Modeling Magnetic Configurations for Improved Separations of Magnetic and Non-Magnetic Materials

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Abstract: Magnetic separation of magnetic liquid phases/materials from non-magnetic liquid phases/materials are needed in biomedical and microfluidic applications, as well as for cleaning up oil spills.

Magnetic fluids (also called ferrofluids), in a magnetic field, experience a magnetic force density. COMSOL Multiphysics was used to show that magnetic energy density of a particular permanent magnet configuration is a more intuitive way of understanding how a magnetic fluid moves in a magnetic field. As a result, a novel magnetic separation technique was developed, using permanent magnet edges and Halbach magnet arrays, that separated a variable magnetic volume fraction from a mixture of magnetic and non-magnetic liquid phases.

Keywords: magnetic separations, Halbach arrays, ferrofluid, oil spills, Deepwater Horizon, magnetic fluid

1. Introduction

Oil spills have devastating energy and environmental consequences as evident by the 2010 Deepwater Horizon disaster in the Gulf of Mexico. Current oil spill technologies use boom and oleophilic surface skimmers to contain and remove the oil floating on the sea water. The oil recovery efficiency of such technology can be as low as 50% depending on factors such as the type and thickness of oil, presence of waves, etc [1].

The basic premise of this work was to develop an improved oil water separation system for oil spills using magnetic forces. It involved magnetizing either the oil or water phase of the oil spill (using magnetic nanoparticles or micron sized particles), magnetically separating the two phases, recovering the magnetic component of the magnetic phase for reuse and repeating the process to obtain sufficient separation to discard the water phase while storing the oil phase for transport to a refinery facility. The entire process had to be continuous, utilize minimal energy and not be harmful to the marine environment [2].

It became quickly obvious that the main difficulty would be the magnetic separation of the magnetic and non-magnetic phases. In particular, it was difficult to make a permanent magnet system to magnetically separate a variable magnetic volume fraction from a liquid mixture.

With the help of COMSOL Multiphysics, several magnet configurations were investigated first by thinking in terms of the magnetic energy density experienced by a magnetic fluid in a magnetic field. This proved to be a more effective way of designing permanent magnet configurations to be used for magnetic separation.

It resulted in building a simple test unit to demonstrate magnetic separation of oil-based ferrofluid (colloidal suspension of oleophilic magnetic nanoparticles of ≈10 nm in diameter) from water. The resulting magnetic separation technique could be applied to other applications such as biomedical and microfluidic applications.

2. Governing Equations

The magnetic force density **F** [N/m^3], given in Eq. (1)

$$\mathbf{F} = \mu_0 \left(\mathbf{M} \bullet \nabla\right) \mathbf{H} \qquad (1)$$

where $\mu_0 = 4\pi \times 10^{-7}$ [H/m] is the magnetic permeability of free space, **M** [A/m] is the material's magnetization and **H** [A/m] is the applied magnetic field. The magnetic force density depends on the strength of the magnetization **M** as well as, the spatial gradient of the applied magnetic field **H**.

When it comes to designing a magnetic force density necessary for magnetic separation, it is difficult to think in terms of spatial gradients. A better way is to think in terms of magnetic energy density and relate it to magnetic force density.

Magnetic force density, exerted by an external magnetic field on the ferrofluid, is related to magnetic energy density given in Eq. (2) [3].

$$W_m = -\int_0^{\xi} \mathbf{F_m} \bullet \mathbf{d\xi} \tag{2}$$

where magnetic energy density W_m [J/m^3] is given in Eq. (3) [3].

$$W_m = \int_0^{B} \mathbf{H} \bullet \mathbf{dB} \tag{3}$$

In chemistry, an electron prefers the lowest energy level and is "attracted" to that state (energy minimization). Work has to be done to lift the electron out of that "energy potential."

The same can be applied to ferrofluids. An attractive force that a ferrofluid experiences represents a negative potential energy well (hence the negative sign in Eq. (2)), that the fluid wants to be in, and positive work has to be done to overcome the force of magnetic attraction and remove the ferrofluid from this energy potential well.

