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Parameter Identification in Partial Integro-Differential Equations for Physiologically Structured Populations

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Temporal mismatch

Population dynamics of G. pulex

Identifiability of parameters

Future Applications

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Climate change

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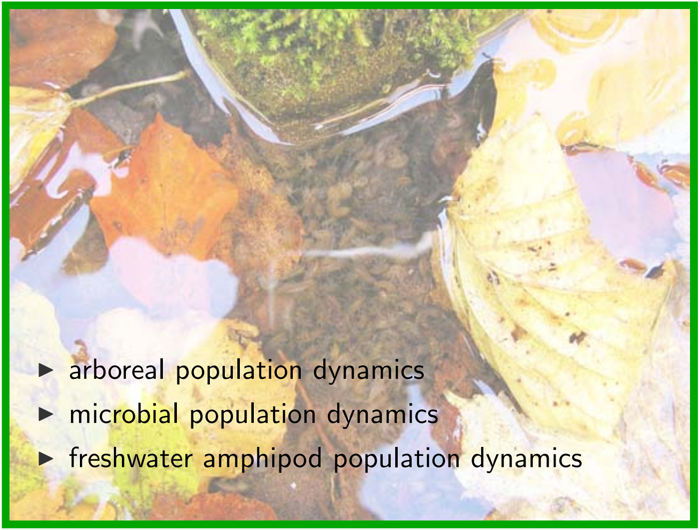

- arboreal population dynamics
- microbial population dynamics
- freshwater amphipod population dynamics

Interdependency in ecology

- birth
- growth
- reproduction
- death

Interdependency in ecology

- birth
- growth
  - food availability
  - temperature conditions
  - ...
- reproduction
- death

Interdependency in ecology

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Interdependency in ecology

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- reproduction
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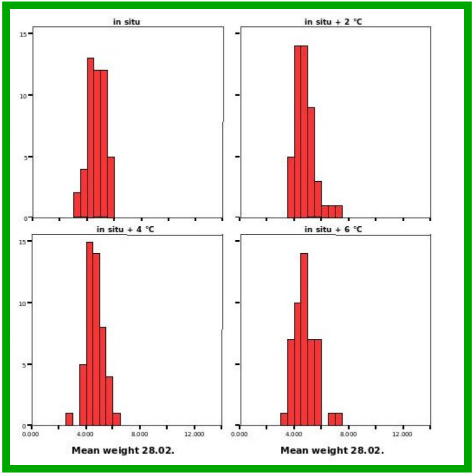
  - ...
- death
  - food availability
  - temperature conditions
  - age
  - ...

Measuring interdependency in ecology

- in-situ, highly dynamic
- in lab, highly specific

Measuring I



Suhling et al., 2008

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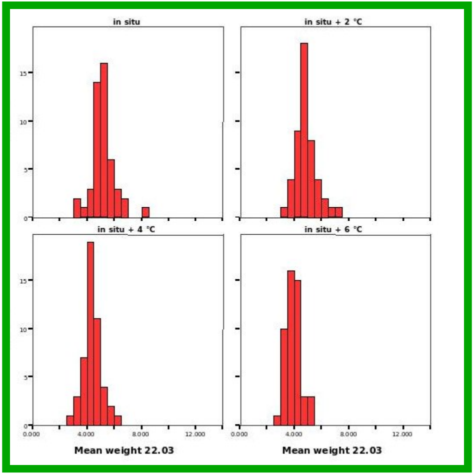

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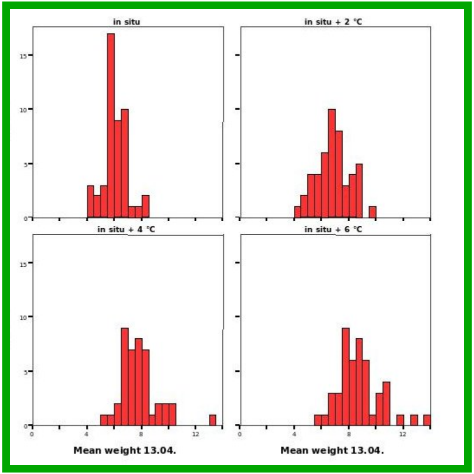

