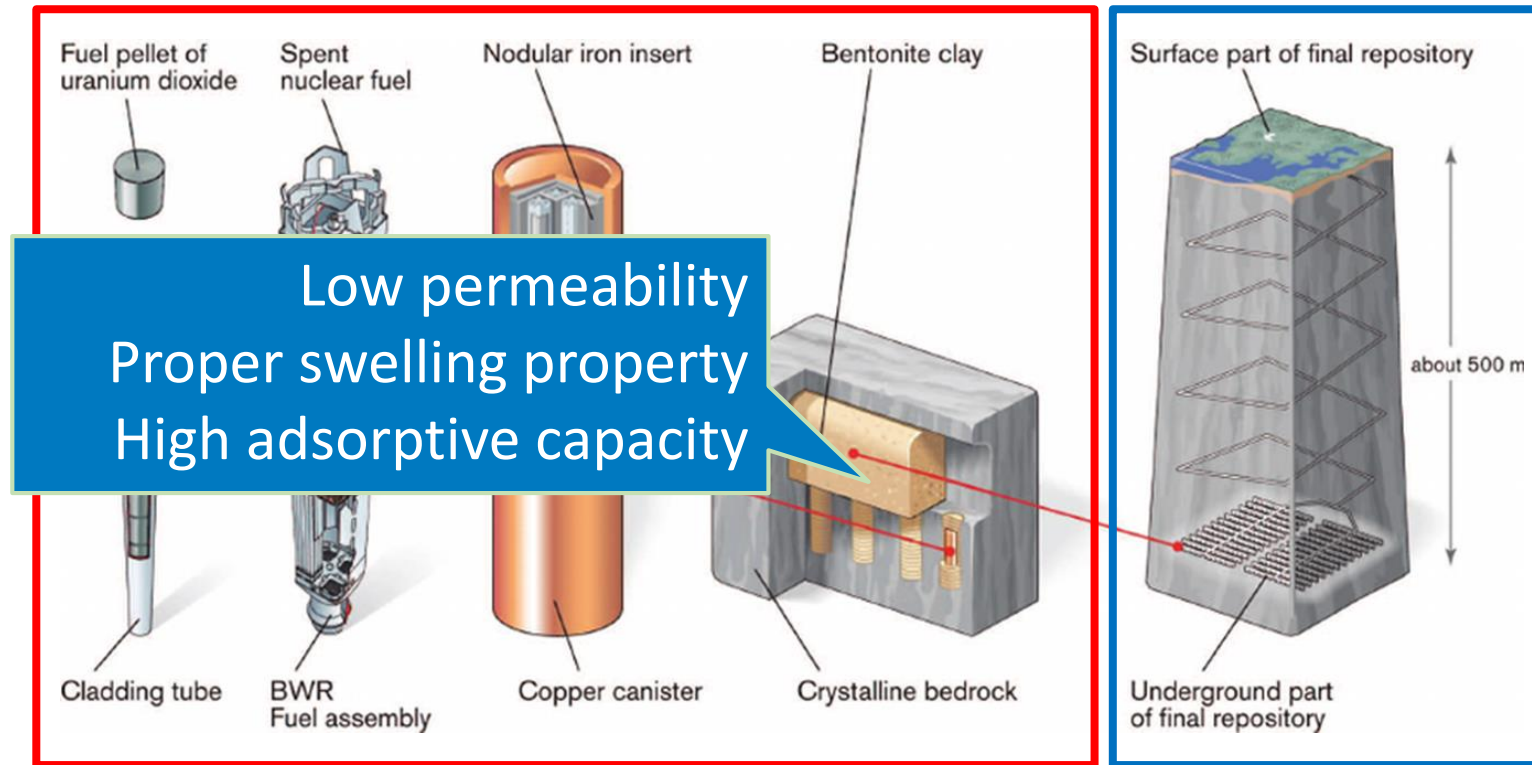
Distribution of Nuclear Power Plants

(From Wikipedia)