3. Thinking In Magnetic Energy Density

3.1. COMSOL Modules Used

COMSOL Multiphysics 4.2 was used to simulate different permanent magnet configurations. The AC/DC module was used to simulate the applied magnetic field by the permanent magnets. The AC/DC module was sufficient in many cases to derive the magnetic force densities experienced by the ferrofluid. To do this, Eq. (1) was resolved in Cartesian directions and were used as variables in COMSOL. In post-processing, it was easy to display arrow surface plots corresponding to these resolved magnetic force density variables.

Laminar Two Phase flow in the CFD module was also used for some simulations to simulate the magnetic fluid flow under the influence of the permanent magnets' magnetic fields. The resolved magnetic force densities of Eq. (1) were then used as Volume Forces in the Laminar Two Phase flow module to simulate the ferrofluid.

The following case studies demonstrate using the magnetic energy density method to describe the magnetic forces acting on a ferrofluid.

3.2. Case Study 1: Two magnets (attraction configuration)

Two magnets are placed in a configuration to attract each other across a channel that would hypothetically contain ferrofluid as illustrated in Figure 1.

Figure 2 is a surface plot distribution of the magnetic energy density, given in Eq. (3), with height of the 3D image corresponding to energy density magnitude. The ferrofluid from the opposite ends of the channel would get attracted to the saddle shaped potential well, in the center of the channel, corresponding to the uniform field region between the two magnets. Although the magnetic fluid would get attracted to this saddle shaped potential well, it would more likely be pulled to the sharp energy troughs corresponding to the edges of the two cubic magnets.

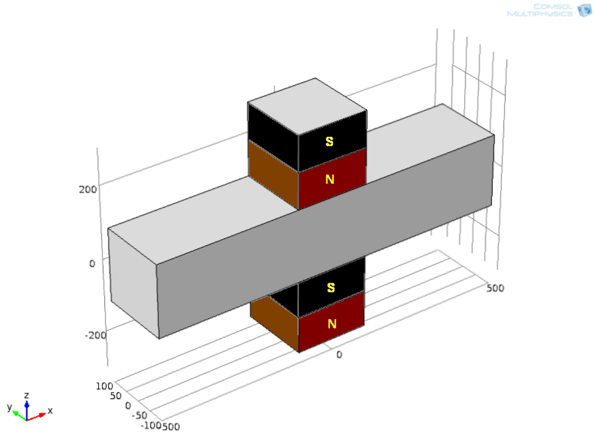


Figure 1. Two cubic magnets placed on top of a channel. The magnets are placed such that they would attract each other. Black represents south pole and red represents north pole.

Figure 3 is the 2D version of the 3D magnetic energy density surface plot of Figure 2, but also includes magnetic field streamlines and arrows representing magnetic force density vectors. The magnetic force density vectors follow the spatial gradient of the magnetic energy density as given in Eq. (2). When comparing Figure 2 & Figure 3 it becomes intuitively easier to think in terms of a ferrofluid approaching a potential well (Figure 2) than to think in terms of spatial gradients that generate magnetic forces. This approach makes it easier to design magnetic configurations to attract magnetic fluids.

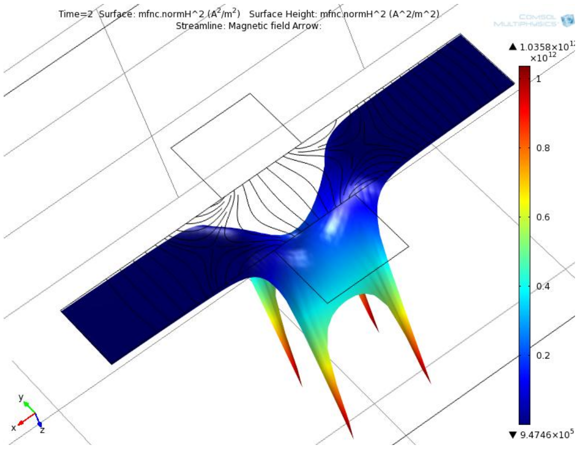


Figure 2. A surface plot of the magnetic energy density distribution, with energy density magnitude corresponding to the height of the 3D image, for two magnets in the attractive configuration of Figure 1. Magnetic field streamlines (in black) are also plotted.

