Simulated Annealing and Genetic Algorithm Optimization using COMSOL Multiphysics: Applications to the Analysis of Ground Deformation in Active Volcanic Areas

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Abstract: We combine the potentiality of COMSOL with Monte Carlo optimization procedures, referred to as Simulated Annealing and Genetic Algorithm, in order to analyze and interpret ground deformation measured in active volcanic areas. Through MATLAB® subroutines, we use FE models that include complexities like topography, vertical and lateral heterogeneities and time-dependent material properties. Preliminary results of an application to a real case show that our approach may help to better represent volcanic processes and lead to a more accurate interpretation and understanding of surface deformation in active volcanic areas.

Keywords: Simulated Annealing, Genetic Algorithm, Optimization procedures, Ground deformation, Earth Science.

1. Introduction

Ground deformation signals in volcanic areas are the expression of near-surface and/or deep-seated physical processes. As most of the geophysical analysis, the interpretation of the deformation data is usually performed setting up inverse problems, which often use Monte Carlo optimization techniques like the Simulated Annealing and the Genetic Algorithm, in order to constrain the nature of the causative sources at depth (Dzurisin, 2006). Usually, these methods exploit the problem's solution space iterating forward analytical models, which consider simplified geometries and homogeneous isotropic distribution of the material properties. This approach is preferred because of the straightforward analytical relationships, and also because up to recent times the quality of geodetic data was not high enough to justify more complex models.

Nowadays, surface deformations can be measured by means of a range of geodetic techniques, reaching in most cases sub-centimetric accuracy. Especially space-based remote sensing techniques as continuous GPS and Differential SAR Interferometry (DInSAR) increased the spatial and temporal resolution of our observation of deformation processes related to events as earthquakes and volcanic unrest (Cervelli et al., 2001). In addition, several recent studies have shown that oversimplified forward models may lead to misinterpretations of the retrieved source parameters (e.g. Manconi et al., 2007).

Finite Element (FE) method is a powerful numerical tool that allows implementing models with complex geometries, material heterogeneities, as well as time dependent physical processes. For this reason, FE models are a suitable candidate to fill the gap between the accuracy achieved on the observation of ground deformation in volcanic areas and the models used for its interpretation. Nevertheless, due to several limitations, as for example the large computational capabilities often needed for such procedures, the use of FE models within optimization analyses is still a challenging task.

In this work, we present an implementation of COMSOL models within Monte Carlo optimization procedures, referred to as Simulated Annealing and Genetic Algorithm, through MATLAB® subroutines. After a brief description of the optimization algorithms and of the procedure herein adopted for their implementation within COMSOL analyses, we present and discuss preliminary results relevant to the interpretation of the deformation pattern observed via DInSAR on Tenerife, Canary Islands.

2. Optimization problem in surface deformation analyses

As mentioned in the introduction, usually the analyses of the surface deformation in volcanic areas try to constrain parameters of a causative source iterating forward analytical models, thus exploiting the problem's solution space. Indeed, the "optimum" solution is selected evaluating the misfit between the observation and the synthetic modeled displacements through an arbitrary cost function. Derivative-based algorithms, as the Levenberg-Marquardt or the Newton method, offer an efficient approach to solve such an optimization problem. However, since these algorithms move "down-hill" depending on the gradient of the misfit space, they can get easily trapped in the first local minimum and never find the global solution. For this reason, these algorithms work well only when initial guess of the parameters is constrained by a priori information. Since the misfit space in surface deformation analysis presents often several local minima, the so-called "Monte Carlo algorithms" are preferred. In fact, this class of optimization's procedures includes an element of randomness that allows "escaping" from local minima, increasing the chance to achieve a more accurate solution of the problem. For a detailed treatment of optimization of surface deformation's source parameters, we refer the reader to the paper of Cervelli et al., 2001. In the following, we describe the basic features of two of the most popular algorithms of the Monte Carlo class used in the analysis and interpretation of geodetic signals: the Simulated Annealing and the Genetic Algorithm.