# 1 Introduction

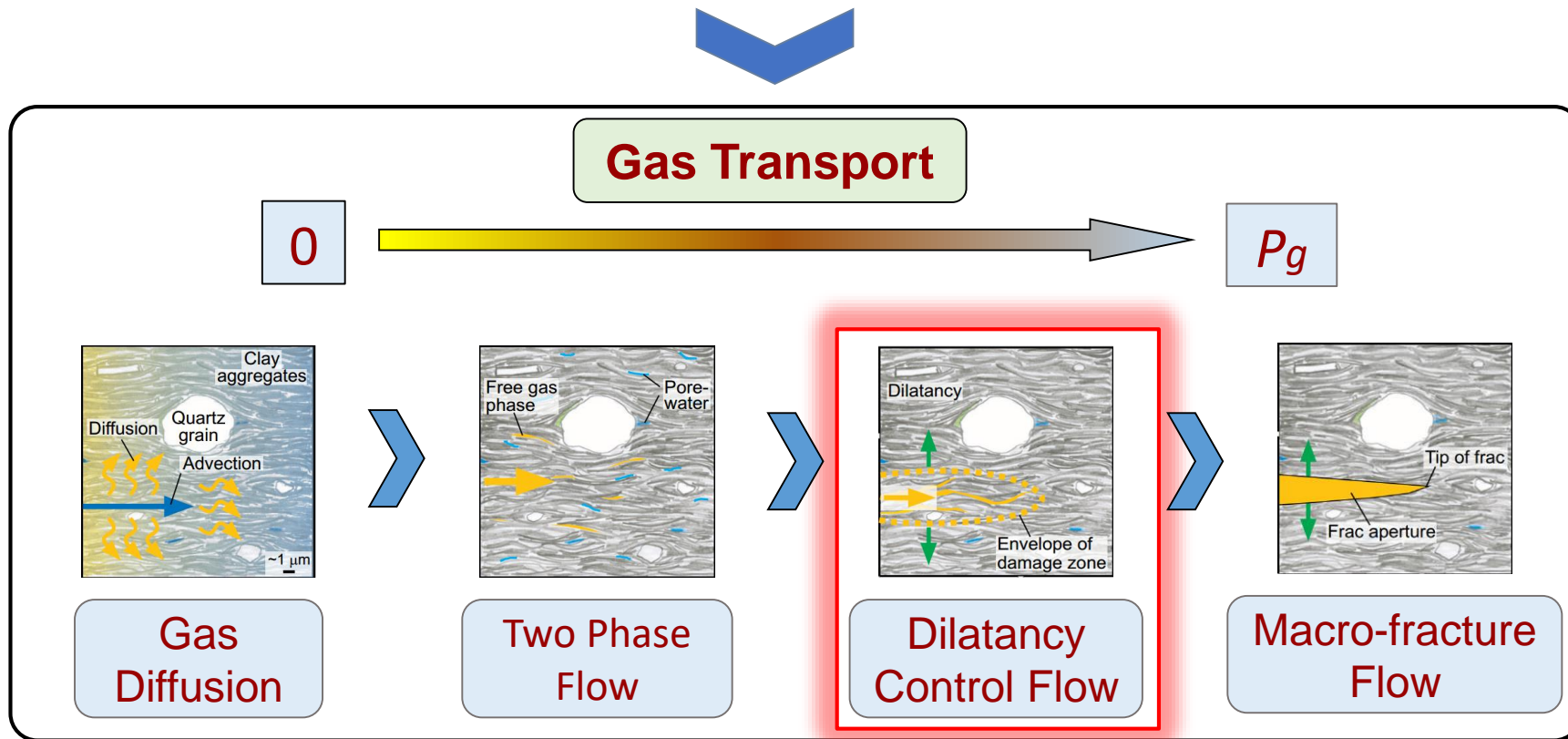
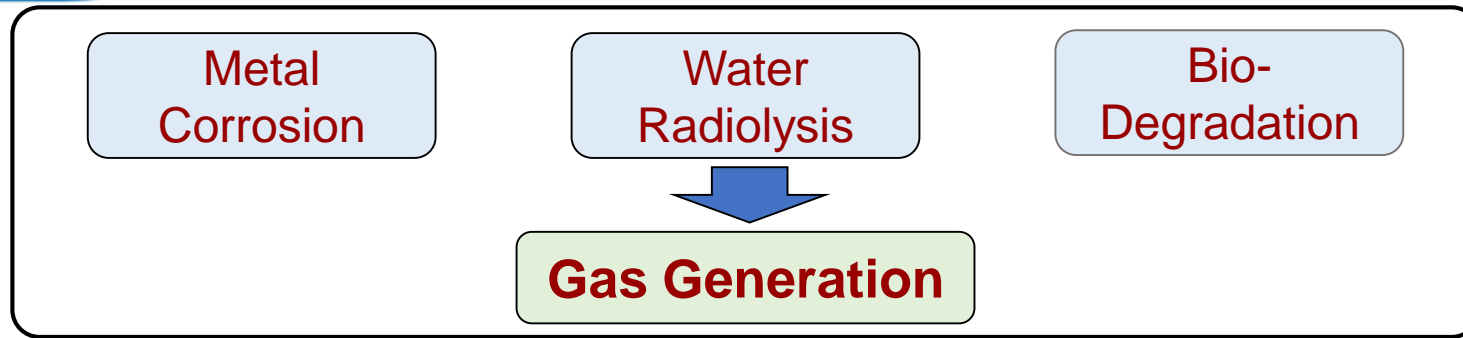
## Engineering Barrier System