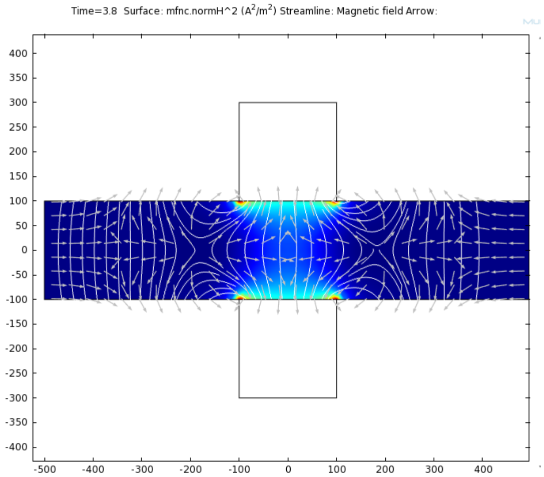


Figure 3. 2D Colored surface plot representing the magnetic energy density experienced by ferrofluid of Figure 2. The streamline distribution represents the magnetic field distribution while the arrows represent the spatial magnetic force density distribution (arrow size is normalized and not proportional to magnitude). The magnetic force density distribution can be interpreted as the spatial derivative of the magnetic energy density plot in Figure 2.

3.3. Case Study 2: Two magnets (repulsion configuration)

When two magnets are positioned in a repulsive configuration as shown in Figure 4, the resulting magnetic energy distribution is illustrated in Figure 5. When thinking energetically, a ferrofluid from far away is pulled into the region with the two magnets but extra energy is necessary for the ferrofluid to "climb" the potential hill in the center of the channel (the field should cancel out in the center) and settle there. The ferrofluid is more attracted to the energy troughs at the edges of the magnets and would settle near those regions rather than in the center.

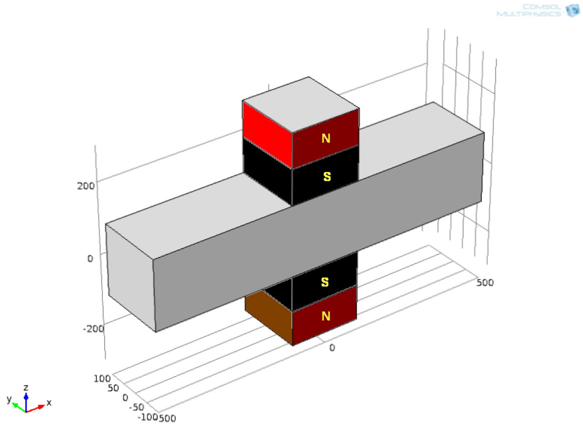


Figure 4. Two cubic magnets placed on top of a channel that hypothetically contains ferrofluid. The magnets are placed such that they would repel each other. Black represents south pole and red represents north pole.

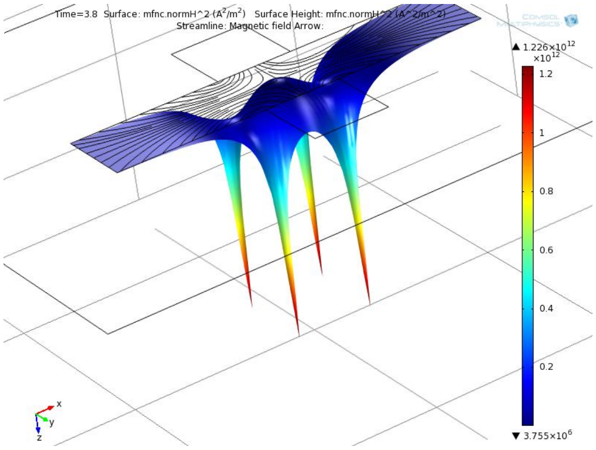


Figure 5. A surface plot of the magnetic energy density distribution, with energy density magnitude corresponding to the surface height of the 3D image, for the two magnets in a repulsive configuration of Figure 4. Magnetic field streamlines (black) are also plotted.