2.1 Simulated Annealing (SA)

SA optimization algorithm is based on the concept of annealing in metallurgy, a technique involving heating and controlled cooling of a material. At first, bounds for the parameters to be optimized are imposed, and an initial model accordingly generated. By analogy with the annealing physical process, each step of the SA replaces the current solution by a random "nearby" solution, chosen with a probability that depends on the cost function values and on a global parameter referred to as Temperature (Temp, see also Figure 1). The latter is decreased during the process following a predefined cooling scheme. The dependency is such that the current solution changes almost randomly when Temp is large, but as Temp goes to zero solutions with lower cost are favored.

Details about the SA strategy and applications can be found in Kirkpatrick et al., 1983. In our work, we use an adapted version of the MATLAB® implementation of SA provided by and J. Vandekerckhove, (2006) available at http://www.mathworks.com/matlabcentral/fileexchange/10548.

2.2 Genetic Algorithm (GA)

GA optimization approach is based on the theories of biological evolution. By analogy, the algorithm starts with an initial set of models (population), which is randomly generated considering the a priori imposed parameter's bounds. Among this population, the "best" is selected as the model that minimizes an arbitrary cost function. Numerical operators as mutation and chromosome's crossover (recombination) act on best individuals, and consent to breed a new population of "evolved" individuals, i.e. only models that survived the precedent selection may reproduce and go ahead to the next step (generation). The procedure is thus iterated until a maximum number of generations allowed (see also Figure 1).

Details about the GA strategy and applications can be found in Holland, 1975. In our work we use an adapted version of the MATLAB® implementation of GA provided by K. Burjorjee, (2007) and available at http://www.mathworks.com/matlabcentral/fileexchange/15164.

3. Implementation of GA and SA within COMSOL analyses

Intrinsically, the accuracy of the final result of an optimization process is related on the forward model considered for the analysis. For this reason, an oversimplification of the problem, such as the standard homogeneous and isotropic assumption, might lead to misinterpretations. For this reason, we propose to use the FE model constructed with COMSOL as substitute of the standard analytical forward models in the

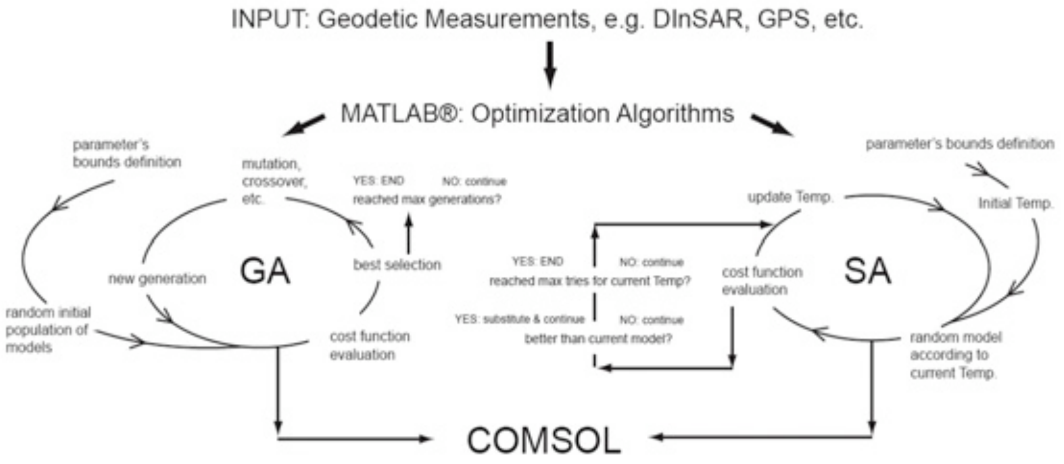

Figure 1. Flowchart representing the steps of optimization strategies as SA and GA and their coupling with COMSOL analysis.

optimization problem. The flowchart in Figure 1 explains into details the steps herein considered.

The main advantage, concerning the analysis of surface deformation, is represented by the possibility to exploit the potential of FE analysis for the simulation and joint consideration of complex features, such as topography, mechanical heterogeneities and time dependent processes. Disadvantages come form the fact that geophysical data to constrain complex features in active volcanic areas are still rare and the resolution is often poor. Moreover, calculation of analytical solutions is usually very fast, while the simulation of a single COMSOL model may require several hours of computation depending on the machine used, on the discretization assumed and on peculiar model complexities. For this reason, either great computational capability and/or a proper scaling of the model space may help to obtain a good balance between resolution and computational time.