## Natural Barrier System



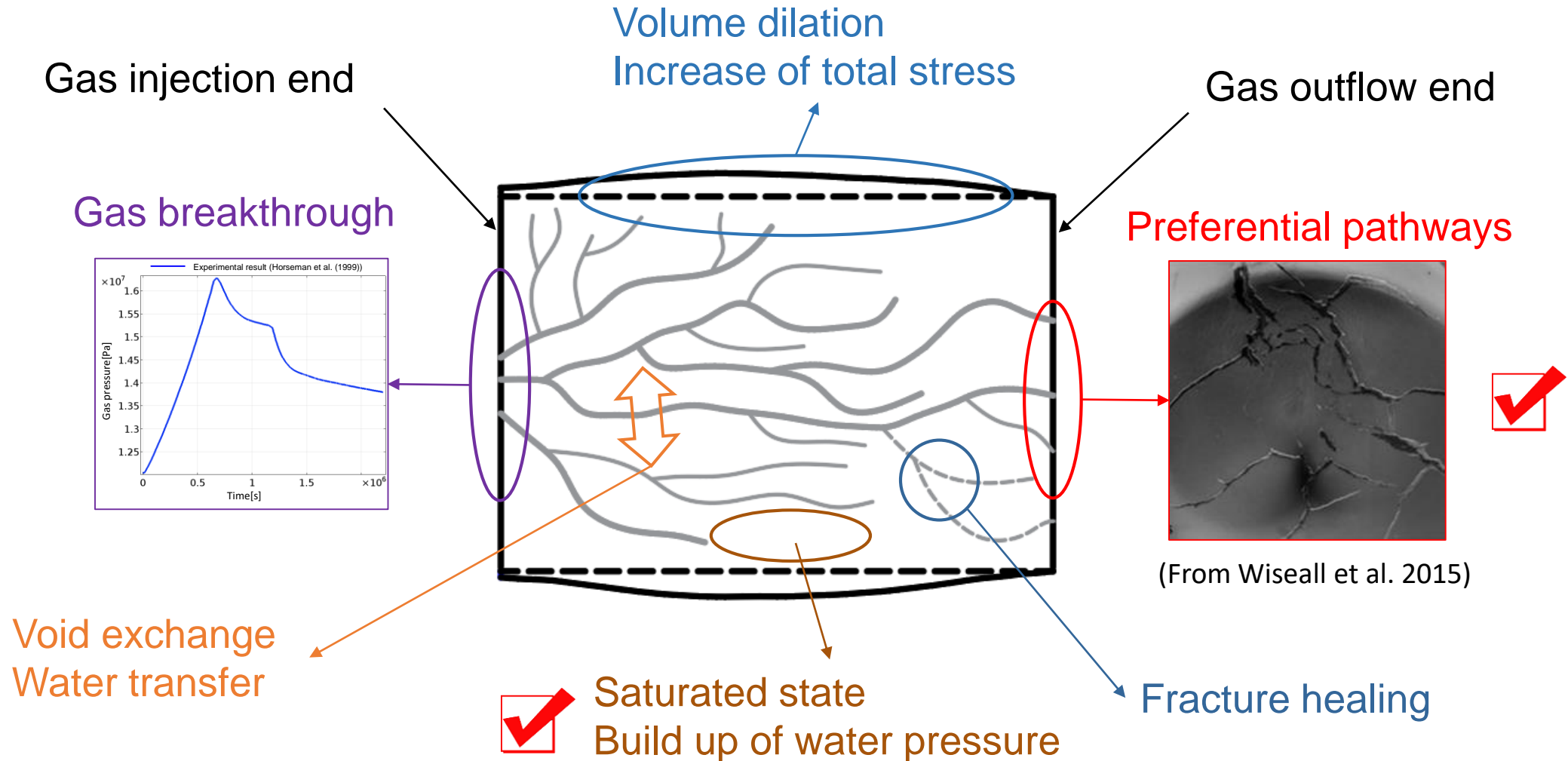
## Multi-barrier system of KBS-3 repository

(From Harrington, J., and Horseman, S. 2003)