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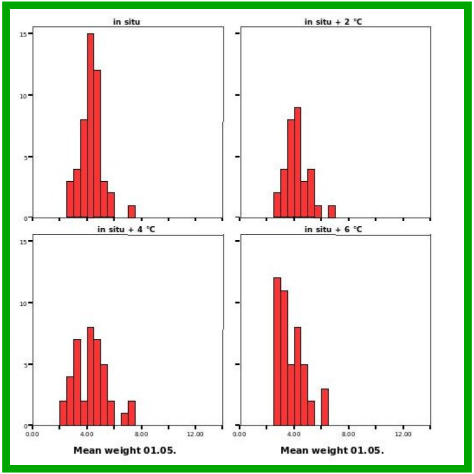

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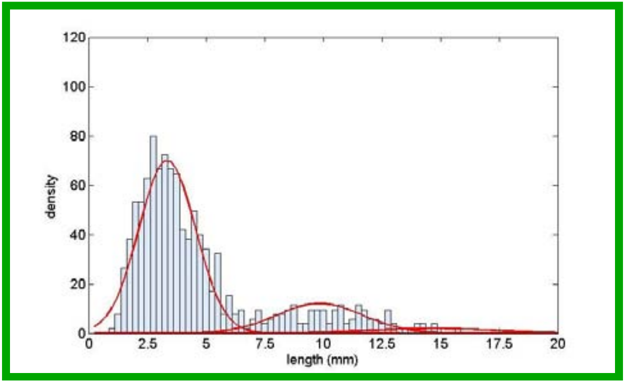
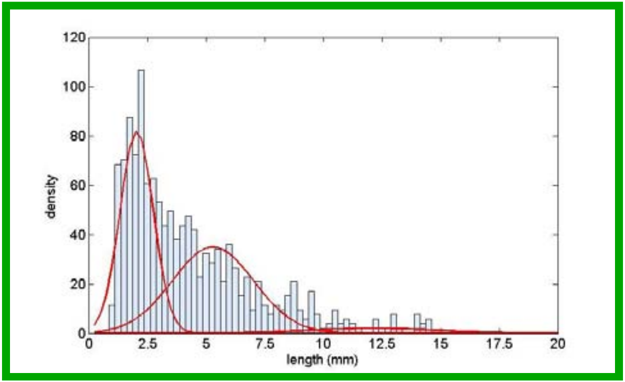
Measuring II



Schneider et al., 2008

Measuring II



Schneider et al., 2008

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Individual growth $g(F, T, w)$

$$\frac{dw}{dt} = g(F, T, w)$$

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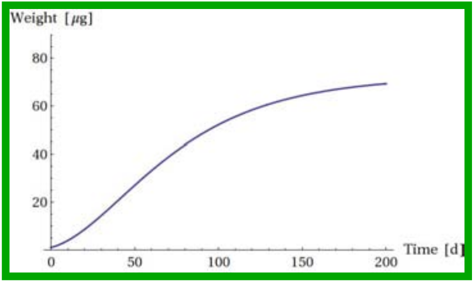
$$\frac{dw}{dt} = \gamma w^{\frac{2}{3}} - \rho w$$



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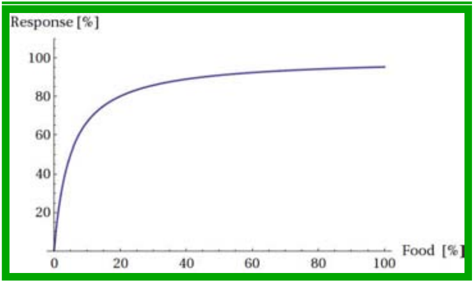

$$\frac{dw}{dt} = \gamma w^{\frac{2}{3}} - \rho w$$

$$\frac{dw}{dt} = \gamma \frac{F}{F+F_{\mathrm{h}}} w^{\frac{2}{3}} - \rho w$$

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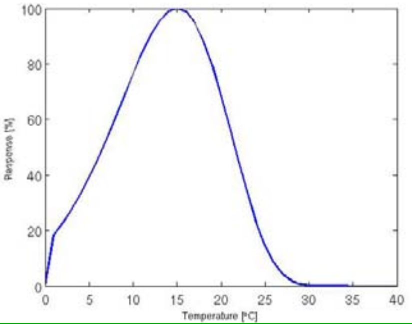