In the next section, we present a case study where the optimization's procedures above detailed have been successfully applied to a real case of interpretation of surface deformation in an active volcanic area.

4. Tenerife, Canary Islands: A case study

Tenerife is the largest shield volcanic complex among the Canary Islands, a volcanic hotspot located off the West African coast. Most of Tenerife is built upon a basement of submarine extrusive rocks, which form the common sub-stratum of the island. The oldest rocks which outcrop in Tenerife have been dated to ~ 8.5 Ma. Recent volcanic activity has been confined to the northwestern quadrant of the island, and the last eruption occurred in 1909.

Recently, Fernández et al., 2009 analyzed 55 satellite images acquired in the area of Tenerife from 1992 to 2005 by the ERS sensors of the European Space Agency (ESA). The data were exploited via the SBAS-DInSAR algorithm (Berardino et al., 2002). This approach allows detecting Earth's surface displacements and to analyze their temporal evolution by generating mean deformation velocity maps and time series along the radar line-of-sight (LOS). The SBAS-DInSAR measurements have a spatial resolution of ~ 100 m with an accuracy of about 0.1 cm/yr for the deformation velocity and 1 cm for surface displacements (Casu et al., 2006). In the case of Tenerife, the SBAS-DInSAR analysis allowed detecting a surface subsidence in the order of 0.35 cm/yr along the satellite LOS.

In order to explain the observed displacements, Fernández et al., 2009 proposed a simplified analytical forward model (point source embedded in a homogeneous and viscoelastic half-space), leading to the interpretation of the deformation pattern as the gravitational sinking caused by the denser core of the island on the weaker lithosphere. However, while such model seems to well explain the observed LOS displacements and is substantially in agreement with the considered geodynamic scenario, the analytical solutions fail on the interpretation of the spatial variability of the deformation signal.

In the following, we will show that the consideration of lateral variations of densities and/or viscosities may help to better interpret the measured surface deformation.

4.1 Use of COMSOL

We applied the procedure explained in section 3 to the analysis of the surface deformation observed at Tenerife. To this end, we selected LOS velocities along a profile of the island with 500 m resolution, considered representative of the overall measured SBAS-DInSAR data. This represents our input data for the optimization.

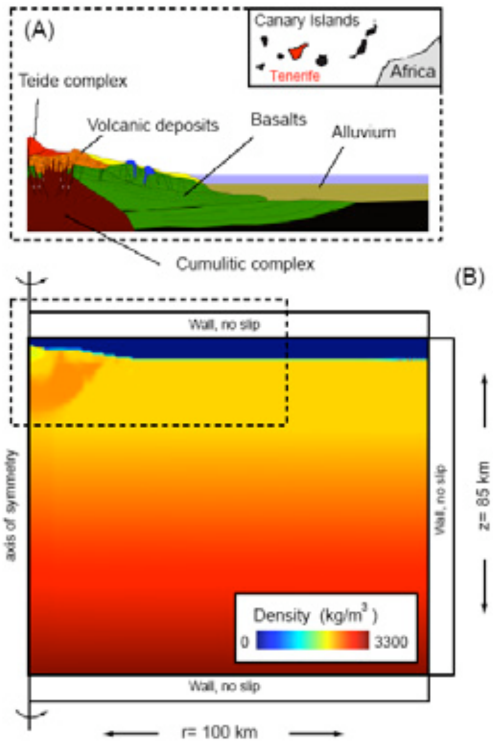

Figure 2. (A) Axisymmetric geological sketch of Tenerife, adapted from Fernández et al., 2009. (B) COMSOL model setup. The density distribution is extrapolated from the results of Gottsmann et al., 2008 and included as (r,z) function within the FE model.