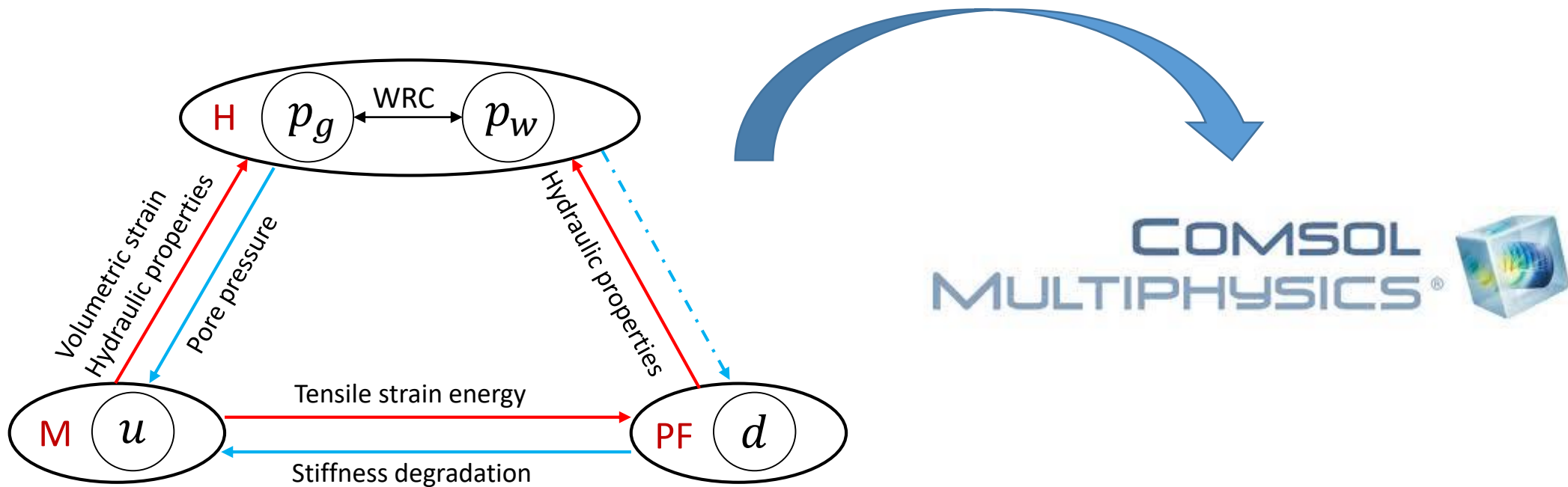


\*Images from Marschall et al. (2005)

# 2 Conceptual Coupled HM-PF Model



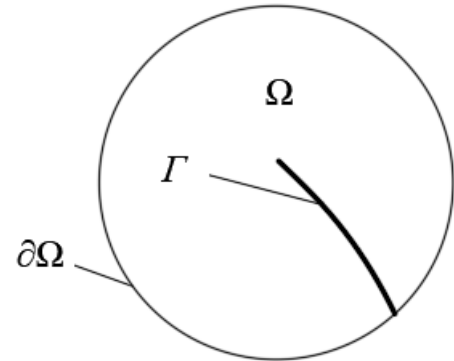
## ➤ Couplings Between Different Physical Field



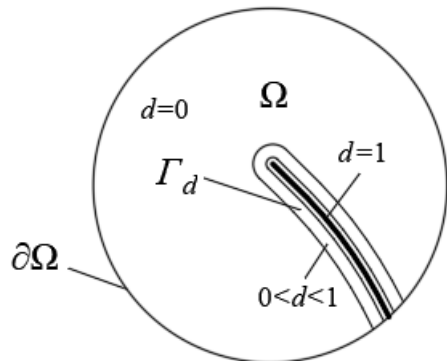
(Modified from Guo, G. & Fall, M. 2019)

# 3 Numerical Model and Implementation

## ➤ Phase Field Method



$$\gamma(d, \nabla d) = \frac{1}{2l} d^2 + \frac{l}{2} |\nabla d|^2$$



### Variational Principle of Free Energy Minimization

$$(1-d)H_M^+ - (d - l^2 \nabla^2 d) = 0 \quad H_M^+ = \max_{\tau \in [0, t]} \left\{ \left\langle \frac{\psi_0^{e+}}{\psi_{cr}} - 1 \right\rangle_+ \right\}$$

Equation

Show equation assuming:

Study 1, Time Dependent

$$e_a \frac{\partial^2 d_{pf}}{\partial t^2} + d_a \frac{\partial d_{pf}}{\partial t} + \nabla \cdot (-c \nabla d_{pf} - \alpha d_{pf} + \gamma) + \beta \cdot \nabla d_{pf} + a d_{pf} = f$$

$$\nabla = \left[ \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right]$$

Diffusion Coefficient

c 1

Isotropic

Absorption Coefficient

a  $(1+H_w)/L_{ph}^2$

Source Term

f  $H_w/L_{ph}^2$

Equation

Show equation assuming:

Study 1, Time Dependent

$$e_a \frac{\partial^2 W_m}{\partial t^2} + d_a \frac{\partial W_m}{\partial t} = f$$

Source Term

f  $W_m - \text{nojac}(\text{if}(W_{plus} < W_m, W_m, W_{plus}))$

Damping or Mass Coefficient

$d_a$  0

Previous Solution 1



## ➤ Mechanical Model

- Solid Mechanics (fcon)
  - Linear Elastic Material 1
    - External Stress 1
    - Initial Stress and Strain 1
    - Equation View

$$\nabla \cdot \left[ g(d) \boldsymbol{\sigma}'^+ (\boldsymbol{\varepsilon}) + \boldsymbol{\sigma}'^- (\boldsymbol{\varepsilon}) - \bar{p} \mathbf{I} \right] + \rho \mathbf{g} = \mathbf{0}$$

External Stress

Stress input:

Pore pressure

Absolute pressure:

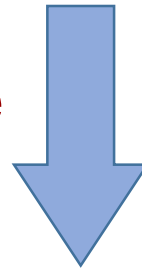
$p_A$  User defined

p\_ave

- Solid Mechanics (fcon)
  - Linear Elastic Material 1
    - External Stress 1
    - Initial Stress and Strain 1
    - Equation View

$$\boldsymbol{\sigma}'^\pm (\boldsymbol{\varepsilon}) = \sum_{a=1}^3 \left[ \lambda \langle \text{tr}(\boldsymbol{\varepsilon}) \rangle_{\pm} + 2\mu \langle \varepsilon_a \rangle_{\pm} \right] \mathbf{n}_a \otimes \mathbf{n}_a$$

Replace



|           |   |                  |
|-----------|---|------------------|
| fcon.Jel  | $\text{sqrt}(\text{fcon.Cel11} * \text{fcon.Cel22} * \text{fcon.Cel33})$  | 1                |
| fcon.SI11 | $\text{fcon.Sil11} + \text{fcon.D11} * \text{fcon.eel11} + \text{fcon.D12} * \text{fcon.eel12} + \text{fcon.D13} * \text{fcon.eel13}$ | N/m <sup>2</sup> |
| fcon.SI12 | $\text{fcon.Sil12} + \text{fcon.D14} * \text{fcon.eel11} + \text{fcon.D15} * \text{fcon.eel12} + \text{fcon.D16} * \text{fcon.eel13}$ | N/m <sup>2</sup> |
| fcon.SI13 | $\text{fcon.Sil13} + \text{fcon.D16} * \text{fcon.eel11} + \text{fcon.D17} * \text{fcon.eel12} + \text{fcon.D18} * \text{fcon.eel13}$ | N/m <sup>2</sup> |
| fcon.SI22 | $\text{fcon.Sil22} + \text{fcon.D12} * \text{fcon.eel11} + \text{fcon.D13} * \text{fcon.eel12} + \text{fcon.D14} * \text{fcon.eel13}$ | N/m <sup>2</sup> |
| fcon.SI23 | $\text{fcon.Sil23} + \text{fcon.D15} * \text{fcon.eel11} + \text{fcon.D16} * \text{fcon.eel12} + \text{fcon.D17} * \text{fcon.eel13}$ | N/m <sup>2</sup> |
| fcon.SI33 | $\text{fcon.Sil33} + \text{fcon.D13} * \text{fcon.eel11} + \text{fcon.D17} * \text{fcon.eel12} + \text{fcon.D18} * \text{fcon.eel13}$ | N/m <sup>2</sup> |

Strain

- Principal strain directions
  - Principal strain direction 1 (material and geometry frames)
  - Principal strain direction 2 (material and geometry frames)
  - Principal strain direction 3 (material and geometry frames)

Principal strains

- fcon.ep1 - First principal strain
- fcon.ep2 - Second principal strain
- fcon.ep3 - Third principal strain