Figure 6 illustrates that the vector magnetic force density distribution (arrows) closely matches the spatial derivative of the magnetic energy distribution of Figure 5. The magnetic fluid is attracted away from the central region, where the repulsive magnetic field cancel each other, to the edges of the magnet again. The magnetic energy density diagram of Figure 5 illustrates the magnetic fluid would favor settling at the sharp potential troughs and is a more intuitive diagram than that of Figure 6.

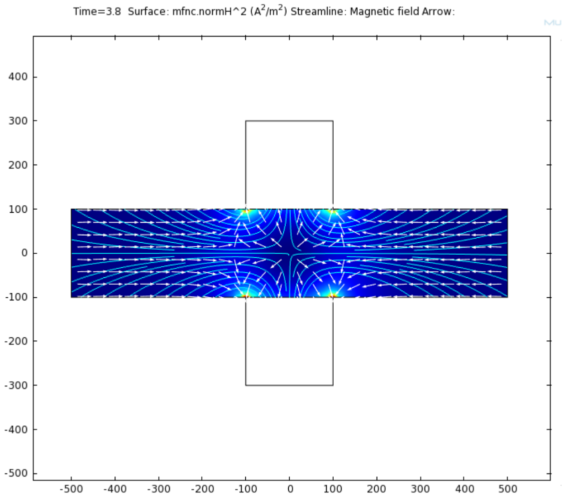


Figure 6. 2D colored surface plot representing the magnetic energy density experienced by ferrofluid of Figure 5. Magnetic field distribution (streamlines) and magnetic force density vector distribution (arrows) are also represented. The magnetic fluid experiences a force towards the edges of the two magnets but also does not prefer to stay in the center region between the two magnets.

3.4. Case Study 3: Investigating Flows with no Magnet Edges - Two Magnets with Curved Edges

Unlike the previous case studies, this case study investigates the magnetic forces without the strong non-uniform field of the permanent magnet edges.

Two magnets are positioned to attract each other on either side of a channel and placed at an angle of 10 degrees with respect to the x axis. Ferrofluid is positioned at the inlet (right side of the channel). Figure 7 illustrates the position of the magnets, streamlines of the magnetic field distribution and also shows the starting volume fraction (red=100%) of the magnetic phase. The edges of the magnets are rounded out, to remove any sharp edges, that would create a very strong spatial gradient.

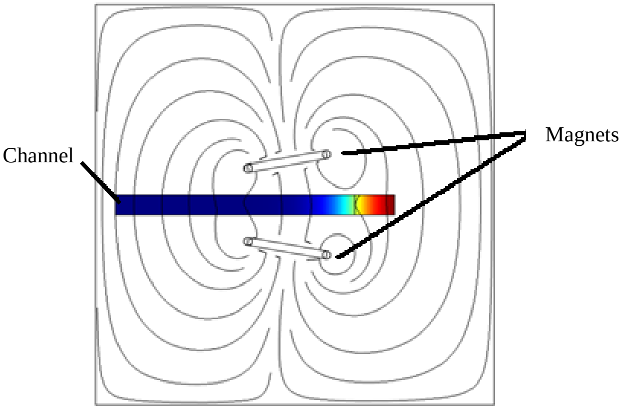
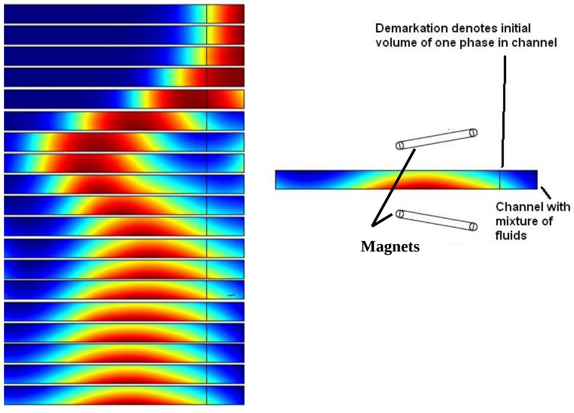


Figure 7. Two magnets placed above and below (attractive configuration) a channel containing a mixture of magnetic and non-magnetic phases. The magnetic field streamlines are illustrated as well as the colored surface plot representing the magnetic phase as a volume fraction. Red representing a 100% volume fraction of magnetic phase with blue representing 0%.