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$$\frac{dw}{dt} = \gamma \frac{F}{F+F_{\mathrm{h}}} w^{\frac{2}{3}} - \rho w$$

$$\frac{dw}{dt} = \Phi(T)(\gamma \frac{F}{F+F_{\mathrm{h}}} w^{\frac{2}{3}} - \rho w)$$

Weight structured population $n(w, t)$

Population density n varying with time t and weight w:

$$\frac{\partial n(w,t)}{\partial t} = -\frac{\partial g(F,T,w)n(w,t)}{\partial w}$$

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$$\frac{\partial n(w,t)}{\partial t} = -\frac{\partial g(F,T,w)n(w,t)}{\partial w} + \lambda \frac{\partial^2 n(w,t)}{\partial w^2} - p_{-}(F,T,w,\ldots)$$

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Population density n varying with time t and weight w:

$$\begin{aligned} \frac{\partial n(w,t)}{\partial t} &= -\frac{\partial g(F,T,w)n(w,t)}{\partial w} + \lambda \frac{\partial^2 n(w,t)}{\partial w^2} \\ &\quad - p_-(F, T, w, \dots) \\ &\quad + p_+(F, T, w, \dots) \end{aligned}$$

Mortality and Fertility

Mortality

$$p_-(F, T, w, \ldots) = \mu(F, T, w, \ldots)n(w, t)$$

Fertility

$$p_+(F, T, w, \ldots) = B(F, T, w, \ldots)\Pi(w)$$

Mortality and Fertility

Mortality

$$
\begin{aligned}
p_{-}(F, T, w, \ldots) &= \mu(F, T, w, \ldots) n(w, t) \\
1 - \mu(F, T, w, \ldots) &= (1 - \mu_0)
\end{aligned}
$$

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p_{+}(F, T, w, \ldots) = B(F, T, w, \ldots) \Pi(w)
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Mortality and Fertility

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$$p_{-}(F, T, w, \ldots) = \mu(F, T, w, \ldots)n(w, t)$$
$$1 - \mu(F, T, w, \ldots) = (1 - \mu_0)(1 - \mu_T(1 - \Phi(T)))$$

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Mortality and Fertility

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Parameter identification

- Minimization of

$$L(\theta) = \sum_j \sum_i (n_i(t_j) - \hat{n}_i(t_j, \theta))^2$$

- through
  - direct simulation with *Comsol* in *Matlab*
  - optimization toolbox of *Matlab*

- with model efficiency *ME*

$$ME = 1 - \frac{\sum_j \sum_i (n_i(t_j) - \hat{n}_i(t_j))^2}{\sum_j \sum_i (n_i(t_j) - \bar{n}(t_j))^2}$$

Process analysis

- Statistical method for comparison of nested models: Likelihood ratio test

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- The test statistic is $-2\log(\frac{L_{\text{complex}}(\theta)}{L_{\text{simple}}(\theta)})^{\frac{n}{2}}$.
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Process analysis

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- The test statistic is $-2\log(\frac{L_{\text{complex}}(\theta)}{L_{\text{simple}}(\theta)})^{\frac{n}{2}}$.
- It is χ^2 distributed with the difference in numbers of parameters as degrees of freedom.
- A significance level of 0.05 is applied.

Accounting for continuity

- Experiments on temperature response are based on homogeneity assumption over intervals.

O'Neill temperature response function:

$$\Phi(T) = k\left(\frac{T_{\max}-T}{T_{\max}-T_{\mathrm{opt}}}\right)^p e^{p\frac{T-T_{\mathrm{opt}}}{T_{\max}-T_{\mathrm{opt}}}}$$

with

$$p = \frac{1}{400}W^2\left(1+\sqrt{1+\frac{40}{W}}\right)^2$$

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- Our idea: dynamic parameter identification under controlled temperature regimes.