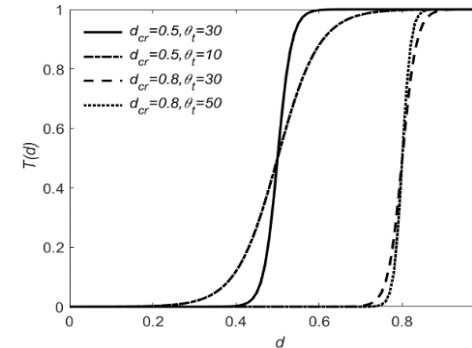
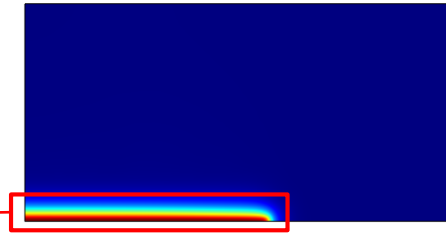
## ➤ Hydraulic Model

$$\rho_{\kappa} \phi \left( \frac{S_{\kappa}}{K_{\kappa}} - \frac{\partial S_e}{\partial p_c} \right) \frac{\partial p_{\kappa}}{\partial t} + \nabla \cdot (\rho_{\kappa} \mathbf{v}_{\kappa}^D)$$

$$= -S_{\kappa} \rho_{\kappa} \frac{\partial \varepsilon_v}{\partial t} - \rho_{\kappa} \phi \frac{\partial S_e}{\partial p_c} \frac{\partial p_{\kappa}'}{\partial t}$$

- Gas migration (pg)
  - Fluid and Matrix Properties 1
  - No Flow 1
  - Initial Values 1
  - Storage Model 1
  - Mass Source 1

$$\mathbf{v}_{\kappa}^D = -\frac{k_{in} k_{r\kappa}}{\mu_{\kappa}} (\nabla p_{\kappa} - \rho_{\kappa} \mathbf{g})$$