Figure 8 is a 5 second time lapse simulation illustrating the movement of the magnetic phase in the channel driven by the magnetic force density generated due to the two magnets. It can be seen that the magnetic phase accelerates and overshoots the position of the magnet but then gets pulled back and eventually settles in the center of the two magnets.

This time lapse plot can be easily understood when looking at the energy density plot (Figure 9) for the magnet configuration of Figure 7. The magnetic fluid experiences a potential well near the inlet of the channel due to the magnets and accelerates down the channel until it reaches the position on the channel corresponding to the bottom of the potential well. The momentum of the accelerating ferrofluid helps push itself up the left side of the channel, effectively doing work to climb out of the potential well. The outlet (left) side of the energy well is steeper than the inlet side (right) and the ferrofluid experiences a drastic "pull back" and eventually settles at the position in the channel corresponding to the bottom of the potential well. Ideally without friction (viscosity), the ferrofluid would be able to climb out of the potential well and reach the left side of the channel.



(a) (b)

Figure 8. Time lapse simulation of magnetic phase flow due to force generated by the two magnets. a) An initial magnetic volume fraction of 100% (red) is shown at the right most of the channel (demarked by the small rectangle that makes up a small portion of the entire channel). The magnetic phase (100% magnetic volume fraction=red, 0%=blue) as a function of time starting from t=0 can be seen to be attracted to the magnet. Each frame represents 5 seconds of the simulation. b) The final state of the magnetic volume fraction after 90s and gives perspective of the position of the magnets with respect to the magnetic phase.

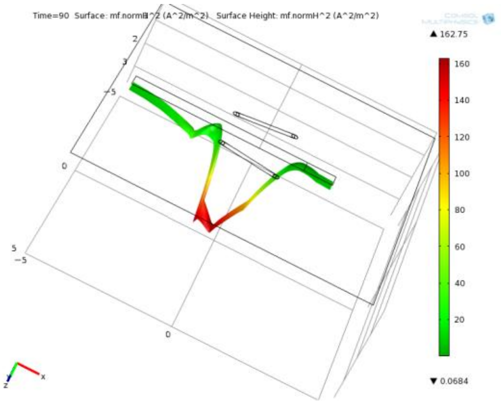


Figure 9. Magnetic energy density plot for the magnet configuration of Figure 7. A potential well in the channel can be seen explaining the "pull back" experienced by the magnetic fluid. The magnetic fluid experiences a pulling force into the well and a "pull back" towards the left of the channel because of the steeper energy slope making it difficult for the fluid to escape the potential well.

4. Magnetic Field Requirements for Magnetic Separation

Some conclusions can be made from the three case studies of Section 3.

1) A magnetic fluid in no magnetic field is at a higher energy state than a magnetic fluid in the presence of a magnetic field. Extra work has to be done on a magnetic fluid to pull it out of a magnetic field (through a pressure gradient etc).

2) Increasing the size of the potential well, increases the likelihood and velocity of the magnetic fluid being pulled by the magnetic field.

For magnetic separation of an unknown variable volume fraction (ϕ) of magnetic phase from non-magnetic phase, it is necessary to separate the magnetic phase from the channel flow region. If the magnetic phase is separated in the same channel flow region as the non-magnetic phase it would result in contamination of either collected phase especially if there is only one phase either the magnetic phase (ϕ=100%) or no magnetic phase (ϕ=0%).

Using the magnetic energy density method approach of designing magnetic fields, potential wells can be designed to divert magnetic phase from the channel and a bigger potential well could be used to collect the diverted magnetic phase. From the case studies of Section 3, the large potential wells are created at magnet edges and Halbach arrays.

5. Designing a Magnetic Separator

From Section 3, it was evident that permanent magnet edges create sharp energy troughs and magnetic fluids are drawn to those regions. As a result, magnet edges can be used to divert the magnetic phase from the channel containing both magnetic and non-magnetic phases. The magnetic energy density surface plot of an axially magnetized cylindrical magnet is shown in Figure 10. It can quickly be determined that magnetic fluid will be attracted away from the central region of the magnet towards the cylindrical edges corresponding to the sharp energy troughs. As a result, cylindrical magnets could be used to divert the magnetic phase from the flow region that consists of both magnetic and non-magnetic phases.