Since the physical processes under study might be well represented in a fluid-dynamic context, the module of COMSOL Multiphysics to solve the incompressible Navier-Stokes set of differential equations represents an appropriate choice. For simplicity, we performed our analysis on an axisymmetric domain, 100 km wide in r direction and 85 km high in the z direction, and discretized the model space in about 6,000 triangular elements (Figure 2B). The accuracy of the discretization has been tested through resolution tests. Subdomain density *(rho)* is extrapolated from the gravity analysis of Gottsmann et al., 2008, and included within the COMSOL model as *(r,z)* function. Such function has 500 m resolution on the first 20 km of the subsurface, and is then linearly interpolated up 85 km considering average values of density for the lower Earth's mantle. The spatial distribution of density contrast also allowed the definition of an approximated topographic profile of the island. Body forces on the subdomain are due to lithostatic loading, thus:

$$\begin{aligned} F(r) &= 0 \\ F(z) &= -rho(r,z)\cdot g \end{aligned} \tag{1}$$

where g is the gravity acceleration (9.81 m/s). Boundary conditions are "wall, no slip" type on all model sides, excluding the symmetry axis. Since the area of interest is far enough from the bounds, this assumption did not affect the final results. Pressure reference level (equal to zero) was set on volcano topographic surface. The simulations were then performed considering the stationary case.

The viscosity distribution was evaluated through the iterations of the optimization algorithms, and included within the COMSOL model as *(r,z)* function. We allowed for two possible configurations: (1) variable crust thickness, vertical variation of viscosity for crust and mantle; (2) variable crust thickness, vertical and lateral variation of viscosities. The displacements are extrapolated from the topographic surface, projected along the LOS of the satellite and evaluated using the root mean square error (RMS) as cost function. The best-fit viscosity distribution is finally selected as the model with minimum cost among both, GA and SA optimizations.

4.2 Results

In Figure 3A we show the results of the optimization analysis. We note that the solutions for the case (1), considering both SA and GA optimization, shows an overall good fitting of the deformation pattern, however are not able to explain the lateral variability of the deformation signal (blue line in Figure 3A, viscosity distribution in Figure 3B). This is in agreement with the analytical solutions proposed by Fernández et al., 2009. Strikingly, the best-fit models for case (2), where also the lateral heterogeneities are included, show a more accurate representation of the observed deformation pattern, confirmed by a lower RMS value (red line in Figure 3A, viscosity distribution in Figure 3C).

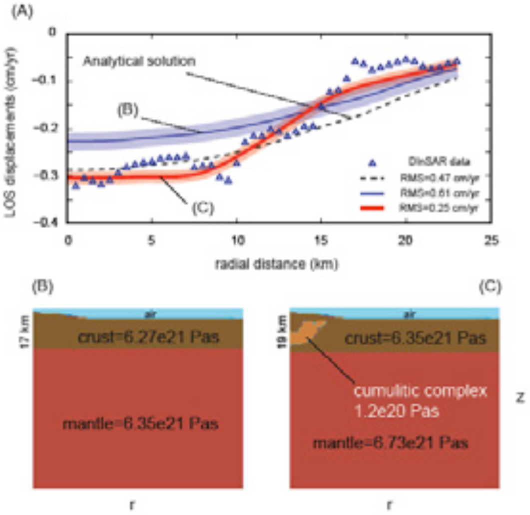

Figure 3. Results of the analysis of the surface displacements measured on Tenerife via SBAS-DInSAR. (A) Measured data (blue triangles) vs. solutions of the optimization algorithms for different viscosity distributions. Shaded area represents solutions with low RMS. Solid lines (bleu for model B, red for model C) represent the best model among SA and GA optimizations.

5. Conclusions

We presented an implementation of COMSOL models within SA and GA optimization procedures through MATLAB® subroutines, in order to interpret surface deformation in active volcanic areas. The results of an application of this approach to the analysis of the deformation pattern revealed on Tenerife, Canary Islands, demonstrate that the use of more complex models, which can take into account in this particular case also for the lateral variability of the material heterogeneities, may help to better interpret the surface deformation signal. This approach, which is straightforwardly extendable for FE models that consider of 3D geometries, as well as time and temperature dependent phenomena, is particularly suitable for a more accurate representation of physical processes occurring in active volcanic areas, as well as an appropriate interpretation of the measured deformation signals. Moreover, the same optimization approach might be applied also starting from a range of different input data (e.g. temperature measurements, aquifer water level changes, etc.) and used in combination with the Multiphysics capabilities of COMSOL in different geophysical studies for the analysis and interpretation of peculiar characteristics of the subsurface.

6. References

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7. Acknowledgements

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