$$k_{fin} = C_k k_{pin} *$$

$$k_{frw} = \frac{1}{2} S_w^2 (3 - S_w)$$

$$k_{frg} = S_g^3 + \frac{3}{2M} S_g (1 - S_g^2)$$

$$S_{fe} = \left[ 1 + \left( \frac{p_c}{p_{fgev}} \right)^n \right]^{-m}$$

$$k_{pin} *$$

$$k_{prw} = \sqrt{S_e} \left[ 1 - (1 - S_e^{1/m})^m \right]^2$$

$$k_{prg} = \sqrt{1 - S_e} (1 - S_e^{1/m})^{2m}$$

$$S_{pe} = \left[ 1 + \left( \frac{p_c}{p_{pgev}} \right)^n \right]^{-m}$$

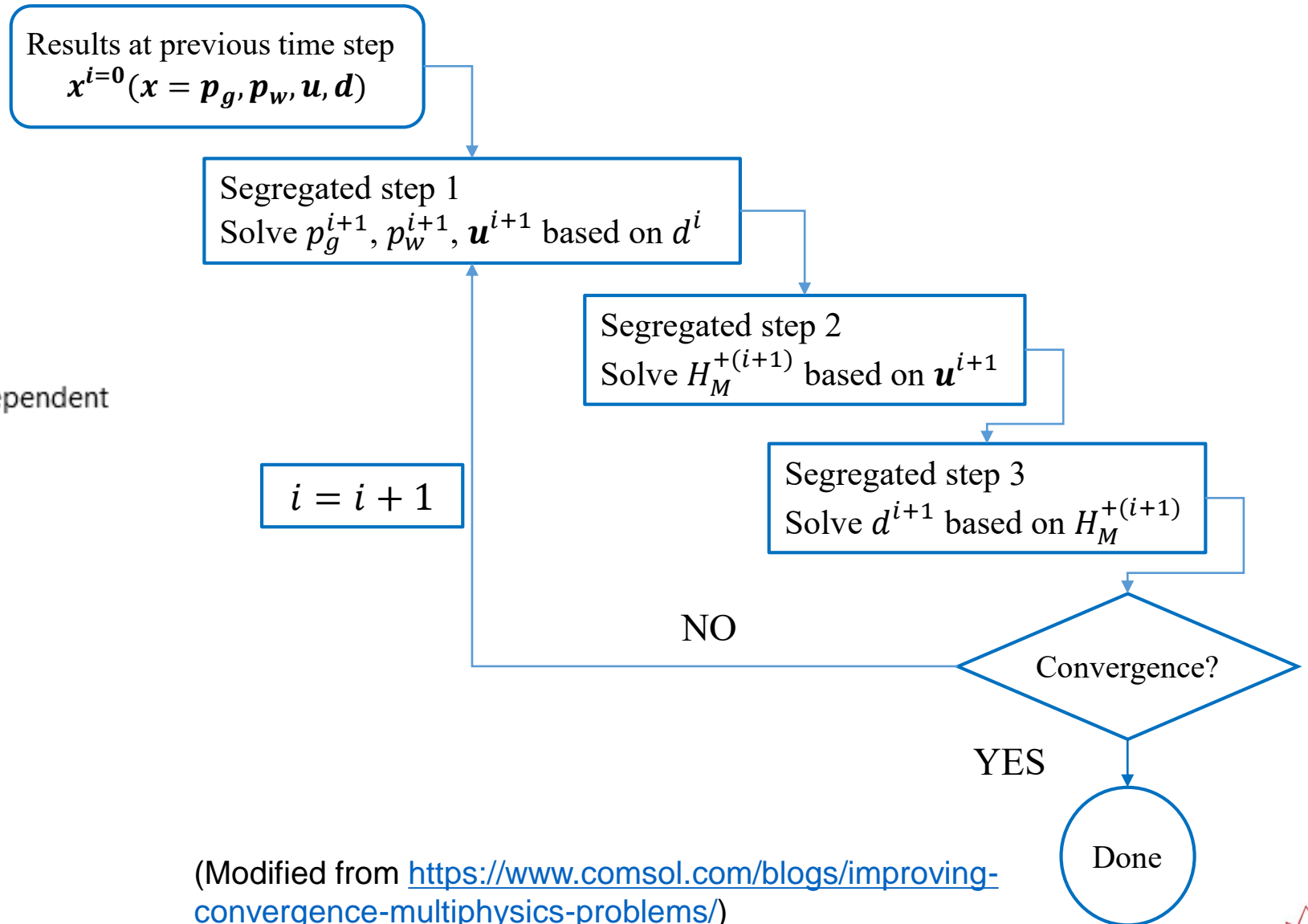
Transition function

$$T(d) = \frac{1}{2} \left\{ \tanh \left[ \theta_t (d - d_{cr}) \right] - \tanh (-d_{cr} \theta_t) \right\}$$

\* (From Guo, G. & Fall, M. 2019)

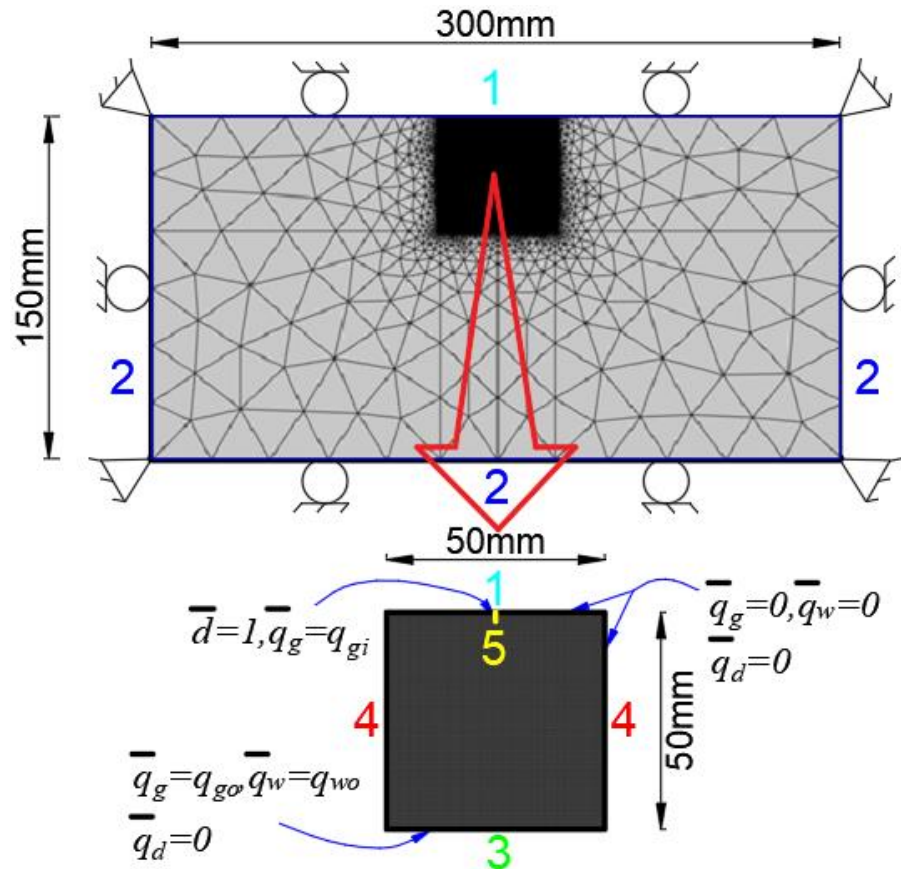
## ➤ Solver Settings

- Study 1
  - Step 1: Time Dependent
    - Solver Configurations
      - Solution 1 (sol1)
        - Compile Equations: Time Dependent
          - Dependent Variables 1
            - Time-Dependent Solver 1
              - Direct
              - Advanced
              - Fully Coupled 1
              - Segregated 1
                - Segregated Step 1
                - Segregated Step 2
                - Segregated Step 3
              - Previous Solution 1