The step after separating the magnetic phase from the non-magnetic phase in the flow channel is to collect the magnetic phase by designing a large potential well that will attract and confine the magnetic fluid to that region. A Halbach array creates this large one-sided potential well.

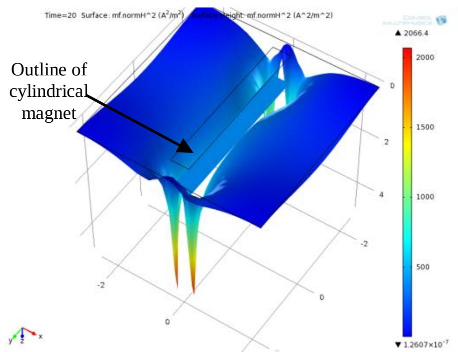


Figure 10. Magnetic energy density surface plot near a cylindrical permanent magnet magnetized in the axial direction.

A Halbach array is a special arrangements of permanent magnets that result in a one-sided magnetic flux. It is conceptually explained in Figure 11.

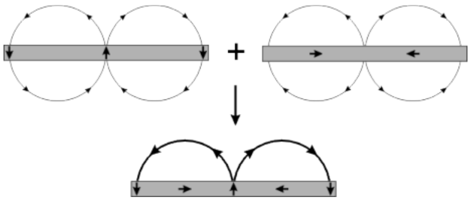


Figure 11. Halbach array of magnets resulting in one sided magnetic flux. Image taken from [4]. The arrow represents the orientation of the magnetic north and south pole with the arrow head denoting the magnetic north pole. The Halbach array is constructed using three vertically oriented magnets (top left) and two horizontally oriented magnets (top right) such that the magnetic field add on top of the magnet array and cancel below it.

A COMSOL simulation of a Halbach array made of 13 cubic magnets was performed and the magnetic field distribution plotted in Figure 12. Figure 13(a) illustrates the potential well on the top of the Halbach array (strongest magnetic field region) while Figure 13(b) shows the magnetic energy density below the Halbach array (weakest field region). The potential well on top of the Halbach array is significantly deeper than the potential well on the bottom of the array (the potential well at the edges are due to incomplete termination of the Halbach array).

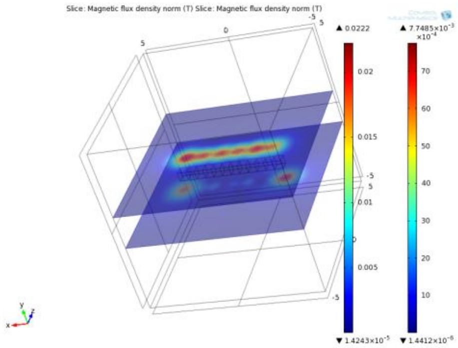


Figure 12. Linear Halbach array simulated in COMSOL 4.2. Two 2D slices above and below the magnet array (cubic magnets sandwiched between the two slices) show the magnetic field intensity being very strong on top of the array and negligible under the bottom of the array except at the edges due to incomplete cancellation at the ends of the linear Halbach array.

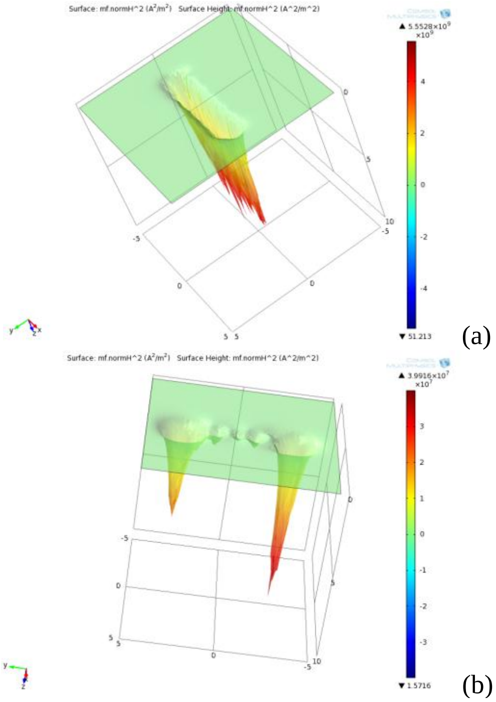


Figure 13. Energy density plot taken (a) at top slice of the Halbach array (strongest field region) from Figure 12 showing deep potential well experienced by the ferrofluid where it will collect. (b) Bottom slice of the Halbach array with negligible potential wells in the middle region of the Halbach array and relatively small magnitude potential wells at the ends of the array due to incomplete magnetic field cancellation there. The ferrofluid would not collect at the bottom of the Halbach array and would climb to the top surface with the strongest magnetic field strength.