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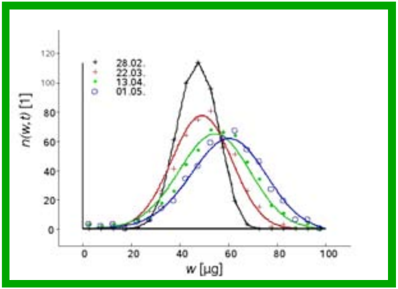
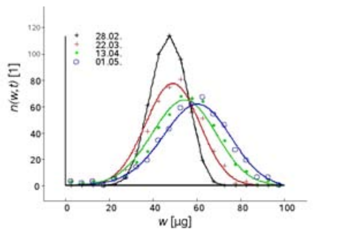


Accounting for continuity

	γ	ρ	T_{opt}	q_{10}	
A	0.32	0.06	18.3	2.58	97.9
B	0.30	0.07	17.7	2.04	98.5
C	0.23	0.05	17.6	2.11	99.1
D	0.15	0.03	17.7	1.75	99.2
	0.16	0.04	17.5	1.7	



Accounting for unmanageability

- Experiments on food response would require food control.

Litter dynamics

$$\frac{dF}{dt} = L(t, T) - I_{\text{max}}\Phi(T)\frac{F}{F + F_{\text{h}}}\int_0^{w_{\text{max}}} w^{\frac{2}{3}} n(w, t)dw$$

Possible approximation

$$F(t) = F_0 e^{-(t/t_1)^{p_1}}(1 - e^{-(t/t_2)^{p_2}})$$

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- Food control is unjustifiably expensive.

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- Our idea: dynamic parameter identification including food dynamic.

Litter dynamics

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Possible approximation

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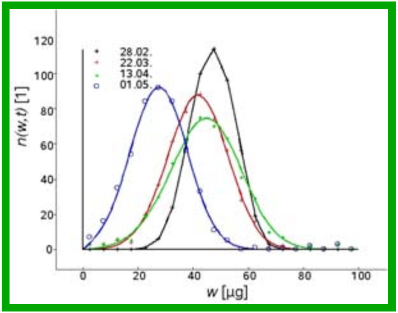
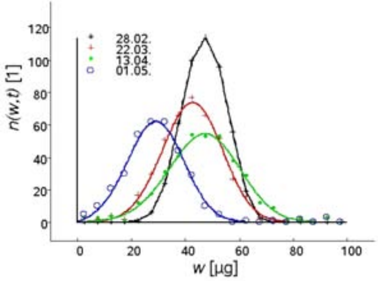
Accounting for unmanageability

F_h	p_1	p_2	t_1	t_2
4.27	18.4	25.1	84.0	97.9
4.7	20	25.0	82	98



Identifying processes

- Mortality depends on multiple processes.
- Effect of individual process often unknown.
- Adequate simplifications can be tested.



Identifying processes

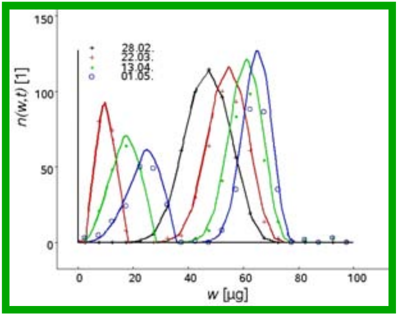
μ_0	μ_T	μ_F	RMSE	ME (%)
•			268	99.0
	•		364	98.7
		•	285	99.0
•	•		263	99.0
•		•	264	99.0
	•	•	266	99.0
•	•	•	260	99.0
0.002	0.002	0.002		

Identifying processes

μ_0	μ_T	μ_F	RMSE	Λ
0.004			268	•
	0.01		364	
		0.008	285	○
0.003	0.004		263	•
0.003		0.002	264	•
	0.005	0.005	266	•
0.002	0.003	0.004	260	
0.002	0.002	0.002		

Completing the circle

μ_0	r_{max}	ME (%)
0.006	0.007	94.1
0.01	0.008	



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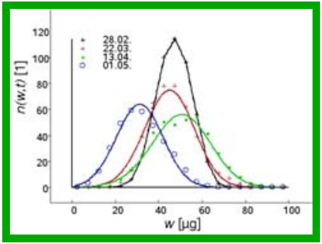
What we learned



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