(Modified from <https://www.comsol.com/blogs/improving-convergence-multiphysics-problems/>)

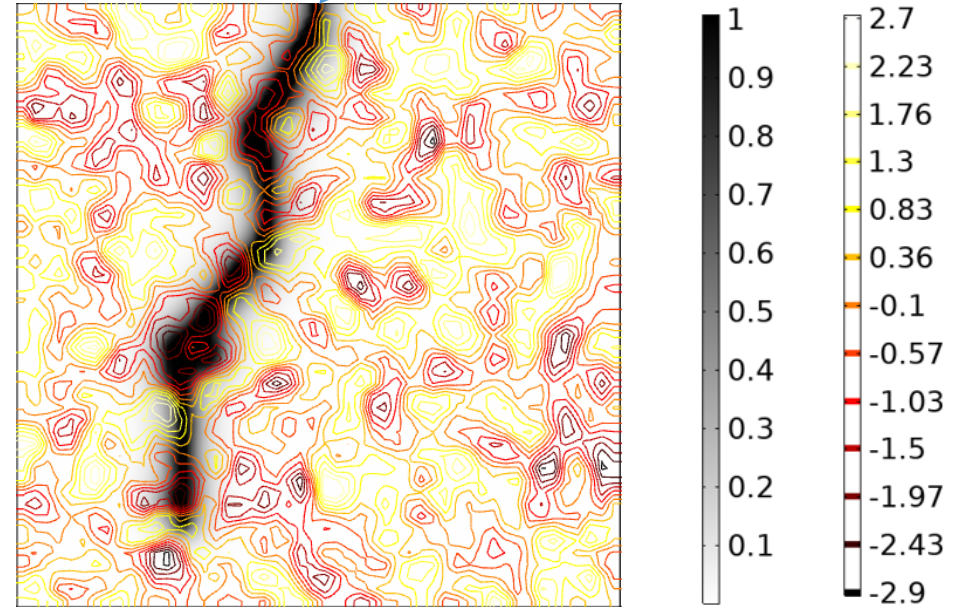
# 4 Simulation Results



- Meshing and boundary conditions

(Modified from Guo, G. & Fall, M. 2019)

Preferentially propagate through areas of low resistance



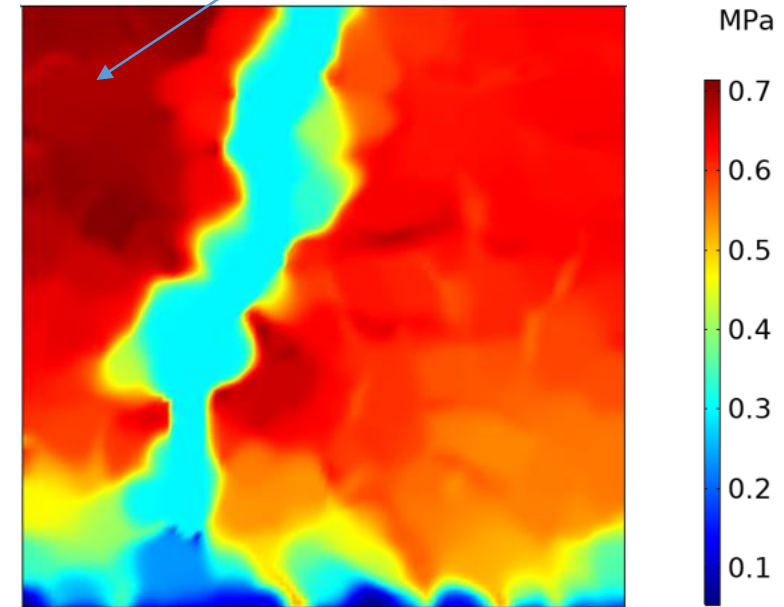
- Fracture trajectory (Phase field) in the heterogeneous domain

Preferential gas flow in the developed fracture



- Distribution of gas pressure (scaled by its degree of saturation)

Rise of water pressure during fracturing process



- Distribution of water pressure

## 4 Conclusions

- The developed coupled HM-PF model is successfully implemented into COMSOL by using Solid Mechanics Module, Darcy's Law Module, Coefficient Form PDE, Domain ODEs and DAEs and the Previous Solution Node.
- The developed model has satisfactorily described some HM behaviors observed in experiments, such as the development of preferential pathways, the localized gas flow and the rise of water pressure.

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Thank you for your attention!