6. Experimental Results

By thinking in terms of magnetic energy density the individual parts of the magnetic separator were identified and simulated using COMSOL in the previous section. A magnetic separator, to separate the magnetic from the non-magnetic liquid phase, was built consisting of cylindrical magnets and a Halbach array as seen in Figure 14 [5]. The magnetic separation process is demonstrated in Figure 15.

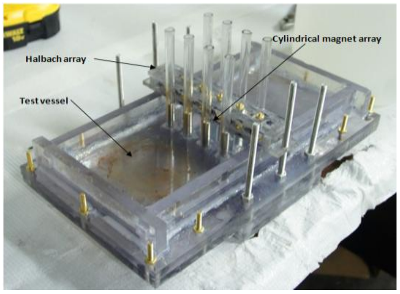
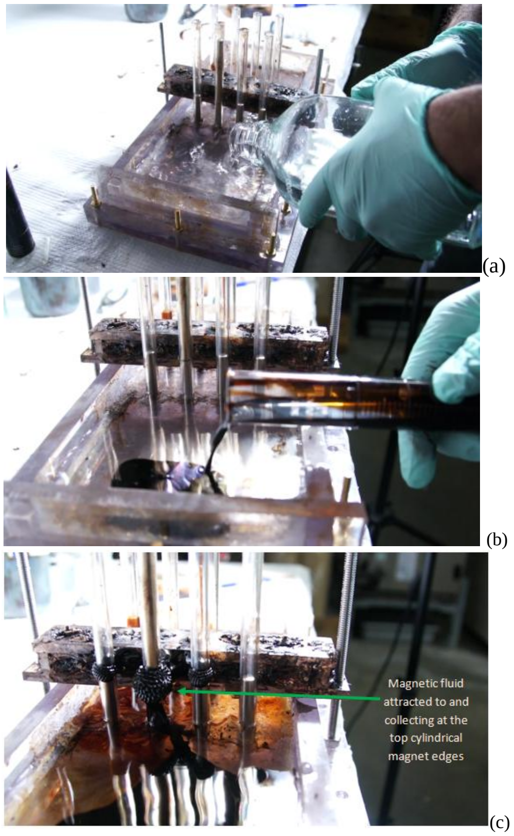


Figure 14. Constructed magnetic separator unit consisting of an array of cylindrical magnets to attract the magnetic phase from the test vessel and then collect on top of the Halbach array (strongest field region).



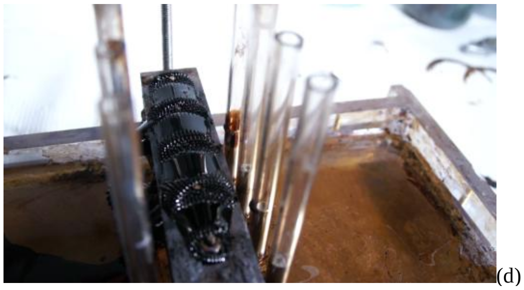


Figure 15. (a) Water is first added to the test vessel. (b) Oil based magnetic fluid (EFH1 ferrofluid) poured into test vessel and (c) is attracted to and collects at the top edge of the cylindrical magnets. (d) Magnetic oil phase jumps over to the Halbach array and starts to collect on the surface with the strongest magnetic field (the top surface). The top surface can be seen here with collected magnetic fluid.

7. Conclusions

COMSOL Multiphysics was used to demonstrate a novel energetic way of designing magnetic fields and forces to be used in magnetic separation of magnetic and non-magnetic liquid phases. Using this technique individual parts of the magnetic separation concept were tested before building the complete unit which worked exactly as predicted.

8. References

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